Area Moments Defined by Example Subdivision Curves

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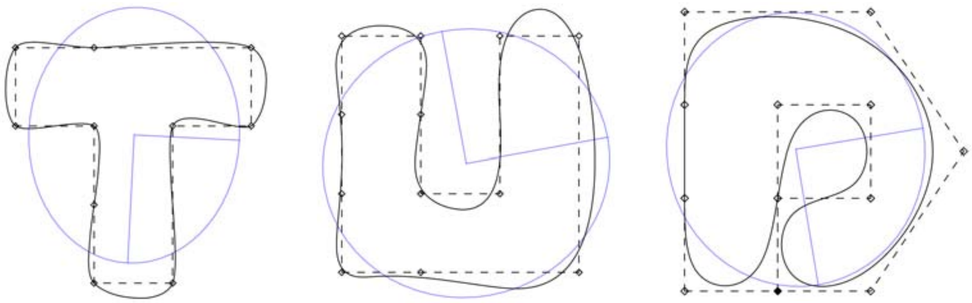

Figure: Three subdivision curves as black, continuous lines. The sequence of control points are the diamonds connected by dashed lines. The blue circumference marks the ellipsoid at the centroid of the area enclosed by the subdivision curve that has equivalent inertia as the area. The principal axes of the ellipsoid are also shown. ■

Abstract

We list examples of subdivision curves together with their exact area, centroid, and inertia. We assume homogeneous mass-distribution within the space bounded by the curve, therefore the term 'area moments' is used. The subdivision curves that we consider are generated by 1) the low order B-spline schemes, 2) the generalized, interpolatory C^1 four-point scheme, as well as 3) the more recent, dual C^2 four-point scheme.

The derivation of the $(d+1)$-linear form that computes the area moment of degree $p+q=d$ based on the initial control points for a given subdivision scheme is deferred to a publication in the near future.

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Introduction

Subdivision of curves is an iterative refinement procedure for polygons. Over the course of the iteration, the increasingly dense point cycle typically converges to a piecewise smooth curve.

Our article is restricted to subdivision of polygons with a finite number of control points $(\mathrm{px}_k, \mathrm{py}_k) \in \mathbb{R}^2$ for $k = 0, 1, 2, \ldots, n-1$ in the 2-dimensional plane. If the resulting subdivision curve is compact, and not self-intersecting, we denote with $\Omega \subset \mathbb{R}^2$ the set in the interior of the curve. Then, the area moments of degree $p+q=d$ of the set Ω with respect to the x- and y-axis are well defined by the following integral

$$M(p, q) := \int_\Omega x^p y^q \, dx \, dy$$

In a future publication we will show that the integral $M(p, q)$ can be substituted by a $(d+1)$-linear form via the divergence theorem. The input to the multi-linear forms are the coordinates of the polygon $(\mathrm{px}_k, \mathrm{py}_k)$. The coefficients of the multi-linear forms depend only on the subdivision rules, and subsequently apply universally to any choice of control points. The derivation of the multi-linear forms does not require the basis functions.

In [Hakenberg et al. 2014], the derivation of the trilinear forms that compute the volume enclosed by subdivision surfaces (=moment of degree 0) has been presented. That article briefly mentions moments of higher degrees of the 3-dimensional sets. However, the authors conclude that establishing the forms is not tractible by today's computational means due to the large number of unknown coefficients. Therefore, for moments of higher degree we focus on the simpler, 2-dimensional case. Here, much fewer coefficients are required, and the forms can be solved for even in the presence of a tension parameter. For instance, the form that computes the centroid (=moment of degree 1) for curves generated by the C^1 four-point scheme with parameter ω can be expressed with variable ω.

Our article is structured as follows: We review the area, centroid, and inertia for sets bounded by polygons. Then, the four families of subdivision are introduced that generate the curves in the examples. Specific curves and the stated area moments might help to verify implementations of the formulas for the moments.

Area moments defined by polygons

The area moments of the 2-dimensional set $\Omega \subset \mathbb{R}^2$ enclosed by a polygon with n control points $(\mathrm{px}_k, \mathrm{py}_k) \in \mathbb{R}^2$ for $k = 0, 1, 2, \ldots, n-1$ can be found for instance in [Bourke 1988]. The moment of degree 0 is the area A, which is determined by the well known alternating bilinear form, that is the determinant of 2×2 matrices

$$M(0,0) = A = \frac{1}{2} \sum_{k=0}^{n-1} \det\begin{pmatrix} x_k & y_k \\ x_{k+1} & y_{k+1} \end{pmatrix} = \frac{1}{2} \sum_{k=0}^{n-1} (x_k\, y_{k+1} - x_{k+1}\, y_k)$$

The indices of the control points are taken modulo n. For instance, index $k = n$ corresponds to index $k = 0$.

The centroid (c_x, c_y) of the set Ω requires the two moments of degree 1, and corresponds to the following trilinear form

$$c_x = \frac{1}{A} M(1,0) = \frac{1}{A} \frac{1}{6} \sum_{k=0}^{n-1} (x_k + x_{k+1}) \det\begin{pmatrix} x_k & y_k \\ x_{k+1} & y_{k+1} \end{pmatrix}$$

$$c_y = \frac{1}{A} M(0,1) = \frac{1}{A} \frac{1}{6} \sum_{k=0}^{n-1} (y_k + y_{k+1}) \det\begin{pmatrix} x_k & y_k \\ x_{k+1} & y_{k+1} \end{pmatrix}$$

The area inertia of Ω can be found in [Juhlnet 2011]. The values are determined by 4-linear forms such as

$$M(2,0) = I_{xx} = \frac{1}{12} \sum_{k=0}^{n-1} \left(x_k^2 + x_k\, x_{k+1} + x_{k+1}^2\right) \det\begin{pmatrix} x_k & y_k \\ x_{k+1} & y_{k+1} \end{pmatrix}$$

$$M(1,1) = I_{xy} = \frac{1}{12} \sum_{k=0}^{n-1} (x_k - x_{k+1}) \left(3\, x_{k+1}\, y_{k+1}^2 + x_k\, y_{k+1}^2 + 2\, x_{k+1}\, y_k\, y_{k+1} + 2\, x_k\, y_k\, y_{k+1} + x_{k+1}\, x_k^2 + 3\, x_k\, y_k^2\right)$$

$$M(0,2) = I_{yy} = \frac{1}{12} \sum_{k=0}^{n-1} \left(y_k^2 + y_k\, y_{k+1} + y_{k+1}^2\right) \det\begin{pmatrix} x_k & y_k \\ x_{k+1} & y_{k+1} \end{pmatrix}$$

The coefficients in the multi-linear forms are generally not uniquely determined.

The area moments of polygons can be used to approximate the moments defined by subdivision curves. Thereby, the formulas help to validate the implementation of the exact forms.

The piecewise linear boundary of a polygon is reproduced by linear subdivision. Using our general framework to establish multi-linear forms for the computation of area moments we reproduce the forms stated above.

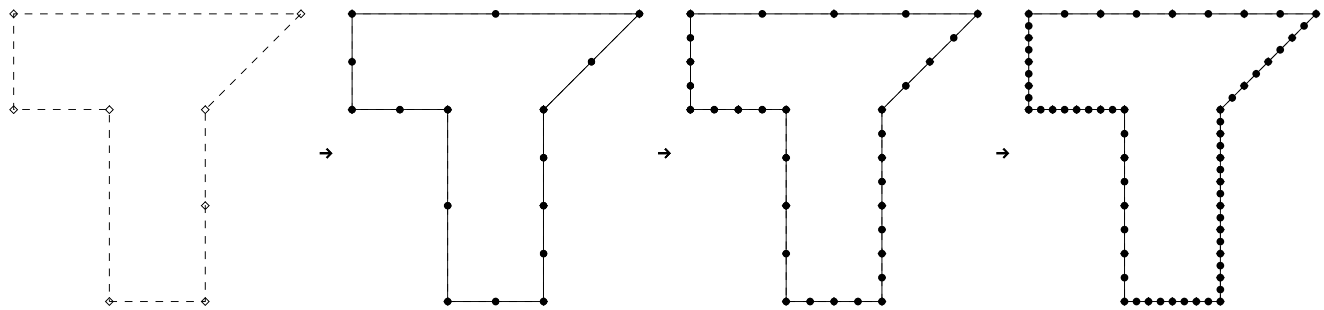


Figure: Three iterations of a T-shaped control point sequence defined by the cycle [(1, 0), (2, 0), (2, 1), (2, 2), (3, 3), (0, 3), (0, 2), (1, 2)] using linear subdivision. The enclosed area is $9/2 = 4.5$. The centroid is located at $\frac{1}{27}(37, 50)$. ■

The rules of linear subdivision are vertex interpolation, and mid-edge insertion

The basis functions that parameterize the curve between two successive control points are the linear polynomials $B_1(t) = 1 - t$, and $B_2(t) = t$ for $t \in [0, 1]$.

Schemes for curves

We briefly review the subdivision schemes that are used in the upcoming examples.

Quadratic B-spline

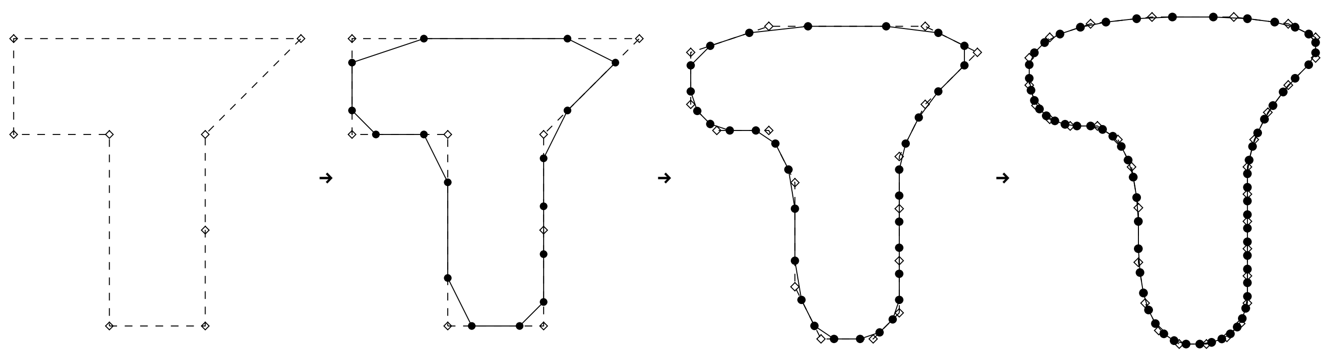


Figure: Three iterations of the T-shaped control point sequence defined above using quadratic B-spline subdivision. The area enclosed by the limit curve is $101/24 = 4.20833\ldots$, the centroid is located at $(139/101, 928/505)$. ■

Quadratic B-spline subdivision for curves is also referred to as *Chaikin's* scheme, or *corner-cutting* scheme. The scheme is *dual*, i.e. two output control points are inserted between a pair of input control points. The weights for the insertion are symmetric

The basis functions that piecewise parametrize the curve are the quadratic polynomials

$$B_1(t) = \frac{1}{2}(t-1)^2,\ B_2(t) = \frac{1}{2} + t - t^2, \text{ and } B_3(t) = \frac{1}{2}t^2 \text{ for } t \in [0, 1].$$

