

Building a Foolproof Navigation System: Fuzzy Logic Emulating the Brain

Subhajit Ganguly

Email: gangulysubhajit63@gmail.com

© 2013 Subhajit Ganguly

Abstract:

Our aim is to help build a machine that can reduce the possibility of mishaps in navigation to zero. For that devise a new system of numbers, in which the real numbers are represented on the y-axis and complex numbers on the x-axis. Inside such a system, we incorporate the equivalent Ideal Fuzzy Logic that can be used by the machine to predict and avoid mishaps.

Introduction:

Natural processes are vastly emergent phenomena and each new result is always the source of new emergence. To cope with nonlinear control problems, binary logic is no longer sufficient. What we need is an ideal Fuzzy Logic that not only can process complex numbers with utmost efficiency, but also 'thinks' in terms of complex numbers. Such a system also needs to be as simple as possible for us and for machines to work with.

A perfectly efficient navigation system will be able to receive inputs that have all sets of possible values. These values may be real or imaginary. Such a system will be particularly efficient in dealing with imaginary numbers and will be able to reduce the probability of a mishap to zero.

The Fuzzy logic that we have today is yet to incorporate imaginary numbers with the desired level of satisfaction, and many serious problems remain in this.

We develop a new system of numbers that can make use of the equivalent Fuzzy Logic inside it. A machine that runs on such a system will be able to predict and avert mishaps completely.

Let us consider a system of numbers, where all complex numbers are real and all real numbers are complex, i.e.,

$$R \Leftrightarrow C \quad \dots(1),$$

R and C being real numbers and complex numbers, respectively.

In such a system, all individual positive numbers are negative and all individual negative numbers are positive, for complex numbers, while all original real numbers are multiplied by i.

Thus,

$$C=x+iy \Rightarrow C'=-x+\sqrt{1}(-y) = -(x\pm y) \in R \quad \dots(2.1)$$

And

$$R \Rightarrow R'=-iR =-(0+iR) \in C \quad \dots(2.2).$$

Thus, in such a system, we have two lines of numbers:

1. The set of originally complex numbers, transformed into real ones, represented on the x-axis.
2. The set of originally real numbers, transformed into complex ones, represented on the y-axis.

The original system transforms, therefore, into two separate and mutually perpendicular number lines, with individual numbers being points on the corresponding lines.

An Ideal Navigation System:

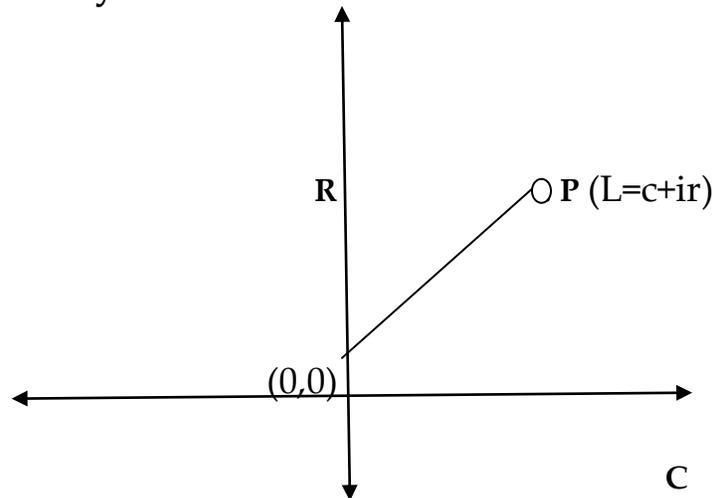


Fig. 1: A simple illustration of signal-processing by a machine running on the new system.

Fig. 1 shows a simple illustration of signal processing by a machine. The set of complex numbers on the x-axis and the set of real numbers on the y-axis give rise to the parameter L for any given point P in the super-imaginary complex plane of decision

making. The parameter L represents both the real and the imaginary parts of the signal and incorporates them into the abstract decision making/perception plane.

