

On the Confinement of Quarks without Applying the Bag Pressure

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Abstract

We explain herein the fatal error in formulation of strong interaction (quantum chromodynamics). We postulate that quarks are tachyons and do not obey Yang-Mills theory. By applying this correction to the dynamics of quarks, we can confine quarks in hadrons. We seek to show why quarks do not obey the Pauli exclusion principle and why we cannot observe free quarks. In addition, we obtained the correct sizes of hadrons and derive appropriate formulations of strong interaction. Instead of several discrete QCD methods, we derive a united formulation that enables us to solve the strong interaction for all energy values.

1 Introduction

In contrast to the observed spin-statistic behavior of quarks, it is a well-established fact that two electrons with identical quantum numbers cannot exist in a hydrogen atom, because each electron is subluminal and its phase velocity is superluminal. When there are two electrons with identical quantum numbers in a hydrogen atom or with identical energy levels in a cubic box, the second electron exists at every location (space-time coordinates) with exactly identical wave function characteristics to those of the first electron. In other words, the two electrons simultaneously exist at an exact point at the same time. This phenomenon is a consequence of the probabilistic characteristics of wave functions and quantum mechanics. Specifically, the wave equation does not provide us with more information about the exact location of each electron. The energy and absolute value of the momentum of each electron are exactly determined, but the electrons do not have specific locations. At a given time, they are ubiquitous at every location where the wave function does not vanish. However, as we know we can have three identical quarks with identical spin states and quantum numbers in baryons. To explain this phenomenon, we propose a strange theorem:

Theorem. *Quarks are superluminal particles.*

First, let's explain the foundations of quantum mechanics somewhat further. Any specific change in the state of a wave function in its associated Hilbert space will propagate in space-time coordinates with the phase velocity of the wave function in spacetime. Specifically, entangled particles communicate with each other at their entangled phase velocity. We

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postulate that quarks are superluminal. As a result, because each quark is superluminal, its phase velocity must be subluminal; thus, If we change the wave function of the second quark, this change will propagate at less than c to the other space-time locations in the bag. In other words, the first quark is unaware of the spin and characteristics of the second quark, because their phase velocities are subluminal. The phase velocity is not measured in a space-like region and quarks with identical spins can occupy the same energy level in hadrons. Specifically, two quarks with identical energies and momenta are located at different points in the bag. Quantum mechanics postulates that, at a specific time, a subluminal particle with a specific energy-momentum does not have a specific location. In other words, it is ubiquitous in the bag. However, because the phase velocity of a superluminal particle is subluminal, a superluminal particle is no longer ubiquitous. These particles are somehow uncollapsed localized wave functions. Thus, two superluminal particles (quarks) that are confined in a hadron no longer exist at the exact space-time points and obey Fermi-Dirac statistics, so it is not necessary for them to obey the Pauli exclusion principle (they are not ubiquitous). The exclusion principle applies to two identical particles with identical wave function characteristics [1](ubiquitous at some region of the space-time coordinate).

Theoretically, the wave function of a tachyon such as a hypothetical superluminal neutrino cannot collapse, because the phase velocity of collapse is subluminal and obeys causality. Before the wave function collapses, the particle does not have a specific location. We can create its location by performing an experiment and measuring its location. However, after we determine the location of a particle, the particle should not be detected in other locations, even in notably far space-like locations that have no causal relation with the location of the collapsed particle. When ψ_{space} of a subluminal particle collapses, it communicates at its phase velocity (at infinite velocity in the reference frame of the collapsed wave function) to other locations in spacetime that the wave function should not collapse at other locations of the universe. Thus, a particle cannot be detected in two space-like locations, although the two locations do not have a causal relation with each other. However, if the particle is superluminal, its phase velocity is subluminal, and it cannot perform this communication in space-like regions of spacetime. The phase velocity must be superluminal to allow for the collapse of the wave function[2]. Because quarks are superluminal, we never observe free quarks. Note that, although we can identify quarks in hadrons using deep inelastic scattering, before scattering, the wave functions of quarks are confined in hadrons, and it is not necessary for the wave functions to communicate with the entire universe to be able to collapse. The above argument is applicable to free quarks.