Cubic B-spline

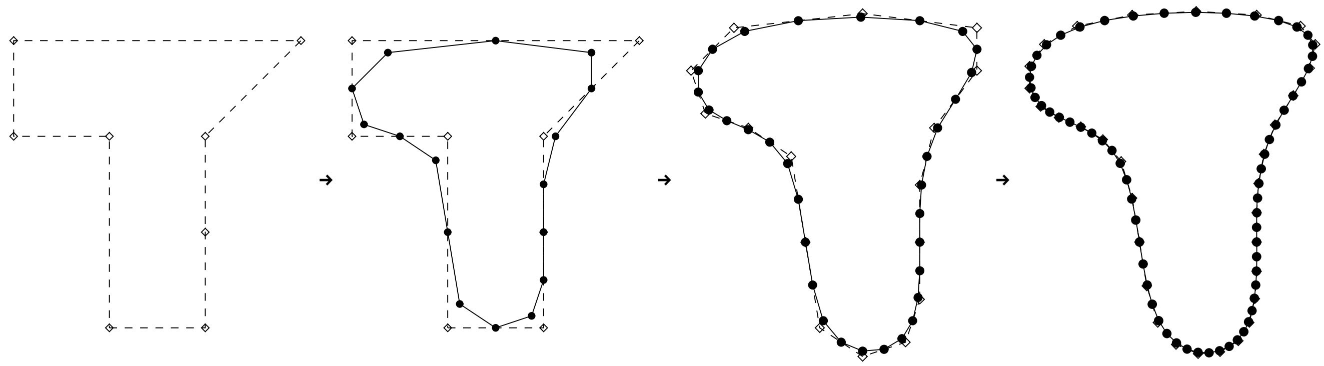


Figure: Three iterations of cubic B-spline subdivision applied to the T-shaped control point sequence with coordinates defined above. The area enclosed by the limit curve is $\frac{59}{15} = 3.93333...$. The centroid is located at $\left(\frac{41077}{29736}, \frac{432751}{237888}\right)$. ■

A very popular polygon refinement algorithm is cubic B-spline subdivision with the following averaging mask and mid-edge insertion

The basis functions that parametrize the curve between a pair of successive control points are the following cubic polynomials

$$B_1(t) = -\frac{1}{6}(t-1)^3,\ B_2(t) = \frac{1}{6}\left(4 - 6t^2 + 3t^3\right),\ B_3(t) = \frac{1}{6}\left(1 + 3t + 3t^2 - 3t^3\right), \text{ and } B_4(t) = \frac{1}{6}t^3 \text{ for } t \in [0, 1].$$

C^1 four-point scheme

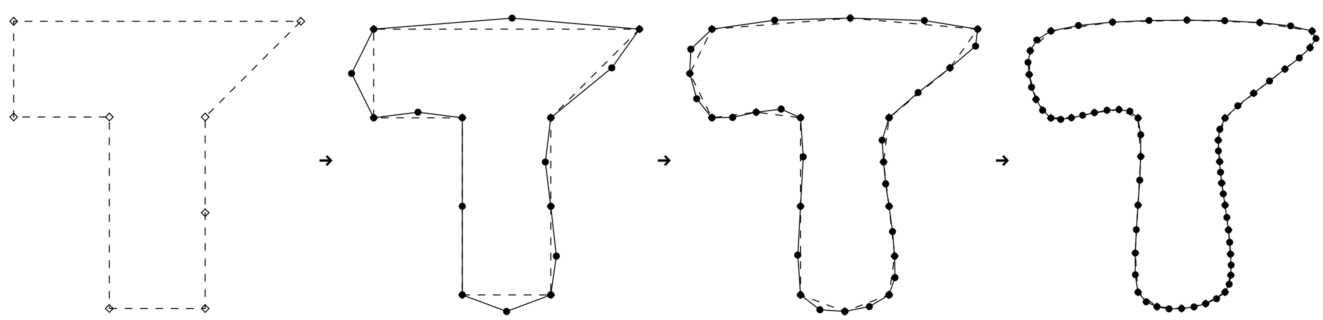
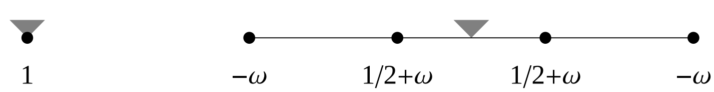


Figure: Three iterations of the C^1 four-point scheme with tension parameter $\omega = 1/16$. The area enclosed by the limit curve is $\frac{27-25\omega+171\omega^2+88\omega^3+224\omega^4+320\omega^5}{6-18\omega+54\omega^2-48\omega^3+96\omega^4}$ for general ω, and $\frac{85625}{16632} = 5.14821...$ for $\omega = 1/16$. ■

The interpolatory four-point scheme was conceived by [Dubuc 1986], and generalized later in [Dyn/Gregory/Levin 1987] who introduced the tension parameter $\omega \in \mathbb{R}$. Dubuc's original scheme corresponds to $\omega = 1/16 = 0.0625$.



[Hechler/Moessner/Reif 2008] prove that the scheme produces C^1 curves when $\omega \in (0, \omega^*)$ with ω^* as the unique real solution of the cubic polynomial $32\omega^3 + 4\omega - 1 = 0$, namely

$$\omega^* = \frac{1}{12}\sqrt[3]{27+3\sqrt{105}} - \frac{1}{2}\sqrt[-3]{27+3\sqrt{105}} = 0.1927292492648120252062865923267567418137 63\ldots$$

C^2 four-point scheme

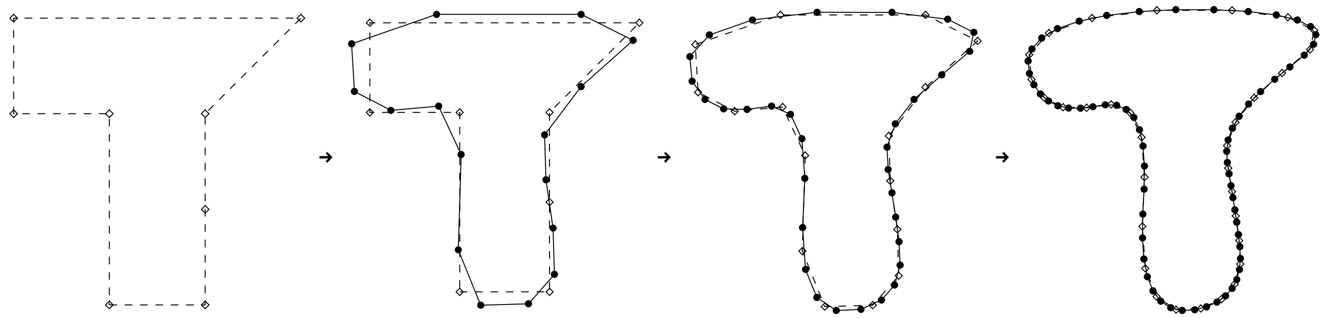


Figure: Three iterations of the C^2 four-point scheme with tension parameter $\omega = 1/128$. The area enclosed by the limit curve is the fraction $\frac{8\,235\,644\,399\,709}{1\,634\,922\,512\,365} = 5.03733\ldots$. ■

The C^2 four-point scheme was introduced by [Dyn/Floater/Hormann 2005] and uses the tension parameter $\omega \in \mathbb{R}$. Smoothness is guaranteed for parameters in the interval $\omega \in (0, 1/48]$, but possibly also for values beyond $\omega > 1/48 = 0.0208333\ldots$. The default choice is $\omega = 1/128 = 0.0078125$.

The scheme is dual, i.e. the output control points are located between the input control points. The weights are

For the choice $\omega = 0.013723\ldots$ the scheme is called “tightest”. For that parameter value, the basis function sampled at the integers $k \in \mathbb{Z}$ are closest to the Kronecker sequence $\delta_{0,k}$ in the least square sense. The limit curves are almost, but not quite, entirely unlike interpolatory.

Remarks

The subdivision weights are applied coordinatewise.

In order to establish the area moments refinement through subdivision is not required. In fact, less refinement means faster evaluation of the formula. Despite that, we give a visual impression of the subdivision curves by subdiving the input polygon about 6-7 times.

For the linear, quadratic, cubic, etc. B-spline subdivision schemes, the area moments can be derived by solving the integral expression via the divergence theorem. That is because the basis functions are polynomials.

Examples

For all example curves that follow, we state the coordinates of the control points of the polygon that are input to the subdivision iteration. We apply the various subdivision schemes in turn. The limit curves are visualized and can be compared conveniently. For each contour, we state the exact area, centroid, and inertia defined by the limit curve.

The inertia is measured with respect to a) the (previously established) centroid of the area, (because that reference is the most relevant in practice), and b) the x-, and y- axis. We remark that the formula easily permits to compute the inertia with respect to any point in the plane. The principal axes are determined by eigenvalue decomposition of the inertia matrix, and are plotted in the graphics.

Whenever all weights of a subdivision scheme are rational, the coefficients in the multi-linear forms that determine the area moments are also fractions. This allows us to establish the area, centroid, and inertia in exact algebraic form given that also the coordinates of the control points are rational numbers.

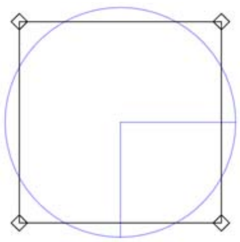
In the upcoming examples, some algebraic expressions exceed the page margins due to their large number of digits. In that case, we restate the value in full length immediately below.

Cube

Curve coordinates ↓

$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

Linear B-spline ↓



Area 1 ($\approx$ 1.00000000000000000000)

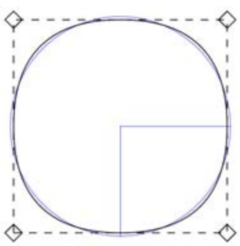
Centroid= $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

Centroid≈ (0.50000000000000000000 0.50000000000000000000)

Inertia = $\begin{pmatrix} \frac{1}{12} & 0 \\ 0 & \frac{1}{12} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 0.083333333333333333333 & 0 \\ 0 & 0.083333333333333333333 \end{pmatrix}$

Quadratic B-spline ↓



Area $\frac{5}{6}$ ($\approx$ 0.83333333333333333333)

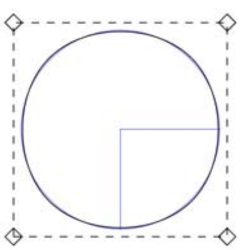
Centroid= $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

Centroid≈ (0.50000000000000000000 0.50000000000000000000)

Inertia = $\begin{pmatrix} \frac{31}{560} & 0 \\ 0 & \frac{31}{560} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 0.055357142857142857143 & 0 \\ 0 & 0.055357142857142857143 \end{pmatrix}$

Cubic B-spline ↓



Area $\frac{61}{90}$ ($\approx$ 0.67777777777777777778)

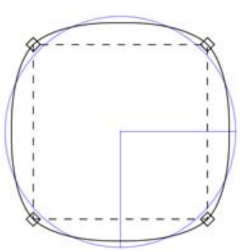
Centroid= $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

Centroid≈ (0.50000000000000000000 0.50000000000000000000)

Inertia = $\begin{pmatrix} \frac{27\,371}{748\,440} & 0 \\ 0 & \frac{27\,371}{748\,440} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 0.036570733792956015178 & 0 \\ 0 & 0.036570733792956015178 \end{pmatrix}$

C^1 Four-Point ω=1/16 ↓



Area $\frac{14\,272}{10\,395}$ ($\approx$ 1.3729677729677729678)