In the complex decision/perception plane:

$$L=c+ir \quad \dots(3).$$

c and r being the complex and real coordinates for the point P .

The basic difference between *classical sets* and *fuzzy sets* is that while classical sets allow only a dual degree of membership, a fuzzy set can incorporate any real value between the dual states concerned. A characteristic membership function assigns 0 to an element that is not a member of a given classical set, while it assigns a value of 1 to an element that is a member of that set. The degree of membership to a fuzzy set can take any value in the real unit interval $[0, 1]$.

In our decision/perception plane a fuzzy set L_F may be defined as:

$$L_F: L \rightarrow [0,1], \quad \dots(4).$$

where L is a domain of elements (universe of discourse).

For every particular value of a variable $L_i \in L$ the degree of membership to fuzzy set

L_F is $L_F(L_i)$.

Equation (4) describes how we can incorporate a fuzzy complex number or FCN in our decision/perception plane.

L_F in the universe of discourse L is defined by the complex membership grade function $\mu_{L_F}(L_i)$. The complex membership grade function or CMG is defined as:

$$\mu_{L_F}(L_i) = L_F(L_i)e^{ic} \quad \dots(5).$$

The Cartesian representation of CMG for $\mu_{L_F}(L_i) = \mu_{L_F}(c_i + ir_i)$ is:

$$\mu(c_i, r_i) = \mu(c_i) + ir_i \quad \dots(6)$$

And, the polar representation is:

$$c_i e^{isr} \quad \dots(7),$$

the scaling factor s being in the interval $(0, 2\pi]$.

The degree of fulfillment or DOF of any given proposition follows CMG and lies in the interval $[0, 1]$.

According to the definition of transformation of coordinates:

$$\mu(c_i, r_i) \Leftrightarrow c_i e^{isr}$$

The operators \wedge and \vee defining t-norm and s-norm respectively and L_i being the set of fuzzy numbers concerned, the fuzzy set of a function of L_i has the membership function:

$$\mu(c'_i, r'_i) = \vee_{c'_i=f(L_i)} [\mu(c_1, r_1) \wedge \mu(c_2, r_2) \wedge \mu(c_3, r_3) \dots \wedge \mu(c_n, r_n)] \quad \dots(8).$$

Operators and Functions in the New Logic:

OR:

The maximum s-norm (smax) may be used to calculate the DOF concerned. P and Q being two possible values under consideration,

$$DOF(L = P + Q) = \text{smax}[DOF(P), DOF(Q)] = \max[DOF(P), DOF(Q)] \quad \dots(9).$$

Truth Table

P	Q	max(L)=max(P,Q)
0	0	0
0	1	1
1	0	1
1	1	1

AND:

The minimum t-norm (tmin) may be used to calculate the DOF concerned. P and Q being two Possible values under consideration,

$$DOF(L = P \cdot Q) = \text{tmin}[DOF(P), DOF(Q)] = \min[DOF(P), DOF(Q)] \quad \dots(10).$$

Truth Table

P	Q	min(L)=min(P,Q)
0	0	0
0	1	0
1	0	0
1	1	1

NOT:

The complement or negation is used to calculate the DOF concerned. \bar{L} being the complement of L ,

$$\text{DOF}(\bar{L}) = 1 - \text{DOF}(L) \quad \dots(11).$$

Truth Table

L	\bar{L}
0	1
1	0

De Morgan's Operations:

De Morgan's Involution holds as:

$$\neg(P \wedge Q) = \neg P \vee \neg Q$$

and

$$\neg\neg L = L \quad \dots(12).$$

where \neg is the NOT operator.