Unfortunately the physical concepts and descriptions that we offer above do not create a firm justification for two facts about tachyonic quarks. First, why is it that quarks do not obey the exclusion principle and why have we not yet observed a single free quark? These results must be expressed in the language of mathematics. However, there is seemingly still not a satisfactory quantum field theory for interacting tachyons and we do not know the statistical laws of tachyons similar to Bose-Einstein or Fermi-Dirac, which apply to traditional particles.

2 Tachyonic field theory

The beginnings of tachyonic quantum field theory were introduced in the Feinberg paper in 1967 [3]. Feinberg introduced the term tachyon for particles that move faster than the speed of light. Before special relativity there were some attempts to describe the specifications of

such particles [4, 5, 6, 7]. Cherenkov radiation was one of the predictions of these authors. After the introduction of special relativity, there was no interest in pursuing these attempts and describing particles that were forbidden to exist until 1962, when the first papers to create a relativistic tachyonic equation and the elimination of its philosophical contradictions with special relativity were published [8]. In that era, in addition to theoretical efforts [9, 10, 11, 12], there were also some attempts to detect tachyons by experiment [13, 14, 15]. In 1985, for the first time, Alan Chodos et al. suggested that an electron neutrino was a tachyon [16]. Later, several experiments to prove that a neutrino mass was imaginary were performed. Yet, the important point about all of the positive results in favor of superluminal neutrinos was that all the conclusions were in the domain of experimental error. Thus, their validity could not be verified. In addition, this fact contradicted well-established neutrino oscillation which considered the real mass for neutrinos. After the Chodos paper appeared, a large number of theoretical papers on the subject began to appear, oriented in such a way that they designed an appropriate tachyon field theory which described superluminal neutrinos. These developments accelerated up until 2011 when CERN reported a neutrino anomaly. Immediately a large number of manuscripts in favor and against that idea were published. Yet, it later became evident that the origin of the anomaly was due to an error in experiment [17, 18, 19].

The main problems for constructing an interacting tachyonic field theory are canonical quantization, microcausality, and the spin-statistics theorem. As we know, in order for a microcausality condition to hold for bilinear observables, the field must either commute or anti-commute for a space-like interval. If the Dirac equation is quantized according to commutation relations, the Hamiltonian does not have ground states. If the Klein-Gordon equation is quantized according to anti-commutation relations the microcausality will not be valid for space-like or time-like intervals. To create a tachyonic Klein-Gordon equation or Dirac equation we must quantize the field equation. For preserving scalar field Lorentz invariance, Feinberg assumed a Fermi-Dirac statistic (anti-commutation relation) for the quantization of tachyonic spinless particles! (Ironically quarks which are spin one-half particles do not obey the exclusion principle, but we created the loophole of color to accommodate this fact.) However, this method created a problem whereby the field vacuum state and particle number were not Lorentz invariant [3, 20]. After the first Feinberg paper, it was clear that the fields with imaginary mass led to instability similar to an unstable equilibrium point in classical mechanics and would lead to tachyonic condensation [21].

3 Wave equation of a hydrogen atom with a superluminal electron

There is a significant difference between an ordinary hydrogen atom and a model with a superluminal electron. In the subluminal model, we have negative potential energy. When we increase the energy of the electron in the subluminal model, the momentum of the electron decreases; thus, the wavelength of the electron increases, and the electron increases its distance from the proton. In the subluminal model, although the energy cannot be less than the mass of the particle, the minimum momentum can be zero.

$$E^2 = c^2 P^2 + m^2 c^4 \tag{1}$$

Thus, the wavelength has no maximum, i.e., according to the Wilson-Sommerfeld rule [22, 23] it can approach infinity, which results in the escape of an electron from the hydrogen

atom . The minimum principal quantum number for the minimum radius of the hydrogen atom is $n = 1$.