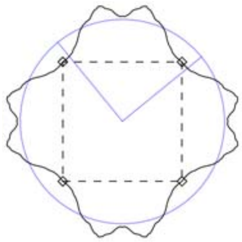
Centroid= $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

Centroid≈ (0.50000000000000000000 0.50000000000000000000)

Inertia = $\begin{pmatrix} \frac{23\,340\,561\,324\,786\,432\,115\,362\,070\,413\,499\,461\,043\,666\,460\,891}{154\,520\,168\,587\,414\,501\,234\,522\,160\,187\,896\,923\,984\,378\,608\,000} & 0 \\ 0 & \frac{23\,340\,561\,324\,786\,432\,115\,362\,070\,413}{154\,520\,168\,587\,414\,501\,234\,522\,160\,187} \end{pmatrix}$

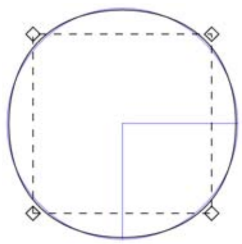
Inertia ≈ $\begin{pmatrix} 0.15105187586940993871 & 0 \\ 0 & 0.15105187586940993871 \end{pmatrix}$

C^1 Four-Point ω=0.192729… ↓



Area	2.25279	(≈ 2.25279)
Centroid≈	(0.5 0.5)	
Inertia ≈	$\begin{pmatrix} 0.425514 & 0 \\ 0 & 0.425514 \end{pmatrix}$	

C^2 Four-Point ω=1/128 ↓



Area $\frac{2\,140\,937\,273\,264}{1\,634\,922\,512\,365}$ (≈ 1.3095038187265667219)

Centroid= $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

Centroid≈ (0.50000000000000000000 0.50000000000000000000)

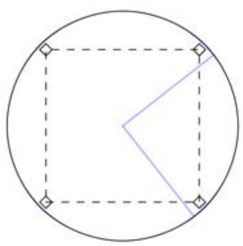
Inertia = $\left(\begin{matrix} \frac{2\,785\,312\,589\,755\,250\,739\,045\,622\,399\,401\,429\,623\,305\,842\,223\,910\,999\,553\,040\,673\,971\,900\,727\,762\,966\,178}{20\,400\,941\,189\,697\,027\,827\,745\,155\,981\,139\,943\,353\,369\,700\,479\,862\,209\,407\,032\,288\,354\,148\,721\,816\,336\,632} \\ 0 \end{matrix} \right.$

Inertia ≈ $\begin{pmatrix} 0.13652863188301829104 & 0 \\ 0 & 0.13652863188301829104 \end{pmatrix}$

The two coefficients along the diagonal of the inertia matrix are of identical value, namely the fraction

2 785 312 589 755 250 739 045 622 399 401 429 623 305 842 223 910 999 553 040 673 971 900 727 762 966 178 378 026 258 070 730 972 668 635 241 671 468-
087 323 418 249 734 743 416 211 419 392 /
20 400 941 189 697 027 827 745 155 981 139 943 353 369 700 479 862 209 407 032 288 354 148 721 816 336 632 345 552 614 291 779 328 958 727 445 153-
359 268 845 237 360 797 485 780 583 257 725

C^2 Four-Point ω=0.013723… (Tightest) ↓



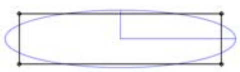
Area	1.78413	(≈ 1.78413)
Centroid≈	(0.5 0.5)	
Inertia ≈	$\begin{pmatrix} 0.253307 & 0 \\ 0 & 0.253307 \end{pmatrix}$	

Rectangle

Curve coordinates ↓

$$\begin{pmatrix} 0 & 4 & 4 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

Linear B-spline ↓



Area 4 $(\approx 4.0000000000000000000)$

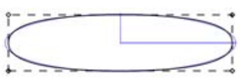
Centroid= $\begin{pmatrix} 2 & \frac{1}{2} \end{pmatrix}$

Centroid≈ $\begin{pmatrix} 2.0000000000000000000 & 0.50000000000000000000 \end{pmatrix}$

Inertia = $\begin{pmatrix} \frac{16}{3} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 5.3333333333333333333 & 0 \\ 0 & 0.33333333333333333333 \end{pmatrix}$

Quadratic B-spline ↓



Area $\frac{10}{3}$ $(\approx 3.3333333333333333333)$

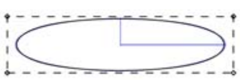
Centroid= $\begin{pmatrix} 2 & \frac{1}{2} \end{pmatrix}$

Centroid≈ $\begin{pmatrix} 2.0000000000000000000 & 0.50000000000000000000 \end{pmatrix}$

Inertia = $\begin{pmatrix} \frac{124}{35} & 0 \\ 0 & \frac{31}{140} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 3.5428571428571428571 & 0 \\ 0 & 0.22142857142857142857 \end{pmatrix}$

Cubic B-spline ↓



Area $\frac{122}{45}$ $(\approx 2.7111111111111111111)$

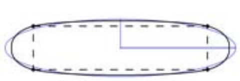
Centroid= $\begin{pmatrix} 2 & \frac{1}{2} \end{pmatrix}$

Centroid≈ $\begin{pmatrix} 2.0000000000000000000 & 0.50000000000000000000 \end{pmatrix}$

Inertia = $\begin{pmatrix} \frac{218968}{93555} & 0 \\ 0 & \frac{27371}{187110} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 2.3405269627491849714 & 0 \\ 0 & 0.14628293517182406071 \end{pmatrix}$

C^1 Four-Point ω=1/16 ↓



Area $\frac{57088}{10395}$ $(\approx 5.4918710918710918711)$

Centroid= $\begin{pmatrix} 2 & \frac{1}{2} \end{pmatrix}$

Centroid≈ $\begin{pmatrix} 2.0000000000000000000 & 0.50000000000000000000 \end{pmatrix}$

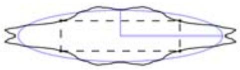
Inertia = $\begin{pmatrix} \frac{23340561324786432115362070413499461043666460891}{2414377634178351581789408752935889437255915750} & 0 \\ 0 & \frac{23340561324786432115362070413 49}{38630042146853625308630540046 97} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 9.6673200556422360777 & 0 \\ 0 & 0.60420750347763975486 \end{pmatrix}$

The obscured value in the inertia matrix is

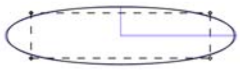
$$\frac{23340561324786432115362070413499461043666460891}{38630042146853625308630540046974230996094652000}.$$

C^1 Four−Point ω=0.192729… ↓



Area	9.01117	(≈ 9.01117)
Centroid≈	(2. 0.5)	
Inertia ≈	$\begin{pmatrix} 27.2329 & 0 \\ 0 & 1.70206 \end{pmatrix}$	

C^2 Four−Point ω=1/128 ↓



Area	$\frac{8\,563\,749\,093\,056}{1\,634\,922\,512\,365}$	(≈ 5.2380152749062668877)
Centroid=	$\begin{pmatrix} 2 & \frac{1}{2} \end{pmatrix}$	
Centroid≈	(2.0000000000000000000 0.50000000000000000000)	
Inertia =	$\begin{pmatrix} \frac{178\,260\,005\,744\,336\,047\,298\,919\,833\,561\,691\,495\,891\,573\,902\,330\,303\,971\,394\,603\,134\,201\,646\,576\,829\,835\,41}{20\,400\,941\,189\,697\,027\,827\,745\,155\,981\,139\,943\,353\,369\,700\,479\,862\,209\,407\,032\,288\,354\,148\,721\,816\,336\,63} \\ 0 \end{pmatrix}$	
Inertia ≈	$\begin{pmatrix} 8.7378324405131706266 & 0 \\ 0 & 0.54611452753207316416 \end{pmatrix}$	

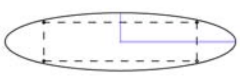
The diagonal of the inertia matrix contains the two values

178 260 005 744 336 047 298 919 833 561 691 495 891 573 902 330 303 971 394 603 134 201 646 576 829 835 416 193 680 516 526 782 250 792 655 466 973-957 588 698 767 983 023 578 637 530 841 088 / 20 400 941 189 697 027 827 745 155 981 139 943 353 369 700 479 862 209 407 032 288 354 148 721 816 336 632 345 552 614 291 779 328 958 727 445 153-359 268 845 237 360 797 485 780 583 257 725

and

11 141 250 359 021 002 956 182 489 597 605 718 493 223 368 895 643 998 212 162 695 887 602 911 051 864 713 512 105 032 282 923 890 674 540 966 685 872-349 293 672 998 938 973 664 845 677 568 / 20 400 941 189 697 027 827 745 155 981 139 943 353 369 700 479 862 209 407 032 288 354 148 721 816 336 632 345 552 614 291 779 328 958 727 445 153-359 268 845 237 360 797 485 780 583 257 725

C^2 Four−Point ω=0.013723… (Tightest) ↓



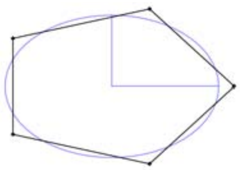
Area	7.13651	(≈ 7.13651)
Centroid≈	(2. 0.5)	
Inertia ≈	$\begin{pmatrix} 16.2117 & 0 \\ 0 & 1.01323 \end{pmatrix}$	

Pentagon

Curve coordinates ↓

$$\begin{pmatrix} 0.927050983124842 & -2.42705098312484 & -2.42705098312484 & 0.927050983124842 & 3.00000000000000 \\ 1.90211303259031 & 1.17557050458495 & -1.17557050458495 & -1.90211303259031 & 0 \end{pmatrix}$$

Linear B-spline ↓

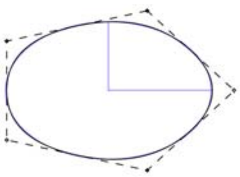


Area 14.2658477444273 (≈ 14.2658477444273)

Centroid≈ (0 0)

Inertia ≈ $\begin{pmatrix} 24.705063660786 & 0 \\ 0 & 10.980028293683 \end{pmatrix}$

Quadratic B-spline ↓

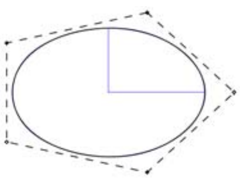


Area 12.6229380190550 (≈ 12.6229380190550)

Centroid≈ (0 0)

Inertia ≈ $\begin{pmatrix} 19.024305188341 & 0 \\ 0 & 8.455246750374 \end{pmatrix}$

Cubic B-spline ↓

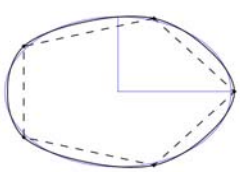


Area 11.0557098070166 (≈ 11.0557098070166)

Centroid≈ (0 0)

Inertia ≈ $\begin{pmatrix} 14.590618131429 & 0 \\ 0 & 6.484719169524 \end{pmatrix}$

C^1 Four-Point ω=1/16 ↓

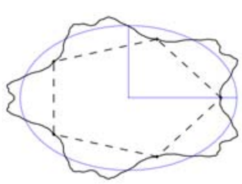


Area 17.8084379590431 (≈ 17.8084379590431)

Centroid≈ (0 0)

Inertia ≈ $\begin{pmatrix} 37.903145548965 & 0 \\ 0 & 16.845842466207 \end{pmatrix}$

C^1 Four-Point ω=0.192729… ↓

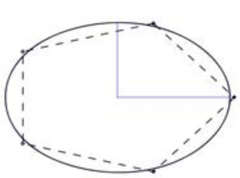


Area 25.4077 (≈ 25.4077)