Also, De Morgan's laws hold as:

$$\text{NOT}(P \text{ AND } Q) \Leftrightarrow (\text{NOT } P) \text{ OR } (\text{NOT } Q)$$

And

$$\text{NOT}(P \text{ OR } Q) \Leftrightarrow (\text{NOT } P) \text{ AND } (\text{NOT } Q) \quad \dots(13).$$

Now, the relations (13) also translate to:

$$1 - \min[DOF(P), DOF(Q)] \Leftrightarrow \max[1 - DOF(P), 1 - DOF(Q)]$$

And

$$1 - \max[DOF(P), DOF(Q)] \Leftrightarrow \min[1 - DOF(P), 1 - DOF(Q)] \quad \dots(14).$$

Chaotic Fuzziness:

In order to incorporate chaotic instances, the ideal Fuzzy Logic may start with a basic starting measure as the standard reference, from which it will calculate the required differences with respect to other values. It may even take one attractor as its reference. However, the manners of functionality of the fuzzy operations that both will follow will be just the same.

For a given transport of parameterization of the degree of membership between an initial and a final point in consideration, let the trajectory of the initial point of reference $L_o = L(o)$ be denoted by,

$$L(t) = f^t(L_o)$$

Expanding $f^t(L_o + \delta L_o)$ to linear order, the evolution of the distance to a neighbouring trajectory $L_i(t) + \delta L_i(t)$ is given by the Jacobian matrix J ,

$$\delta L_i(t) = \sum_{j=1}^d J^t(L_o)_{ij} \delta L_{oj},$$

$$J^t(L_o)_{ij} = \frac{\delta L_i(t)}{\delta L_{oj}} \quad \dots(15).$$

A trajectory of the shift in degree of membership as moving on the decision/perception plane is specified by two position coordinates and the direction of motion. The Jacobian matrix describes the deformation of an infinitesimal neighbourhood of $L(t)$ along the shift.

Holding the hyperbolicity assumption (i.e., for large n the prefactors a_i , reflecting the overall size of the system, are overwhelmed by the exponential growth of the unstable eigenvalues Λ_i , and may thus be neglected), to be justified, we may replace the magnitude of the area of the i th strip $|B_i|$ by $\frac{1}{|\Lambda_i|}$ and consider the sum,

$$[n = \sum_i^n \frac{1}{|\Lambda_i|};$$

Where the sum goes over all periodic points of period n . We now define a generating function for sums over all periodic orbits of all lengths,

$$[z = \sum_{n=1}^{\infty} [n z^n \quad \dots(16).$$

For large n , the n th level sum tends to the limit $[n \rightarrow e^{-n\gamma}$, so the escape rate γ is determined by the smallest $z = e^\gamma$ for which $[z$ diverges,

$$[z \approx \sum_{n=1}^{\infty} (ze^{-\gamma})^n = \frac{ze^{-\gamma}}{1-ze^{-\gamma}} \quad \dots(17).$$

Making an analogy to the Riemann zeta-function, for periodic orbit cycles,

$$[\zeta = -z \frac{d}{dz} \sum_p \ln(1 - t_p);$$

$[\zeta(z)$ is a logarithmic derivative of the infinite product

$$\frac{1}{\zeta(z)} = \prod_p (1 - t_p), t_p = \frac{z^{n_p}}{|\Lambda_p|}$$

This represents the dynamical zeta function for the escape rate of the trajectories of quantum-transport. The fraction of initial x whose trajectories remain within B at time t may decay exponentially,

$$[t = \frac{\int_s dx dy \delta[y - f^t(x)]}{\int_s dx} \rightarrow e^{-\gamma t} \quad \dots(18).$$

Considering a collection of such points and applying a statistical approach, the logistic equation (due to May, 1967) for L can be written as,

$$L_{t+1} = KL_t[1 - L_t] \quad \dots(19).$$

where K is a constant.

Also, the quadratic map (due to Lorentz, 1987) can be written as:

$$L_{t+1} = K - (L)_t^2 \quad \dots(20).$$

All trajectories described by the quadratic map become asymptotic to $-\infty$ for $K < -0.25$ and $K > 2$.

As we deal with the flow of a given measure towards a given reference, the expression for the attractor for each such point can be written as,

$$L^* = \left(1 - \frac{1}{K}\right) \quad \dots(21).$$

where $0 < K < 4$.