However, in the superluminal model, although the minimum amount of relativistic energy is zero, the momentum has a non-zero minimum: It cannot be less than the mass of the electron, namely, $m_s c$ [8, 9].

$$c^2 P^2 = E^2 + m_s^2 c^4 \quad (2)$$

$$E = \frac{m_s c^2}{\sqrt{\beta_s^2 - 1}} \quad \beta_s > 1 \quad (3)$$

$$P = \frac{m_s v}{\sqrt{\beta_s^2 - 1}} \quad \beta_s > 1 \quad (4)$$

We see that the electron has a maximum wavelength $\lambda = h/cm_s$. Thus, by the Wilson-Sommerfeld rule, the electron cannot have an infinite wavelength and thus cannot escape the hydrogen atom. This fact sets a limit on the maximum radius of the bag. Thus, the electron in the superluminal model is confined. For the superluminal model, the principal quantum number of the maximum radius of the bag is $n = 1$.

$$\frac{(m_s^2 c^4 + E^2)^{1/2}}{hc} 2\pi r = 1 \quad (5)$$

When the electron energy increases, its momentum increases too, but its wavelength decreases; thus, it becomes increasingly confined. The electron falls deeper into the hydrogen atom or bag, which is in contrast to our observation in the subluminal model.

It is at this point that , we seek to derive and solve the wave function of a confined superluminal electron in the hydrogen bag. First, we study the radial Dirac equation. The Dirac equation for a subluminal particle with real mass leads to the following [24]

$$\hbar c \frac{dg(r)}{dr} + (1 + \kappa) \hbar c \frac{g(r)}{r} - [E + m_o c^2 + \frac{Z\alpha}{r}] f(r) = 0 \quad (6)$$

$$\hbar c \frac{df(r)}{dr} + (1 - \kappa) \hbar c f(r) r + [E - m_o c^2 + \frac{Z\alpha}{r}] g(r) = 0 \quad (7)$$

The normalized solutions are proportional to

$$f(r) \approx -\frac{1}{\Gamma(2\gamma + 1)} (2\lambda r)^{\gamma-1} e^{-\lambda r} \times \left\{ \left(\frac{(n' + \gamma) m_o c^2}{E} - \kappa \right) F(-n', 2\gamma + 1; 2\lambda r) + n' F(1 - n', 2\gamma + 1; 2\lambda r) \right\} \quad (8)$$

$$g(r) \approx \frac{1}{\Gamma(2\gamma + 1)} (2\lambda r)^{\gamma-1} e^{-\lambda r} \times \left\{ \left(\frac{(n' + \gamma) m_o c^2}{E} - \kappa \right) F(-n', 2\gamma + 1; 2\lambda r) - n' F(1 - n', 2\gamma + 1; 2\lambda r) \right\} \quad (9)$$

For normalizable wave functions, γ should be positive. κ is the Dirac quantum number, and

$$\lambda = \frac{(m_o^2 c^4 - E^2)^{1/2}}{\hbar c} \quad (10)$$

$$q = 2\lambda r \quad (11)$$

$$\gamma = +\sqrt{\kappa^2 - (Z\alpha)^2} = +\sqrt{\left(j + \frac{1}{2}\right)^2 - (Z\alpha)^2} \quad (12)$$

To terminate the hypergeometric series, we should discard the negative values of n' :

$$n = n' + |\kappa| = n' + j + \frac{1}{2} \quad n = 1, 2, 3 \quad (13)$$

The solution for the hydrogen atom is a hypergeometric function, which is an associated Laguerre polynomial and is characteristic of a wave function in the Coulomb potential.

$$L_n^m(x) = \frac{(n+m)!}{n!m!} F(-n, m+1, x) \quad (14)$$

where $L_n^m(x)$ is the associated Laguerre function.

To create a superluminal Dirac equation for quarks, we can use imaginary mass or substitute the following matrix $\beta_s = i\beta$ (imaginary mass Dirac equation) to calculate $f(r)$ and $g(r)$.

$$H\psi = c(\alpha.p)\psi + i\beta mc^2\psi \quad (15)$$

However, when we want to construct the Dirac current, we will encounter a problem. The other method is to consider the following non-Hermitian matrices, where $\beta_s = \beta\gamma_5$ [25, 16] (tachyonic Dirac equation)

$$H\psi = c(\alpha.p)\psi + \beta_s m_s c^2 \psi = c(\alpha.p)\psi + \beta\gamma_5 m_s c^2 \psi \quad (16)$$

$$\alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix} \quad \beta_s = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \quad (17)$$