Centroid≈ (0 0)

Inertia ≈ $\begin{pmatrix} 80.444 & 1.13062 \times 10^{-10} \\ 1.13062 \times 10^{-10} & 35.7529 \end{pmatrix}$

C^2 Four-Point ω=1/128 ↓

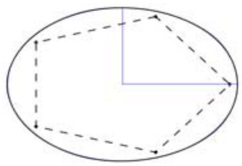


Area 17.4134714578743 (≈ 17.4134714578743)

Centroid≈ (0 0)

Inertia ≈ $\begin{pmatrix} 36.196731214164 & 0 \\ 0 & 16.087436095184 \end{pmatrix}$

C^2 Four-Point ω=0.013723… (Tightest) ↓



Area 21.8187 (≈ 21.8187)

Centroid≈ (0 0)

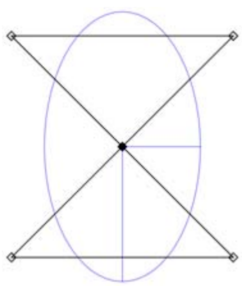
Inertia ≈ $\begin{pmatrix} 56.8265 & 0 \\ 0 & 25.2562 \end{pmatrix}$

Hourglass

Curve coordinates ↓

$\begin{pmatrix} 0 & 1 & -1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 0 & -1 & -1 \end{pmatrix}$

Linear B-spline ↓



Area 2 (≈ 2.0000000000000000000)

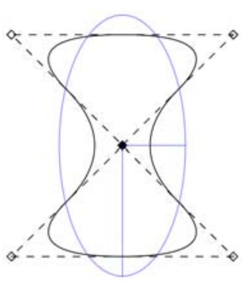
Centroid= (0 0)

Centroid≈ (0 0)

Inertia = $\begin{pmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 0.33333333333333333333 & 0 \\ 0 & 1.0000000000000000000 \end{pmatrix}$

Quadratic B-spline ↓



Area $\frac{11}{6}$ (≈ 1.8333333333333333333)

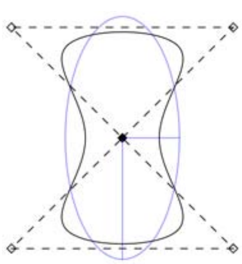
Centroid= (0 0)

Centroid≈ (0 0)

Inertia = $\begin{pmatrix} \frac{41}{240} & 0 \\ 0 & \frac{1229}{1680} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 0.17083333333333333333 & 0 \\ 0 & 0.73154761904761904762 \end{pmatrix}$

Cubic B-spline ↓



Area $\frac{301}{180}$ (≈ 1.6722222222222222222)

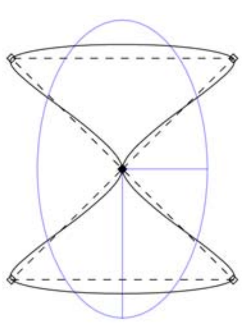
Centroid= (0 0)

Centroid≈ (0 0)

Inertia = $\begin{pmatrix} \frac{70\,657}{598\,752} & 0 \\ 0 & \frac{1\,607\,567}{2\,993\,760} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 0.11800712147934370157 & 0 \\ 0 & 0.53697256961145850035 \end{pmatrix}$

C^1 Four-Point ω=1/16 ↓



Area $\frac{78\,223}{33\,264}$ (≈ 2.3515812890812890813)

Centroid= (0 0)

Centroid≈ (0 0)

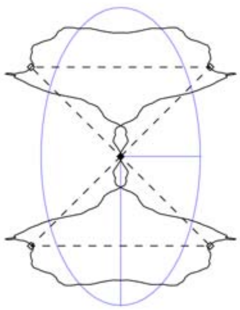
Inertia = $\begin{pmatrix} \frac{191\,993\,747\,418\,159\,360\,630\,614\,549\,273\,144\,936\,221\,187\,466\,707\,213\,503}{402\,145\,727\,814\,833\,116\,364\,899\,760\,275\,939\,474\,160\,984\,005\,367\,808\,000} & \\ 0 & \frac{7\,683\,231\,983\,224\,363\,149\,428\,53\ldots}{5\,227\,894\,461\,592\,830\,512\,743\,69\ldots} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 0.47742331731685671039 & 0 \\ 0 & 1.4696608815785930104 \end{pmatrix}$

The obscured 2nd entry in the diagonal of the inertia matrix is

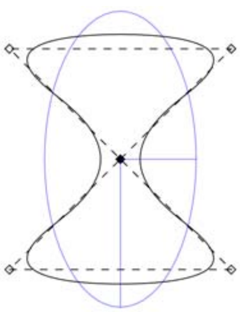
$$\frac{7\,683\,231\,983\,224\,363\,149\,428\,538\,701\,183\,500\,632\,071\,770\,497\,720\,770\,137}{5\,227\,894\,461\,592\,830\,512\,743\,696\,883\,587\,213\,164\,092\,792\,069\,781\,504\,000}.$$

```
C^1 Four-Point ω=0.192729… ↓
```



```
Area         3.07754    (≈ 3.07754 )
Centroid≈    ( 0  0 )
Inertia ≈    ( 0.920878     0     )
             (    0      3.23252  )
```

```
C^2 Four-Point ω=1/128 ↓
```



Area $\frac{3\,800\,007\,905\,001}{1\,634\,922\,512\,365}$ (≈ 2.3242740107016398989)

Centroid= (0 0)

Centroid≈ (0 0)

Inertia = $\left(\begin{array}{c} \frac{6\,862\,068\,724\,607\,274\,687\,678\,214\,948\,362\,249\,014\,157\,710\,242\,195\,107\,762\,632\,817\,165\,914\,455\,190\,786\,087}{20\,400\,941\,189\,697\,027\,827\,745\,155\,981\,139\,943\,353\,369\,700\,479\,862\,209\,407\,032\,288\,354\,148\,721\,816\,336\,632} \\ 0 \end{array}\right.$

Inertia ≈ $\begin{pmatrix} 0.3363603992973023480 6 & 0 \\ 0 & 1.2928624163176987046 \end{pmatrix}$

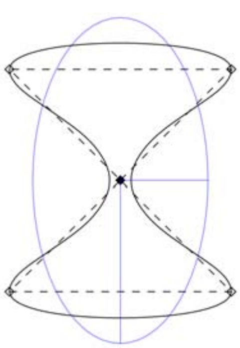
The inertia matrix is a diagonal matrix. The two entries are the fractions

6 862 068 724 607 274 687 678 214 948 362 249 014 157 710 242 195 107 762 632 817 165 914 455 190 786 087 437 833 147 569 041 930 410 483 922 128 912-
445 351 336 138 298 930 437 242 917 267 /
20 400 941 189 697 027 827 745 155 981 139 943 353 369 700 479 862 209 407 032 288 354 148 721 816 336 632 345 552 614 291 779 328 958 727 445 153-
359 268 845 237 360 797 485 780 583 257 725

and

1 758 374 008 111 131 086 315 270 628 215 542 288 289 983 833 039 059 628 198 169 083 992 587 975 440 998 619 879 027 934 368 728 929 411 607 340 632-
574 214 169 679 161 206 712 431 867 063 /
1 360 062 745 979 801 855 183 010 398 742 662 890 224 646 698 657 480 627 135 485 890 276 581 454 422 442 156 370 174 286 118 621 930 581 829 676 890-
617 923 015 824 053 165 718 705 550 515

```
C^2 Four-Point ω=0.013723… (Tightest) ↓
```



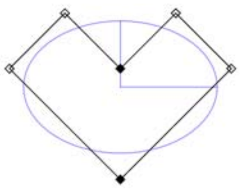
```
Area         2.75318    (≈ 2.75318 )
Centroid≈    ( 0  0 )
Inertia ≈    ( 0.568256     0     )
             (    0      1.95662  )
```

Axis Heart

Curve coordinates ↓

$$\begin{pmatrix} 0 & 1 & \frac{1}{2} & 0 & 0 & -\frac{1}{2} & -1 & 0 \\ 0 & 1 & \frac{3}{2} & 1 & 1 & \frac{3}{2} & 1 & 0 \end{pmatrix}$$

Linear B-spline ↓



Area $\frac{3}{2}$ $(\approx 1.50000000000000000000)$

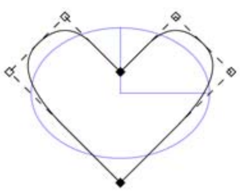
Centroid= $\begin{pmatrix} 0 & \frac{5}{6} \end{pmatrix}$

Centroid≈ $\begin{pmatrix} 0 & 0.83333333333333333333 \end{pmatrix}$

Inertia = $\begin{pmatrix} \frac{5}{16} & 0 \\ 0 & \frac{7}{48} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 0.31250000000000000000 & 0 \\ 0 & 0.14583333333333333333 \end{pmatrix}$

Quadratic B-spline ↓



Area $\frac{11}{8}$ $(\approx 1.37500000000000000000)$

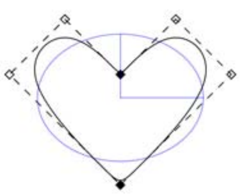
Centroid= $\begin{pmatrix} 0 & \frac{89}{110} \end{pmatrix}$

Centroid≈ $\begin{pmatrix} 0 & 0.80909090909090909091 \end{pmatrix}$

Inertia = $\begin{pmatrix} \frac{2161}{8960} & 0 \\ 0 & \frac{63\,689}{492\,800} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 0.24118303571428571429 & 0 \\ 0 & 0.12923904220779220779 \end{pmatrix}$

Cubic B-spline ↓



Area $\frac{201}{160}$ $(\approx 1.25625000000000000000)$

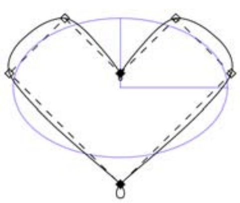
Centroid= $\begin{pmatrix} 0 & \frac{119\,963}{151\,956} \end{pmatrix}$

Centroid≈ $\begin{pmatrix} 0 & 0.78945879070257179710 \end{pmatrix}$

Inertia = $\begin{pmatrix} \frac{6\,063\,109}{31\,933\,440} & 0 \\ 0 & \frac{45\,503\,983\,271}{404\,373\,150\,720} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 0.18986707977593394260 & 0 \\ 0 & 0.11252968499510570077 \end{pmatrix}$

C^1 Four-Point ω=1/16 ↓



Area $\frac{393\,023}{221\,760}$ $(\approx 1.7722898629148629149)$

Centroid= $\begin{pmatrix} 0 & \frac{33\,217\,256\,278\,994\,614\,499}{38\,104\,466\,357\,332\,192\,320} \end{pmatrix}$

Centroid≈ $\begin{pmatrix} 0 & 0.87174180494992907654 \end{pmatrix}$

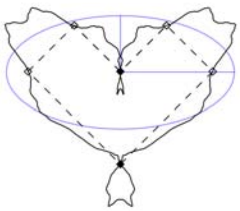
Inertia = $\begin{pmatrix} \frac{12\,424\,508\,062\,106\,609\,074\,560\,017\,673\,148\,847\,656\,454\,406\,522\,513\,968\,101}{27\,882\,103\,795\,161\,762\,734\,633\,050\,045\,798\,470\,208\,494\,891\,038\,834\,688\,000} & \\ 0 & \frac{2\,897\,705\,258\,480\,365\,314\,42}{15\,370\,843\,258\,599\,329\,112\,28} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 0.44560870131552159629 & 0 \\ 0 & 0.18851960232300353528 \end{pmatrix}$