L^* is a point in the desired dimensional plot into which the trajectories seem to crowd. As we do not need to deal with more than one attractor or periodic point, the trajectories will tend to revisit only the attractor point concerned, to the desired level of accuracy of observations and calculations.

In equation (21), for $K \geq 3$, the trajectory behaviour becomes increasingly sensitive to the value of K . There are a few more points to be noted regarding the dependence of the trajectory behaviour on the values of K :

1. For $K \leq 1$, the attractor is a fixed point and has a value 0 .
2. For $1 < K < 3$, the attractor is a fixed point and its value is > 0 but < 0.667 .

3. For $3 \leq K \leq 3.57$, period doubling occurs, with the attractor consisting of 2, 4, 8, etc., periodic points as K increases within that range.
4. For $3.57 < K \leq 4$, we have the region of chaos, where the attractor can be erratic (chaotic with infinitely many points) or stable.

For all calculations, the desired conditions may be placed at the attractor. A trajectory never gets completely and exactly all the way into an attractor though, but only approaches it asymptotically. In the region of chaos, we apply the method of searching for windows or zones of K -values for which iterations from any initial conditions will produce the periodic attractor, instead of a chaotic one. For the logistic equation, the most common such zone lies at $K \approx 3.83$ and for the quadratic map, at $K \approx 1.76$.

Using Lyapunov exponents for the measure L , and replacing $2c \left(\frac{\lambda}{D}\right)$ by a quantity ' τ ',

we have:

$$\frac{d}{d\tau} f^n(L) = \frac{\delta n}{\delta o}$$

i.e.,

$$\frac{\delta n}{\delta o} = \prod_{i=1}^n f'(L_i) \quad \dots(22).$$

$$b = \frac{1}{n} \log_e \left(\frac{\delta n}{\delta o} \right)$$

i.e.,

$$b = \frac{1}{n} \sum_{i=1}^{n-1} \log_e |f'(L_i)| \quad \dots(23).$$

where b is a constant (the local slope of all possible measures), and

$$\Psi = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^{n-1} \log_e |f'(L_i)| \quad \dots(24).$$

where Ψ is a constant.

Let $L_t = c_t + ir_t$ and $L_{t+1} = c_{t+1} + ir_{t+1}$ be corresponding FCN measures with complex membership grade function or CMG as $\mu_{L_F}(L_t)$ and $\mu_{L_F}(L_{t+1})$, respectively. We may then perform the basic arithmetic operations as:

Addition:

$$L_t + L_{t+1} = (c_t + c_{t+1}) + i(r_t + r_{t+1})$$

The corresponding membership function is:

$$\mu_{L_F}(L_t + L_{t+1}) = V_{L_t + L_{t+1}} [\mu_{L_F}(L_t) \wedge \mu_{L_F}(L_{t+1})]$$

Subtraction:

$$L_t - L_{t+1} = (c_t - c_{t+1}) + i(r_t - r_{t+1})$$

The corresponding membership function is:

$$\mu_{L_F}(L_t - L_{t+1}) = V_{L_t - L_{t+1}} [\mu_{L_F}(L_t) \wedge \mu_{L_F}(L_{t+1})]$$

Normalization of L_i :

Considering n number of measures, we have the normalized measure for L_i as:

$$Nor(L_i) = \frac{L_i - \min(L_i)}{\max(L_i) - \min(L_i)} \quad \dots(25).$$

$D(P, Q)$ being the distance measure between two normalized fuzzy sets P and Q , within the measure L_i , the degree of match between them is denoted by:

$$M(P, Q) = 1 - D(P, Q)$$

If, $p \in P$ and $q \in Q$, then the maximum distance between the nearest points in P and Q is the Hausdorff distance between P and Q :

$$H(P, Q) = \max_{p \in P} [\min_{q \in Q} D(p, q)] = \max[\sup_{p \in P} \inf_{q \in Q} D(p, q), \sup_{q \in Q} \inf_{p \in P} D(p, q)],$$

where sup represents the supremum and inf the infimum.