This method satisfies all of the required properties of the superluminal Dirac equation. However, for the sake of simplicity, we mimic the former procedure for the superluminal model with imaginary mass and obtain

$$\hbar c \frac{dg(r)}{dr} + (1 + \kappa)\hbar c \frac{g(r)}{r} - [E + im_\circ c^2 + \frac{Z\alpha}{r}]f(r) = 0 \quad (18)$$

$$\hbar c \frac{df(r)}{dr} + (1 - \kappa)\hbar c f(r)r + [E - im_\circ c^2 + \frac{Z\alpha}{r}]g(r) = 0 \quad (19)$$

We define λ as

$$\lambda = \frac{(m_\circ^2 c^4 + E^2)^{1/2}}{\hbar c} \quad (20)$$

We solve the above equation and exactly mimic the method provided in the reference for the solution of the Coulomb potential [24]. Finally, we obtain

$$g(r) \approx (2\lambda r)^{\gamma-1} e^{-i\lambda r} \times \left\{ \left(\frac{(n' + \gamma)m_\circ c^2}{E} - \kappa \right) F(-n', 2\gamma + 1; 2i\lambda r) - n' F(1 - n', 2\gamma + 1; 2i\lambda r) \right\} \quad (21)$$

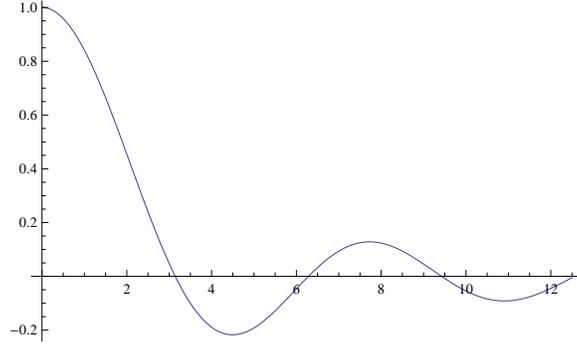


Figure 1: Real part of the function $f(x) = \text{Re}[e^{-ix} F(1, 3, 2ix)]$. The function is exactly similar to a spherical Bessel function of the first type..

$$f(r) \approx -(2\lambda r)^{\gamma-1} e^{-i\lambda r} \times \left\{ \left(\frac{(n' + \gamma)m_0 c^2}{E} - \kappa \right) F(-n', 2\gamma + 1; 2i\lambda r) + n' F(1 - n', 2\gamma + 1; 2i\lambda r) \right\} \quad (22)$$

In the above equations, $F(-n', 2\gamma + 1; 2i\lambda r)$ is normalized for only the negative values of n' if

$$-n' < 2\gamma + 1 \quad (23)$$

For example, for $j = \frac{1}{2}$ (which gives $\gamma = 1$), and $n' = -1$ we have a well-behaved wave function (figure 1). For $-n' = 2\gamma + 1$, the behavior of the wave function $F(-n', 2\gamma + 1; 2i\lambda r)$ is similar to $\cos(r)$. For negative n' , the above hypergeometric equations are similar to the spherical Bessel function of the first type. From (21) and (22), the relation between the hypergeometric series and the Bessel functions is

$$J_\nu(x) = \frac{e^{-ix}}{\nu!} \left(\frac{x}{2}\right)^\nu F\left(\nu + \frac{1}{2}, 2\nu + 1, 2ix\right) \quad (24)$$

The spherical Bessel function of the first type is defined as

$$j_\nu(x) = \sqrt{\frac{\pi}{2x}} J_{\nu+1/2}(x) \quad (25)$$

We observed that the solution for the subluminal hydrogen atom is a Laguerre polynomial. However, we see that $f(r)$ and $g(r)$ for a superluminal electron in the Coulomb potential are similar to the spherical Bessel function of the first type. The spherical Bessel functions appear in only two similar cases. The first case is a particle trapped in an infinite three-dimensional radial well potential. The solutions to this problem are spherical Bessel functions of the first type. Similarly, the solutions to the MIT bag model, which postulated the existence of an unknown pressure and the vanishing of the Dirac current outside the bag, are also spherical Bessel functions of the first type [26, 27].

although we assumed a negative $\frac{\alpha}{r}$ potential, the real shape of the strong interaction is unknown and the other potential will lead to confinement. However, even if (maybe) the force among the particles was repulsive in the above equation or its strength with respect

to distance did not follow a $\frac{1}{r^2}$ law, the factor that determines whether the system is stable and whether the superluminal positron can escape the proton is the energy of the system and not the attractive or repulsive forces among the particles.

it seems from studying the shape of the inter-quark potential that, we can consider the following conjecture:

Conjecture. The strong force is simply the superluminal effect of the electromagnetic force among superluminal particles.