The obscured 2nd entry in the diagonal of the inertia matrix is the value

$$2\,897\,705\,258\,480\,365\,314\,424\,264\,728\,405\,377\,800\,009\,532\,156\,130\,071\,070\,558\,820\,546\,315\,853 / 15\,370\,843\,258\,599\,329\,112\,280\,627\,386\,408\,165\,719\,112\,416\,829\,668\,228\,456\,928\,954\,298\,368\,000 .$$

C^1 Four-Point ω=0.192729… ↓

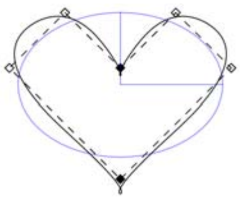


Area 2.39595 (≈ 2.39595)

Centroid≈ (0 0.993494)

$$\text{Inertia} \approx \begin{pmatrix} 0.906361 & 0 \\ 0 & 0.222639 \end{pmatrix}$$

C^2 Four-Point ω=1/128 ↓



$$\text{Area} \quad \frac{5\,678\,342\,886\,549}{3\,269\,845\,024\,730} \quad (\approx 1.7365785973351676572)$$

$$\text{Centroid} = \left(0 \quad \frac{53\,053\,624\,474\,195\,197\,357\,689\,384\,038\,175\,621\,434\,026\,407}{62\,427\,056\,129\,007\,134\,166\,837\,813\,962\,540\,540\,658\,412\,512} \right)$$

Centroid≈ (0 0.84984985299576682505)

$$\text{Inertia} = \left(\frac{21\,883\,914\,125\,926\,199\,290\,897\,329\,651\,922\,234\,075\,255\,396\,838\,046\,690\,475\,830\,627\,883\,775\,708\,367\,676\,744}{54\,402\,509\,839\,192\,074\,207\,320\,415\,949\,706\,515\,608\,985\,867\,946\,299\,225\,085\,419\,435\,611\,063\,258\,176\,897\,686} \quad \ldots \quad 0 \right.$$

$$\text{Inertia} \approx \begin{pmatrix} 0.40225927426165959651 & 0 \\ 0 & 0.20031145533252470337 \end{pmatrix}$$

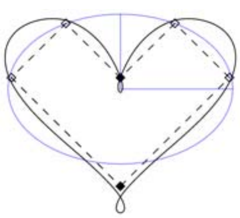
The diagonal of the inertia matrix contains the fraction

21 883 914 125 926 199 290 897 329 651 922 234 075 255 396 838 046 690 475 830 627 883 775 708 367 676 744 876 756 049 929 386 677 720 333 256 811 895-
507 826 272 306 124 176 110 272 529 847 /
54 402 509 839 192 074 207 320 415 949 706 515 608 985 867 946 299 225 085 419 435 611 063 258 176 897 686 254 806 971 444 744 877 223 273 187 075-
624 716 920 632 962 126 628 748 222 020 600

as well as the fraction

37 895 246 662 633 504 056 412 970 938 187 991 671 087 332 124 078 227 671 843 626 526 335 075 569 036 559 247 693 373 817 084 778 358 930 822 249 818-
151 391 419 521 310 555 140 599 866 462 832 807 065 972 431 561 637 253 961 685 227 459 119 /
189 181 625 183 272 420 740 525 479 115 628 291 907 316 890 719 393 159 205 634 158 381 310 704 153 170 589 548 314 213 521 378 745 488 071 570 701-
190 058 207 984 800 224 153 693 913 498 112 925 792 950 947 949 658 082 110 317 702 769 438 600

C^2 Four-Point ω=0.013723… (Tightest) ↓



Area 2.0746 (≈ 2.0746)

Centroid≈ (0 0.895209)

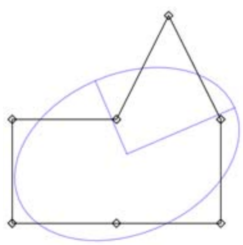
$$\text{Inertia} \approx \begin{pmatrix} 0.59278 & 0 \\ 0 & 0.265732 \end{pmatrix}$$

Dome

Curve coordinates ↓

$$\begin{pmatrix} 0 & 1 & 2 & 2 & \frac{3}{2} & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 & 1 \end{pmatrix}$$

Linear B-spline ↓



Area $\frac{5}{2}$ $(\approx 2.5000000000000000000)$

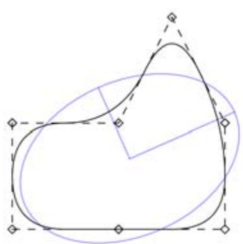
Centroid= $\begin{pmatrix} \frac{11}{10} & \frac{2}{3} \end{pmatrix}$

Centroid≈ $(1.1000000000000000000 \;\; 0.66666666666666666667)$

Inertia = $\begin{pmatrix} \frac{63}{80} & \frac{1}{6} \\ \frac{1}{6} & \frac{17}{36} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 0.78750000000000000000 & 0.16666666666666666667 \\ 0.16666666666666666667 & 0.47222222222222222222 \end{pmatrix}$

Quadratic B-spline ↓



Area $\frac{113}{48}$ $(\approx 2.3541666666666666667)$

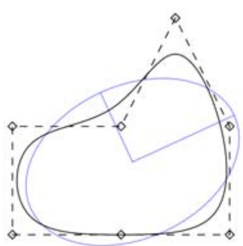
Centroid= $\begin{pmatrix} \frac{499}{452} & \frac{377}{565} \end{pmatrix}$

Centroid≈ $(1.1039823008849557522 \;\; 0.66725663716814159292)$

Inertia = $\begin{pmatrix} \frac{265\,675}{404\,992} & \frac{877\,547}{6\,074\,880} \\ \frac{877\,547}{6\,074\,880} & \frac{996\,637}{2\,531\,200} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 0.65600061235777496839 & 0.14445503450273914876 \\ 0.14445503450273914876 & 0.39374091340075853350 \end{pmatrix}$

Cubic B-spline ↓



Area $\frac{199}{90}$ $(\approx 2.2111111111111111111)$

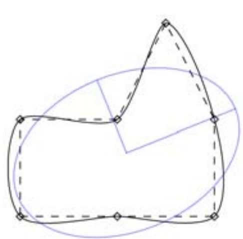
Centroid= $\begin{pmatrix} \frac{147\,869}{133\,728} & \frac{538\,339}{802\,368} \end{pmatrix}$

Centroid≈ $(1.1057444962909787030 \;\; 0.67093777418840232911)$

Inertia = $\begin{pmatrix} \frac{146\,945\,298\,439}{266\,899\,691\,520} & \frac{1\,453\,383\,887}{11\,862\,208\,512} \\ \frac{1\,453\,383\,887}{11\,862\,208\,512} & \frac{1\,092\,313\,165\,117}{3\,202\,796\,298\,240} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 0.55056376274600798759 & 0.12252220027406646888 \\ 0.12252220027406646888 & 0.34104984001550386405 \end{pmatrix}$

C^1 Four-Point ω=1/16 ↓



Area $\frac{106\,445}{38\,016}$ $(\approx 2.8000052609427609428)$

Centroid= $\begin{pmatrix} \frac{791\,020\,381\,158\,472\,310\,627}{722\,405\,799\,402\,161\,616\,000} & \frac{79\,219\,953\,921\,031\,229\,627}{120\,400\,966\,567\,026\,936\,000} \end{pmatrix}$

Centroid≈ $(1.0949806629640761199 \;\; 0.65796775706887425097)$

Inertia = $\begin{pmatrix} \frac{350\,393\,537\,942\,925\,339\,446\,161\,823\,337\,654\,215\,018\,247\,346\,750\,064\,503\,400\,392\,889\,234\,716\,331}{337\,051\,430\,407\,595\,004\,389\,273\,777\,993\,819\,227\,533\,009\,289\,452\,869\,396\,276\,395\,396\,300\,800\,000} & \frac{335\,0}{1\,554} \\ \frac{335\,024\,286\,482\,568\,647\,335\,019\,203\,599\,125\,072\,856\,352\,888\,480\,422\,385\,420\,093\,398\,739\,751\,791}{1\,554\,181\,595\,768\,354\,742\,461\,651\,309\,638\,166\,438\,068\,876\,168\,032\,675\,549\,496\,712\,105\,164\,800\,000} & \frac{376\,8}{635\,8} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 1.0395847824149441392 & 0.21556315387774211530 \\ 0.21556315387774211530 & 0.59270717760526234607 \end{pmatrix}$

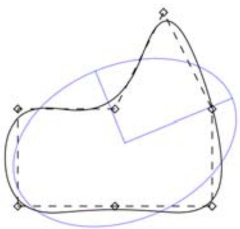
The obscured 2nd entry along the diagonal of the symmetric inertia matrix is the fraction

$$376\,844\,149\,273\,869\,934\,453\,067\,101\,198\,943\,485\,006\,036\,647\,913\,216\,018\,459\,363\,679\,328\,970\,787 / .$$
$$635\,801\,561\,905\,236\,031\,007\,039\,172\,124\,704\,451\,937\,267\,523\,286\,094\,542\,975\,927\,679\,385\,600\,000$$

C^1 Four−Point ω=0.192729… ↓

Area	3.39259	(≈ 3.39259)
Centroid≈	(1.06408 0.648801)	
Inertia ≈	$\begin{pmatrix} 1.80962 & 0.330911 \\ 0.330911 & 0.933294 \end{pmatrix}$	

C^2 Four−Point ω=1/128 ↓



Area $\frac{36\,472\,659\,766\,123}{13\,079\,380\,098\,920}$ (≈ 2.7885618041740867175)

Centroid= $\left(\frac{3\,004\,463\,918\,649\,115\,412\,936\,564\,115\,865\,665\,238\,875\,561}{2\,727\,729\,761\,392\,941\,498\,447\,981\,796\,087\,725\,370\,724\,192} \quad \frac{9\,383\,677\,212\,989\,275\,016\,629\,708\,803\,826\,316}{14\,320\,581\,247\,312\,942\,866\,851\,904\,429\,460\,55} \right.$

Centroid≈ (1.1014521897194306189 0.65525812471822615683)

Inertia = $\left(\begin{matrix} \frac{5\,001\,294\,927\,684\,418\,892\,967\,309\,012\,348\,567\,067\,559\,575\,229\,569\,448\,040\,758\,513\,781\,128\,252\,626\,178\,432\,2}{5\,058\,932\,222\,142\,155\,070\,892\,213\,277\,287\,725\,190\,494\,302\,244\,213\,989\,748\,609\,906\,059\,566\,542\,310\,892\,507\,3} \\ \frac{354\,753\,498\,649\,262\,382\,297\,305\,328\,767\,264\,232\,076\,575\,548\,704\,761\,234\,602\,201\,591\,353\,973\,387\,928\,570\,6}{1\,686\,310\,740\,714\,051\,690\,297\,404\,425\,762\,575\,063\,498\,100\,748\,071\,329\,916\,203\,302\,019\,855\,514\,103\,630\,835\,7} \end{matrix} \right.$

Inertia ≈ $\begin{pmatrix} 0.98860682611926153999 & 0.21037255476356955657 \\ 0.21037255476356955657 & 0.56487370061031578463 \end{pmatrix}$

