A set of Poulet numbers and generalizations of the twin primes and de Polignac's conjectures inspired by this

Marius Coman
Bucuresti, Romania
email: mariuscoman13@gmail.com

Abstract. In this paper I show a set of Poulet numbers, each one of them having the same interesting relation between its prime factors, and I make four conjectures, one about the infinity of this set, one about the infinity of a certain type of duplets respectively triplets respectively quadruplets and so on of primes and finally two generalizations, of the twin primes conjecture respectively of de Polignac's conjecture.

Conjecture 1:

There exist an infinity of Poulet numbers of the form n^2 + 120*n, where n is prime or a composite positive integer.

Note:

In the first case, obviously n is a prime factor of such a Poulet number and the product of the other prime factors is equal to n + 120; for instance, the number 1729 is a part of this set of Poulet numbers because 1729 = 7*13*19 can be written as 13^2 + 13*120 and implicitly 7*19 = 13 + 120. First few such Poulet numbers are:

: 1729 = 7*13*19 = 13^2 + 13*120;
: 4681 = 31*151 = 31^2 + 31*120;
: 6601 = 7*23*41 = 41^2 + 41*120.

Note:

In the second case, obviously n is a product of few prime factors of such a Poulet number and the product of the other prime factors is equal to n + 120. Such a Poulet number is 75361 = 11*13*17*31 = 221^2 + 221*120 and implicitly 11*31 = 13*17 + 120.

Comment:

Interesting (outside the topic of this paper) that there are other pairs of twin primes of the form [m = 10*k + 1, n = 10*k + 3] for which exist primes [p, q] such that m*p = n*q + 120; for instance, 41*227 = 43*241 + 120.

Conjecture 2:

There exist an infinity of duplets of primes [p, q] such that p – q = 120; there also exist an infinity of triplets of primes [p1, p2, q] such that p1*p2 – q = 120; there also exist an infinity of quadruplets of primes [p1, p2, p3, q] such that p1*p2*p3 – q = 120; generally, for any non-null positive integer i there exist i primes p1, p2, ..., pi and a prime q such that p1*p2*...*pi – q = 120.

Examples:

: 151 – 31 = 120;
: 7*19 – 13 = 120;
: 7*17*37 – 4283 = 120.

Conjecture 3:
(generalization of the twin primes conjecture)

For any positive integer i there exist i primes p1, p2, ..., pi and a prime q such that p1*p2*...*pi – q = 2.

Conjecture 4:
(generalization of de Polignac's conjecture)

For any n even positive integer and for any i non-null positive integer there exist i primes p1, p2, ..., pi and a prime q such that p1*p2*...*pi – q = n.