IF-THEN relations may be evaluated using corresponding DOFs. As such relations are of fundamental importance in any logical construct, they are of interest to us here. We may use a weighted scaling measure S to get the DOF of the final result of a given IF-THEN relation. For this, we break the IF-THEN relation into its constituent parts i.e., the condition part (IF) and the result part (THEN). As the IF relation is always of the form *constituent 1 AND constituent 2*, we may write the condition DOF as:

$$DOF_{Condition} = \min[DOF(Constituent 1), DOF(Constituent 2)]$$

The product of the scaling measure S and $DOF_{Condition}$ gives the final result as:

$$DOF_{Result} = S(DOF_{Condition}) \quad \dots(26).$$

The scaling measure S may be taken as the corresponding Hausdorff distance between the constituents.

References

1. A. Connes, Trace formula in noncommutative geometry and the zeros of the Riemann zeta function, *Selecta Math. (NS)* 5 (1999), 29–106.
2. J.B. Conrey, More than two fifths of the zeros of the Riemann zeta function are on the critical line, *J. reine angew. Math.* 399 (1989), 1–26.
3. P. Deligne, La Conjecture de Weil I, *Publications Math. IHES* 43 (1974), 273–308.
4. P. Deligne, La Conjecture de Weil II, *Publications Math. IHES* 52 (1980), 137–252.
5. Browder, Felix, ed. *Mathematical Developments Arising from Hilbert Problems*. American Mathematical Society, 1976.
6. Kantor, Jean-Michel. “Hilbert’s Problems and Their Sequel”, *Mathematical Intelligencer* 18 (1996): 21 – 30.
7. Smale, Stephen. “Mathematical Problems for the Next Century”, *Mathematical Intelligencer* 20 (1998): 7 – 15.

8. Pier, Jean-Paul, ed. *The Development of Mathematics, 1950 – 2000*. Birkhauser, 2000.
9. Arnol'd, Vladimir, Michael Atiyah, Peter Lax, Barry Mazur, eds. *Mathematics Tomorrow*. International Mathematical Union, 2000.
10. C. Deninger, Some analogies between number theory and dynamical systems on foliated spaces, *Proc. Int. Congress Math. Berlin 1998, Vol. I*, 163–186.
11. H.M. Edwards, *Riemann's Zeta Function*, Academic Press, New York - London 1974.
12. S. Haran, Index theory, potential theory, and the Riemann hypothesis, *L-functions and Arithmetic, Durham 1990, LMS Lecture Notes 153 (1991)*, 257–270.
13. G.H. Hardy, *Divergent Series*, Oxford Univ. Press 1949, Ch. II, 23–26.
14. H. Iwaniec and P. Sarnak, *Perspectives on the Analytic Theory of L-Functions*, to appear in proceedings of the conference *Visions 2000*, Tel Aviv.
15. A. Ivíčc , *The Riemann Zeta-Function - The Theory of the Riemann Zeta-Function with Applications*, John Wiley & Sons Inc., New York - Chichester - Brisbane - Toronto - Singapore 1985.
16. N.M. Katz and P. Sarnak, *Random matrices, Frobenius eigenvalues*

and monodromy, Amer. Math. Soc. Coll. Publ. 49, Amer. Math. Soc., Providence RI 1999.

17. E. Landau, Primzahlen, Zwei Bd., IInd ed., with an Appendix by Dr. Paul T. Bateman, Chelsea, New York 1953.

18. N. Levinson, More than one-third of the zeros of the Riemann zetafunction are on $\sigma = 1/2$, Adv. Math. 13 (1974), 383–436.

19. J. van de Lune, J.J. te Riele and D.T. Winter, On the zeros of the Riemann zeta function in the critical strip, IV, Math. of Comp. 46 (1986), 667–681.

20. H.L. Montgomery, Distribution of the Zeros of the Riemann Zeta Function, Proceedings Int. Cong. Math. Vancouver 1974, Vol. I, 379–381.