The 1st entry on the diagonal of the inertia matrix is

5 001 294 927 684 418 892 967 309 012 348 567 067 559 575 229 569 448 040 758 513 781 128 252 626 178 432 289 108 232 905 675 543 587 755 211 179 464-
976 005 733 686 802 349 210 612 505 605 709 120 367 059 416 839 017 991 830 789 566 804 806 453 /
5 058 932 222 142 155 070 892 213 277 287 725 190 494 302 244 213 989 748 609 906 059 566 542 310 892 507 352 256 262 625 843 262 951 440 290 769 854-
639 131 276 134 743 942 788 430 149 704 142 815 952 480 824 985 739 857 456 558 466 711 671 200

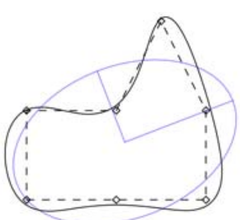
The off-diagonal entry is

354 753 498 649 262 382 297 305 328 767 264 232 076 575 548 704 761 234 602 201 591 353 973 387 928 570 677 608 359 092 064 790 159 054 540 109 304-
077 168 911 011 512 606 863 213 879 984 869 475 492 113 184 437 306 237 567 001 672 879 743 101 /
1 686 310 740 714 051 690 297 404 425 762 575 063 498 100 748 071 329 916 203 302 019 855 514 103 630 835 784 085 420 875 281 087 650 480 096 923 284-
879 710 425 378 247 980 929 476 716 568 047 605 317 493 608 328 579 952 485 519 488 903 890 400

The 2nd entry on the diagonal of the inertia matrix is

714 414 441 364 551 812 354 057 443 518 199 493 357 608 458 428 171 162 629 676 806 864 471 416 687 394 427 071 491 081 006 912 555 931 006 334 081-
893 695 135 088 924 539 659 313 670 754 044 165 328 332 571 410 936 013 320 208 044 528 788 319 /
1 264 733 055 535 538 767 723 053 319 321 931 297 623 575 561 053 497 437 152 476 514 891 635 577 723 126 838 064 065 656 460 815 737 860 072 692 463-
659 782 819 033 685 985 697 107 537 426 035 703 988 120 206 246 434 964 364 139 616 677 917 800

C^2 Four−Point ω=0.013723… (Tightest) ↓



Area	3.14612	(≈ 3.14612)
Centroid≈	(1.09447 0.646055)	
Inertia ≈	$\begin{pmatrix} 1.33832 & 0.273627 \\ 0.273627 & 0.734999 \end{pmatrix}$	

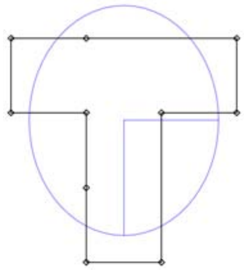
Letter T

The symmetry of the otherwise symmetric T shape is broken deliberately by inserting additional control points to make the resulting curve more interesting, and the values of the area moments less trivial.

Curve coordinates ↓

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 2 & 2 & 3 & 3 & 1 \\ 3 & 2 & 2 & 1 & 0 & 0 & 2 & 2 & 3 & 3 \end{pmatrix}$$

Linear B-spline ↓



Area $5 \quad (\approx 5.0000000000000000000)$

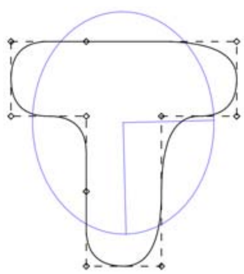
Centroid= $\begin{pmatrix} \frac{3}{2} & \frac{19}{10} \end{pmatrix}$

Centroid≈ $(1.5000000000000000000 \;\; 1.9000000000000000000)$

Inertia = $\begin{pmatrix} \frac{29}{12} & 0 \\ 0 & \frac{217}{60} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 2.4166666666666666667 & 0 \\ 0 & 3.6166666666666666667 \end{pmatrix}$

Quadratic B-spline ↓



Area $\frac{115}{24} \quad (\approx 4.7916666666666666667)$

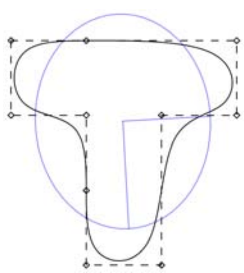
Centroid= $\begin{pmatrix} \frac{343}{230} & \frac{2201}{1150} \end{pmatrix}$

Centroid≈ $(1.4913043478260869565 \;\; 1.9139130434782608696)$

Inertia = $\begin{pmatrix} \frac{317\,137}{154\,560} & -\frac{41\,999}{1\,545\,600} \\ -\frac{41\,999}{1\,545\,600} & \frac{12\,020\,011}{3\,864\,000} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 2.0518698240165631470 & -0.027173266045548654244 \\ -0.027173266045548654244 & 3.1107688923395445135 \end{pmatrix}$

Cubic B-spline ↓



Area $\frac{3307}{720} \quad (\approx 4.5930555555555555556)$

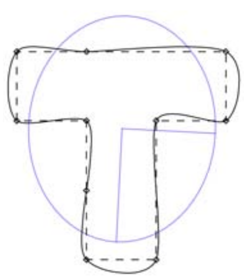
Centroid= $\begin{pmatrix} \frac{309\,716}{208\,341} & \frac{800\,305}{416\,682} \end{pmatrix}$

Centroid≈ $(1.4865820937789489347 \;\; 1.9206613196634363855)$

Inertia = $\begin{pmatrix} \frac{296\,697\,146\,167}{166\,326\,120\,576} & -\frac{85\,372\,363\,553}{1\,663\,261\,205\,760} \\ -\frac{85\,372\,363\,553}{1\,663\,261\,205\,760} & \frac{2\,251\,341\,722\,633}{831\,630\,602\,880} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 1.7838277303619854014 & -0.051328296035132073566 \\ -0.051328296035132073566 & 2.7071415058998940912 \end{pmatrix}$

C^1 Four-Point ω=1/16 ↓



Area $\frac{1\,209\,407}{221\,760} \quad (\approx 5.4536751443001443001)$

Centroid= $\begin{pmatrix} \frac{531\,132\,116\,051\,023\,421\,491}{351\,764\,209\,808\,245\,736\,640} & \frac{73\,698\,801\,254\,482\,029\,211}{39\,084\,912\,200\,916\,192\,960} \end{pmatrix}$

Centroid≈ $(1.5099094826632732252 \;\; 1.8856074404269596632)$

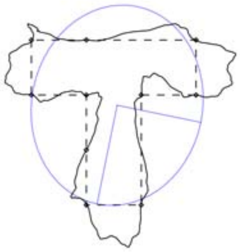
Inertia = $\begin{pmatrix} \frac{3\,511\,953\,009\,278\,964\,672\,443\,332\,726\,489\,749\,752\,644\,148\,310\,086\,453\,380\,371\,891\,928\,787\,218\,111}{1\,135\,176\,644\,594\,510\,071\,341\,126\,197\,225\,937\,849\,613\,152\,860\,817\,930\,543\,916\,813\,570\,572\,288\,000} & \frac{16\,8}{227\,0} \\ \frac{16\,862\,929\,564\,216\,989\,173\,285\,281\,818\,984\,463\,606\,999\,172\,611\,961\,227\,339\,511\,778\,156\,495\,299}{227\,035\,328\,918\,902\,014\,268\,225\,239\,445\,187\,569\,922\,630\,572\,163\,586\,108\,783\,362\,714\,114\,457\,600} & \frac{1\,036}{227\,0} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 3.0937502335008391878 & 0.074274473688808579130 \\ 0.074274473688808579130 & 4.5651291514784276430 \end{pmatrix}$

The obscured 2nd entry along the diagonal of the inertia matrix is

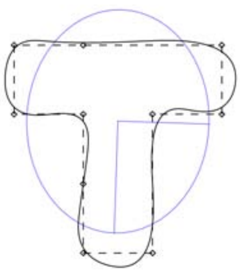
$$1\,036\,445\,598\,463\,172\,877\,534\,857\,207\,623\,597\,743\,838\,420\,702\,748\,655\,127\,893\,779\,358\,698\,083\,967 \,/\, 227\,035\,328\,918\,902\,014\,268\,225\,239\,445\,187\,569\,922\,630\,572\,163\,586\,108\,783\,362\,714\,114\,457\,600.$$

C^1 Four-Point ω=0.192729… ↓



Area	6.56749	(≈ 6.56749)
Centroid≈	(1.56173 1.81404)	
Inertia ≈	$\begin{pmatrix} 5.51705 & 0.408631 \\ 0.408631 & 7.43572 \end{pmatrix}$	

C^2 Four-Point ω=1/128 ↓



Area	$\frac{35\,258\,311\,240\,831}{6\,539\,690\,049\,460}$	(≈ 5.3914346053361312876)
Centroid=	$\left(\frac{2\,595\,192\,806\,879\,677\,216\,806\,254\,291\,916\,252\,312\,428\,731}{1\,730\,472\,612\,928\,113\,491\,882\,092\,552\,374\,416\,198\,817\,147} \quad \frac{1\,473\,655\,849\,938\,034\,908\,325\,932\,516\,629\,081\ldots}{775\,251\,730\,591\,794\,844\,363\,177\,463\,463\,738\,4\ldots}\right.$	
Centroid≈	(1.4997017505456964657 1.9008739894241931993)	
Inertia =	$\left(\frac{12\,400\,009\,816\,773\,174\,860\,159\,783\,643\,349\,405\,143\,545\,841\,537\,621\,604\,266\,638\,385\,623\,343\,017\,894\,309\,041\ldots}{4\,279\,184\,243\,239\,732\,363\,511\,001\,958\,233\,160\,499\,898\,459\,932\,463\,842\,927\,770\,980\,695\,134\,132\,449\,662\,708\ldots} \atop \frac{410\,158\,035\,609\,877\,631\,708\,299\,782\,528\,390\,402\,072\,865\,439\,005\,543\,005\,668\,810\,009\,569\,887\,759\,129\,134\,1\ldots}{8\,558\,368\,486\,479\,464\,727\,022\,003\,916\,466\,320\,999\,796\,919\,864\,927\,685\,855\,541\,961\,390\,268\,264\,899\,325\,416\ldots}\right.$	
Inertia ≈	$\begin{pmatrix} 2.8977508590247653419 & 0.047924792705273964533 \\ 0.047924792705273964533 & 4.3366517354841193862 \end{pmatrix}$	

The *y*-coordinate of the centroid is

$$\frac{1\,473\,655\,849\,938\,034\,908\,325\,932\,516\,629\,081\,348\,622\,423\,799}{775\,251\,730\,591\,794\,844\,363\,177\,463\,463\,738\,457\,070\,081\,856}.$$

The first entry of the inertia matrix is

12 400 009 816 773 174 860 159 783 643 349 405 143 545 841 537 621 604 266 638 385 623 343 017 894 309 041 358 176 665 810 992 419 816 897 795 597 462-
332 554 649 385 821 530 022 679 448 182 242 621 109 761 680 955 221 720 855 949 068 213 296 057 /
4 279 184 243 239 732 363 511 001 958 233 160 499 898 459 932 463 842 927 770 980 695 134 132 449 662 708 316 755 142 573 088 117 731 257 013 449 702-
938 380 270 857 789 883 848 302 292 245 735 192 693 354 823 500 328 193 077 938 775 375 783 100

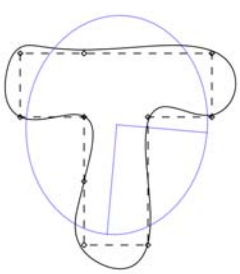
The value of the inertia matrix away from the diagonal is

410 158 035 609 877 631 708 299 782 528 390 402 072 865 439 005 543 005 668 810 009 569 887 759 129 134 130 265 032 162 998 689 792 316 473 888 988-
724 813 142 323 547 134 690 936 601 832 905 909 200 958 084 433 512 019 349 486 165 587 870 627 /
8 558 368 486 479 464 727 022 003 916 466 320 999 796 919 864 927 685 855 541 961 390 268 264 899 325 416 633 510 285 146 176 235 462 514 026 899 405-
876 760 541 715 579 767 696 604 584 491 470 385 386 709 647 000 656 386 155 877 550 751 566 200

The second entry along the diagonal of the inertia matrix is

43 300 440 808 104 394 657 221 077 282 063 941 634 118 855 527 908 850 741 648 039 067 855 153 280 190 894 900 992 565 718 247 820 986 179 468 033 459-
367 600 733 995 872 403 654 267 471 948 141 604 231 740 256 182 733 326 028 985 252 367 746 917 /
9 984 763 234 226 042 181 525 671 235 877 374 499 763 073 175 748 966 831 465 621 621 979 642 382 546 319 405 761 999 337 205 608 039 599 698 049 306-
856 220 632 001 509 728 979 372 015 240 048 782 951 161 254 834 099 117 181 857 142 543 493 900

C^2 Four-Point ω=0.013723… (Tightest) ↓



Area	5.94558	(≈ 5.94558)
Centroid≈	(1.51182 1.87934)	
Inertia ≈	$\begin{pmatrix} 3.86144 & 0.158171 \\ 0.158171 & 5.63275 \end{pmatrix}$	

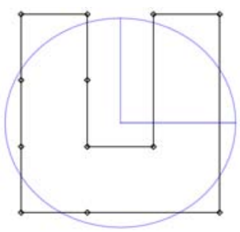
Letter U

The symmetry of the otherwise symmetric letter U is broken deliberately by inserting additional control points to make the result more interesting, and challenging to compute.

Curve coordinates ↓

$$\begin{pmatrix} 0 & 1 & 3 & 3 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 3 & 1 & 1 & 2 & 3 & 3 & 2 & 1 \end{pmatrix}$$

Linear B-spline ↓



Area $7 \quad (\approx 7.0000000000000000000)$

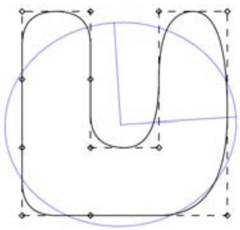
Centroid= $\begin{pmatrix} \frac{3}{2} & \frac{19}{14} \end{pmatrix}$

Centroid≈ $(1.5000000000000000000 \;\; 1.3571428571428571429)$

Inertia = $\begin{pmatrix} \frac{79}{12} & 0 \\ 0 & \frac{457}{84} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 6.5833333333333333333 & 0 \\ 0 & 5.4404761904761904762 \end{pmatrix}$

Quadratic B-spline ↓



Area $\frac{157}{24} \quad (\approx 6.5416666666666666667)$

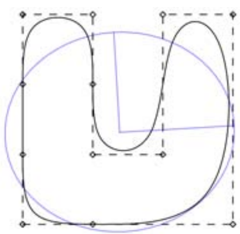
Centroid= $\begin{pmatrix} \frac{2261}{1570} & \frac{2101}{1570} \end{pmatrix}$

Centroid≈ $(1.4401273885350318471 \;\; 1.3382165605095541401)$

Inertia = $\begin{pmatrix} \frac{10\,013\,307}{1\,758\,400} & \frac{823\,757}{10\,550\,400} \\ \frac{823\,757}{10\,550\,400} & \frac{23\,597\,411}{5\,275\,200} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 5.6945558462238398544 & 0.078078271913861085836 \\ 0.078078271913861085836 & 4.4732732408249924173 \end{pmatrix}$

Cubic B-spline ↓



Area $\frac{293}{48} \quad (\approx 6.1041666666666666667)$

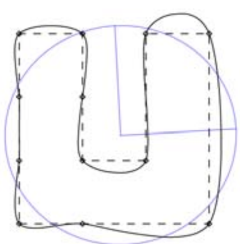
Centroid= $\begin{pmatrix} \frac{382\,973}{276\,885} & \frac{243\,647}{184\,590} \end{pmatrix}$

Centroid≈ $(1.3831482384383408274 \;\; 1.3199360745435830760)$

Inertia = $\begin{pmatrix} \frac{5\,444\,786\,198\,543}{1\,105\,236\,316\,800} & \frac{50\,678\,792\,867}{736\,824\,211\,200} \\ \frac{50\,678\,792\,867}{736\,824\,211\,200} & \frac{41\,852\,421\,617}{11\,164\,003\,200} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 4.9263547675553543664 & 0.068780032057393935972 \\ 0.068780032057393935972 & 3.7488722340208573211 \end{pmatrix}$

C^1 Four-Point ω=1/16 ↓



Area $\frac{1\,063\,193}{133\,056} \quad (\approx 7.9905678811928811929)$

Centroid= $\begin{pmatrix} \frac{412\,867\,896\,656\,157\,348\,923}{257\,697\,398\,751\,797\,512\,800} & \frac{720\,224\,570\,454\,200\,798\,299}{515\,394\,797\,503\,595\,025\,600} \end{pmatrix}$

Centroid≈ $(1.6021422748384551851 \;\; 1.3974230511109825990)$

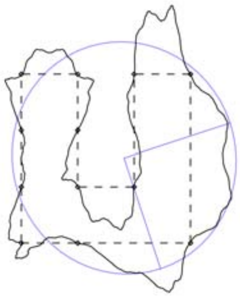
Inertia = $\begin{pmatrix} \frac{398\,116\,069\,029\,356\,818\,404\,123\,802\,723\,767\,844\,491\,757\,011\,170\,194\,856\,506\,058\,957\,892\,714\,163}{47\,977\,735\,174\,551\,332\,336\,020\,001\,196\,360\,176\,617\,380\,254\,472\,439\,720\,648\,001\,807\,216\,640\,000} & \frac{37\,33}{831\,61} \\ \frac{37\,331\,984\,531\,379\,784\,584\,533\,963\,303\,409\,571\,975\,227\,176\,209\,542\,711\,341\,029\,838\,061\,782\,233}{831\,614\,076\,358\,889\,760\,491\,013\,354\,070\,243\,061\,367\,924\,410\,855\,621\,824\,565\,364\,658\,421\,760\,000} & \frac{37\,175\,9}{4\,989\,6} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 8.2979337724246762127 & 0.044890996428094208483 \\ 0.044890996428094208483 & 7.4505679614699399214 \end{pmatrix}$

The obscured 2nd entry along the diagonal of the inertia matrix is

$$\frac{37\,175\,983\,161\,761\,761\,444\,322\,119\,853\,595\,111\,432\,975\,415\,218\,837\,047\,773\,147\,894\,340\,666\,714\,417}{4\,989\,684\,458\,153\,338\,562\,946\,080\,124\,421\,458\,368\,207\,546\,465\,133\,730\,947\,392\,187\,950\,530\,560\,000}.$$

C^1 Four-Point ω=0.192729… ↓



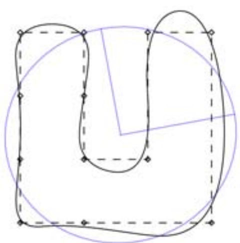
Area 10.2258 (≈ 10.2258)

Centroid≈ (1.82232 1.51285)

Inertia ≈

$$\begin{pmatrix} 12.7238 & -0.372037 \\ -0.372037 & 13.7363 \end{pmatrix}$$

C^2 Four-Point ω=1/128 ↓



Area $\frac{51\,484\,491\,455\,021}{6\,539\,690\,049\,460}$ (≈ 7.8726195072918195158)

Centroid= ($\frac{222\,840\,653\,204\,984\,960\,598\,307\,199\,540\,645\,452\,699\,630\,123}{141\,503\,661\,457\,704\,814\,986\,413\,509\,506\,082\,937\,293\,977\,912}$ $\frac{1\,566\,370\,393\,661\,265\,123\,659\,659\,877\,792\,64}{1\,132\,029\,291\,661\,638\,519\,891\,308\,076\,048\,66}$

Centroid≈ (1.5748048559972536353 1.3836836247956827277)

Inertia = ($\frac{349\,618\,557\,414\,575\,767\,366\,524\,430\,756\,442\,691\,260\,136\,663\,375\,347\,375\,931\,547\,134\,221\,966\,820\,207\,768\,21}{43\,739\,513\,265\,536\,877\,745\,815\,117\,890\,219\,965\,194\,899\,901\,985\,228\,644\,865\,730\,609\,665\,926\,319\,981\,463\,905}$

$\frac{5\,132\,961\,835\,886\,174\,875\,808\,724\,590\,596\,679\,618\,807\,039\,267\,110\,699\,495\,020\,386\,491\,553\,200\,161\,308\,407}{29\,159\,675\,510\,357\,918\,497\,210\,078\,593\,479\,976\,796\,599\,934\,656\,819\,096\,577\,153\,739\,777\,284\,213\,320\,975\,937}$

Inertia ≈

$$\begin{pmatrix} 7.9931972560391133022 & 0.17602945663997104875 \\ 0.17602945663997104875 & 7.0657275161937615431 \end{pmatrix}$$

The obscured *y*-coordinate of the centroid is restated in the following as

$$\frac{1\,566\,370\,393\,661\,265\,123\,659\,659\,877\,792\,646\,163\,614\,634\,589}{1\,132\,029\,291\,661\,638\,519\,891\,308\,076\,048\,663\,498\,351\,823\,296}.$$

The first entry of the inertia tensor is

349 618 557 414 575 767 366 524 430 756 442 691 260 136 663 375 347 375 931 547 134 221 966 820 207 768 211 893 717 603 941 825 059 293 181 924 236-
501 515 634 095 654 754 477 972 825 505 493 855 444 003 517 367 856 964 005 263 962 272 023 443 157 /
43 739 513 265 536 877 745 815 117 890 219 965 194 899 901 985 228 644 865 730 609 665 926 319 981 463 905 567 601 623 750 262 164 532 992 135 504-
784 186 018 341 935 274 215 472 647 107 120 641 400 005 960 807 055 962 957 456 386 281 411 414 700

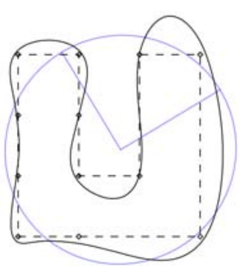
The off-diagonal value of the inertia matrix is

5 132 961 835 886 174 875 808 724 590 596 679 618 807 039 267 110 699 495 020 386 491 553 200 161 308 407 092 217 340 956 371 423 616 134 730 163 573-
134 509 464 928 758 922 090 948 921 882 783 219 543 633 122 850 712 405 306 063 660 560 076 539 /
29 159 675 510 357 918 497 210 078 593 479 976 796 599 934 656 819 096 577 153 739 777 284 213 320 975 937 045 067 749 166 841 443 021 994 757 003-
189 457 345 561 290 182 810 315 098 071 413 760 933 337 307 204 703 975 304 970 924 187 607 609 800

The 2nd entry on the diagonal of the inertia tensor is

309 051 482 425 225 967 186 753 942 213 948 464 797 094 125 184 681 845 112 975 496 293 652 471 233 221 992 238 036 106 526 201 890 255 931 607 323-
266 022 530 890 125 009 918 485 669 462 353 622 952 244 262 369 610 049 122 394 046 831 672 627 469 /
43 739 513 265 536 877 745 815 117 890 219 965 194 899 901 985 228 644 865 730 609 665 926 319 981 463 905 567 601 623 750 262 164 532 992 135 504-
784 186 018 341 935 274 215 472 647 107 120 641 400 005 960 807 055 962 957 456 386 281 411 414 700

C^2 Four-Point ω=0.013723… (Tightest) ↓



Area 9.09471 (≈ 9.09471)

Centroid≈ (1.68885 1.43218)

Inertia ≈

$$\begin{pmatrix} 10.2341 & 0.167818 \\ 0.167818 & 10.0566 \end{pmatrix}$$

Letter D

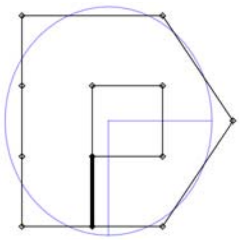
The symmetry of the otherwise symmetric shape D is broken deliberately by inserting additional control points to make the outcome more interesting, and challenging to compute.