21. A.M. Odlyzko, Supercomputers and the Riemann zeta function, Supercomputing 89: Supercomputing Structures & Computations, Proc. 4-th Intern. Conf. on Supercomputing, L.P. Kartashev and S.I. Kartashev (eds.), International Supercomputing Institute 1989, 348–352.

22. B. Riemann, Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse, Monat. der Königl. Preuss. Akad. der Wissen. zu Berlin aus der Jahre 1859 (1860), 671–680; also, Gesammelte math. Werke und wissensch. Nachlass, 2. Aufl. 1892, 145–155.

23. Z. Rudnick and P. Sarnak, Zeros of principal L-functions and random matrix theory, *Duke Math. J.* 82 (1996), 269–322.
24. A. Selberg, On the zeros of the zeta-function of Riemann, *Der Kong. Norske Vidensk. Selsk. Forhand.* 15 (1942), 59–62; also, *Collected Papers*, Springer-Verlag, Berlin - Heidelberg - New York 1989, Vol. I, 156–159.
25. F. Severi, Sulla totalit`a delle curve algebriche tracciate sopra una superficie algebrica, *Math. Annalen* 62 (1906), 194–225.
26. C.L. Siegel, “Über Riemanns Nachlaß zur analytischen Zahlentheorie, *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik* 2 (1932), 45–80; also *Gesammelte Abhandlungen*, Springer-Verlag, Berlin - Heidelberg - New York 1966, Bd. I, 275–310.
27. E.C. Titchmarsh, *The Theory of the Riemann Zeta Function*, 2nd ed. revised by R.D. Heath-Brown, Oxford Univ. Press 1986.
28. R. Taylor and A. Wiles, Ring theoretic properties of certain Hecke algebras, *Annals of Math.* 141 (1995), 553–572.
29. A. Weil, *OEuvres Scientifiques—Collected Papers*, corrected 2nd printing, Springer-Verlag, New York - Berlin 1980, Vol. I, 280–298.
30. A. Weil, *Sur les Courbes Algébriques et les Variétés qui s’en déduisent*, Hermann & Cie , Paris 1948.

31. A. Weil, Sur les "formules explicites" de la th eorie des nombres premiers, Meddelanden Fr an Lunds Univ. Mat. Sem. (dedi e `a M. Riesz), (1952), 252-265; also, OEuvres Scientifiques–Collected Papers, corrected 2nd printing, Springer-Verlag, New York - Berlin 1980, Vol. II, 48–61.
32. A. Wiles, Modular elliptic curves and Fermat’s Last Theorem, Annals of Math. 141 (1995), 443–551.
33. Ganguly, Subhajit (2012): Distribution of Prime Numbers, twin Primes and Goldbach Conjecture. **figshare**.
<http://dx.doi.org/10.6084/m9.figshare.91653>
34. Dave Carr, Jeff Shearer, Nonlinear Control and Decision Making Using Fuzzy Logic in Logix.
35. D.E. Tamir, A. Kandel, Axiomatic Theory of Complex Fuzzy Logic and Complex Fuzzy Classes, Int. J. of Computers, Communications & Control, ISSN 1841-9836, E-ISSN 1841-9844 Vol. VI (2011), No. 3 (September), pp. 562-576.
36. Li Renjun, Yuan Shaoqiang, Li Baowen, Fu Weihai, Fuzzy Complex Number.
37. Xin Fu, Qiang Shen, Fuzzy Complex Numbers and their Application for Classifiers Performance Evaluation.
38. Ramot, D., Milo, R., Friedman, M., Kandel, A., Complex fuzzy sets. IEEE Transactions on Fuzzy Systems 2002, 10(2): p. 171-186.
38. Ramot, D., Friedman, M., Langholz, G., Kandel, A., Complex fuzzy logic. IEEE Transactions on Fuzzy Systems, 2003, 11(4): p. 450-461