Any self-overlapping region contributes double, because the winding number inside these areas is 2.

Curve coordinates ↓

$$\begin{pmatrix} 0 & 1 & 1 & 2 & 2 & 1 & 1 & 2 & 3 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 2 & 1 & 1 & 0 & 0 & \frac{3}{2} & 3 & 3 & 2 & 1 \end{pmatrix}$$

Linear B-spline ↓



Area $\frac{13}{2}$ $(\approx\ 6.5000000000000000000\ )$

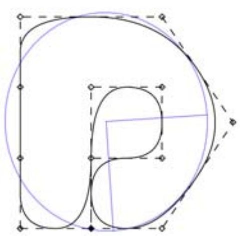
Centroid= $\begin{pmatrix} \frac{16}{13} & \frac{3}{2} \end{pmatrix}$

Centroid≈ $(\, 1.2307692307692307692 \;\; 1.5000000000000000000 \,)$

Inertia = $\begin{pmatrix} \frac{635}{156} & 0 \\ 0 & \frac{239}{48} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 4.0705128205128205128 & 0 \\ 0 & 4.9791666666666666667 \end{pmatrix}$

Quadratic B-spline ↓



Area $\frac{97}{16}$ $(\approx\ 6.0625000000000000000\ )$

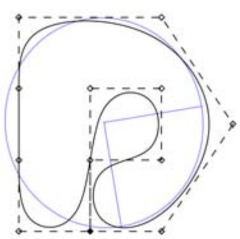
Centroid= $\begin{pmatrix} \frac{588}{485} & \frac{8801}{5820} \end{pmatrix}$

Centroid≈ $(\, 1.2123711340206185567 \;\; 1.5121993127147766323 \,)$

Inertia = $\begin{pmatrix} \frac{7\,695\,031}{2\,172\,800} & -\frac{1\,066\,703}{26\,073\,600} \\ -\frac{1\,066\,703}{26\,073\,600} & \frac{46\,525\,183}{11\,174\,400} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 3.5415275220913107511 & -0.0409112282155130093 27 \\ -0.040911228215513009327 & 4.1635508841638029782 \end{pmatrix}$

Cubic B-spline ↓



Area $\frac{8123}{1440}$ $(\approx\ 5.6409722222222222222\ )$

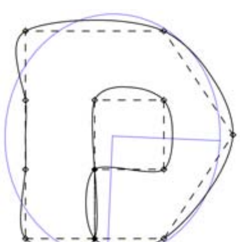
Centroid= $\begin{pmatrix} \frac{116\,437}{97\,476} & \frac{6\,230\,683}{4\,093\,992} \end{pmatrix}$

Centroid≈ $(\, 1.1945196766383520046 \;\; 1.5219089338718785967 \,)$

Inertia = $\begin{pmatrix} \frac{80\,317\,569\,839}{25\,939\,533\,312} & -\frac{25\,457\,520\,163}{389\,092\,999\,680} \\ -\frac{25\,457\,520\,163}{389\,092\,999\,680} & \frac{113\,727\,245\,285\,989}{32\,683\,811\,973\,120} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 3.0963382753630320980 & -0.065427854481928262483 \\ -0.065427854481928262483 & 3.4796199837253128602 \end{pmatrix}$

C^1 Four-Point ω=1/16 ↓



Area $\frac{3\,295\,183}{443\,520}$ $(\approx\ 7.4296153499278499278\ )$

Centroid= $\begin{pmatrix} \frac{15\,864\,686\,104\,274\,849\,177}{12\,610\,872\,019\,886\,438\,160} & \frac{946\,681\,651\,373\,640\,981\,599}{638\,950\,849\,007\,579\,533\,440} \end{pmatrix}$

Centroid≈ $(\, 1.2580165811894197507 \;\; 1.4816188957946137757 \,)$

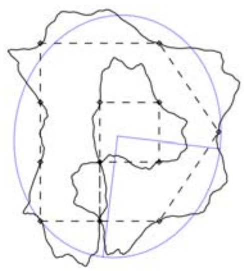
Inertia = $\begin{pmatrix} \frac{5\,119\,570\,881\,316\,376\,495\,134\,772\,393\,219\,119\,571\,246\,714\,354\,845\,017\,257\,106\,656\,498\,413\,223}{980\,638\,226\,812\,736\,897\,685\,221\,616\,789\,466\,994\,709\,668\,756\,047\,878\,282\,149\,877\,792\,768\,000} & \frac{124\,958}{1\,961\,27} \\ \frac{124\,958\,625\,130\,966\,927\,882\,288\,581\,332\,916\,613\,831\,180\,896\,757\,546\,744\,913\,811\,638\,514\,343}{1\,961\,276\,453\,625\,473\,795\,370\,443\,233\,578\,933\,989\,419\,337\,512\,095\,756\,564\,299\,755\,585\,536\,000} & \frac{2\,003\,207}{298\,114\,0} \end{pmatrix}$

Inertia ≈ $\begin{pmatrix} 5.2206519604645312051 & 0.063712907428209547703 \\ 0.063712907428209547703 & 6.7196031139274551357 \end{pmatrix}$

The obscured 2nd entry on the diagonal of the inertia matrix is

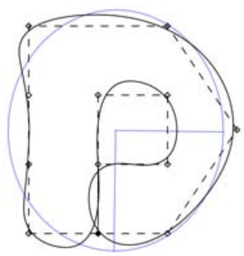
$$\frac{2\,003\,207\,903\,488\,258\,125\,181\,061\,728\,470\,929\,832\,124\,562\,792\,440\,822\,450\,066\,888\,032\,425\,594\,177}{298\,114\,020\,951\,072\,016\,896\,307\,371\,503\,997\,966\,391\,739\,301\,838\,554\,997\,773\,562\,849\,001\,472\,000}.$$

C^1 Four-Point ω=0.192729… ↓



Area	9.42559	(≈ 9.42559)
Centroid≈	(1.29856 1.42927)	
Inertia ≈	(8.48196 0.394433 ; 0.394433 11.7425)	

C^2 Four-Point ω=1/128 ↓



Area $\frac{19\,213\,630\,039\,321}{2\,615\,876\,019\,784}$ (≈ 7.34500790328263 40551)

Centroid= ($\frac{7\,767\,710\,635\,010\,057\,552\,868\,129\,067\,995\,612\,818\,896\,849}{6\,194\,500\,418\,531\,362\,252\,372\,386\,332\,384\,572\,458\,905\,280}$ $\frac{1\,049\,426\,597\,935\,791\,509\,788\,626\,934\,265\,861\ldots}{704\,108\,214\,239\,731\,509\,352\,994\,579\,781\,046\,4\ldots}$

Centroid≈ (1.2539688611162744189 1.4904336815171532621)

Inertia = ($\frac{1\,791\,389\,136\,723\,680\,781\,226\,680\,819\,426\,651\,144\,046\,703\,948\,885\,285\,860\,946\,951\,561\,398\,597\,104\,622\,449\,3\ldots}{351\,037\,885\,383\,366\,517\,314\,997\,234\,149\,245\,960\,790\,429\,060\,922\,900\,160\,719\,502\,093\,486\,428\,041\,510\,389\,9\ldots}$; $\frac{81\,432\,892\,331\,581\,048\,605\,684\,265\,328\,720\,238\,081\,151\,928\,275\,584\,242\,375\,709\,620\,368\,609\,340\,144\,416\,61\ldots}{4\,212\,454\,624\,600\,398\,207\,779\,966\,809\,790\,951\,529\,485\,148\,731\,074\,801\,928\,634\,025\,121\,837\,136\,498\,124\,679\,5\ldots}$

Inertia ≈ (5.1031219458472828180 0.019331458636021731421 ; 0.019331458636021731421 6.4493335709229176875)

The 1st entry of the inertia matrix is

1 791 389 136 723 680 781 226 680 819 426 651 144 046 703 948 885 285 860 946 951 561 398 597 104 622 449 358 211 712 180 167 398 473 118 841 289 211-
000 915 072 810 292 727 757 510 840 877 086 854 612 971 936 808 461 218 836 301 919 123 637 403 /
351 037 885 383 366 517 314 997 234 149 245 960 790 429 060 922 900 160 719 502 093 486 428 041 510 389 964 860 107 508 539 001 725 436 853 793 790-
848 550 326 533 178 648 950 737 049 608 279 458 310 847 893 031 360 180 874 234 436 526 675 800

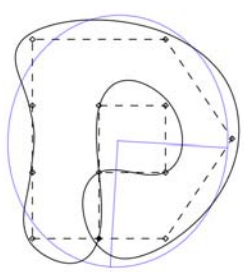
The value outside the diagonal is

81 432 892 331 581 048 605 684 265 328 720 238 081 151 928 275 584 242 375 709 620 368 609 340 144 416 612 018 047 895 501 575 466 329 418 486 192 453-
178 296 628 093 822 537 337 023 658 102 325 228 361 128 392 676 095 115 327 098 594 239 113 /
4 212 454 624 600 398 207 779 966 809 790 951 529 485 148 731 074 801 928 634 025 121 837 136 498 124 679 578 321 290 102 468 020 705 242 245 525 490-
182 603 918 398 143 787 408 844 595 299 353 499 730 174 716 376 322 170 490 813 238 320 109 600

The 2nd entry on the diagonal of the inertia matrix is

842 193 275 819 170 191 210 089 934 503 164 290 224 698 050 925 595 790 191 806 894 858 169 263 098 501 944 729 815 084 782 614 485 131 281 006 821-
596 045 418 419 505 068 860 691 076 410 022 542 498 784 956 735 523 627 222 195 100 808 895 573 969 /
130 586 093 362 612 344 441 178 971 103 519 497 414 039 610 663 318 859 787 654 778 776 951 231 441 865 066 927 959 993 176 508 641 862 509 611 290-
195 660 721 470 342 457 409 674 182 454 279 958 491 635 416 207 665 987 285 215 210 387 923 397 600

C^2 Four-Point ω=0.013723… (Tightest) ↓



Area	8.48031	(≈ 8.48031)
Centroid≈	(1.28296 1.4678)	
Inertia ≈	(6.74801 0.127723 ; 0.127723 8.89297)	

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