4 Quantum Electrodynamics of Superluminal Particles

In this section, we use a heuristic approach for the calculation of cross sections in strong interactions. Although there is not yet a satisfactory theory for interacting tachyonic field theory, we seek to gain insight and a qualitative, not quantitative, sense of the calculation for the cross section of strong interactions and tachyonic particles.

In the superluminal Klein-Gordon equation, the mass term is imaginary, but all other parameters, including the Klein-Gordon current [$j^\mu = (\rho, j)$], are similar to the subluminal ones. To compute the cross sections in the subluminal Dirac and Klein-Gordon equations, we use the flux relation:

$$F = |v_A - v_B|.2E_A.2E_B = 4(|p_A|E_B + |p_B|E_A = 4((P_A.P_B)^2 - m_A^2 m_B^2)) \quad (26)$$

It can be shown that, if we use the superluminal energy-momentum relation (2) instead of (1), the above flux relation remains valid. Thus, we can conclude that the cross section formulas for superluminal and subluminal particles have similar expressions.

In the center-of-mass frame, the $AB \rightarrow CD$ process for spinless particles, has a differential cross section of

$$\frac{d\sigma}{d\Omega}|_{cm} = \frac{1}{64\pi^2(E_A + E_B)(E_C + E_D)} \frac{p_f}{p_i} |\mathcal{M}|^2 \quad (E_A + E_B = E_C + E_D) \quad (27)$$

where for the amplitude,

$$\mathcal{M} = (ie(p_A + p_C)^\mu) \left(\frac{g_{\mu\nu}}{q^2}\right) (ie(p_B + p_D)^\nu) \quad (28)$$

In the superluminal quark model, if quarks exist at the boundary of the bag, then their speeds will approach infinity, their energies will approach zero, and their momenta will reach the minimum value $m_s c$ (non-relativistic region). In contrast, at the center of the bag, their speeds will approach the speed of light, and their energies and momenta will approach infinity (relativistic region).

In the subluminal model, the energy of the system in the denominator of (27) can never be less than the mass of the interacting particles; thus, the cross section for the minimum initial energy of the interacting particles cannot increase dramatically, but in the superluminal model, if quarks exist at the boundary of the bag (non-relativistic limit and infinite velocity, which in QCD is called a large distance), their cross sections can diverge because the energy in the denominator of the above equation (27) can approach zero. Thus, the cross section diverges at the boundary, and a quark cannot escape from the bag.

From equation (27), for the very-high-energy subluminal spinless electron muon interaction, we have

$$\frac{d\sigma}{d\Omega}|_{cm} = \frac{\alpha^2}{4(E_A + E_B)(E_C + E_D)} \left(\frac{3 + \cos\theta}{1 - \cos\theta}\right)^2 \quad e^- + \mu^- \rightarrow e^- + \mu^- \quad (29)$$

where θ is the scattering angle. To obtain this formula, we neglect the mass and equate the energy and momentum in (28). For the superluminal model, the technique is similar and produces a similar result. Thus, equation (29) is applicable to superluminal spinless particles at very high energies too. With this limit, all interactions between the quarks in hadrons, including QCD and QED interactions, are calculated using one superluminal equation (29), which is also related to the subluminal QED formula. Thus, we falsely conclude that, for short distances, the QCD running coupling constant, which is a function of the energy-momentum of the virtual gluons exchanged between quarks $(p_A - p_C)^2$, disappears. Moreover, the QCD interactions between subluminal particles are negligible, and as a result, we have only the subluminal QED result and not QCD (asymptotic freedom). However, there is no change in the running coupling constant, which can be concluded based on our conjecture.

At this stage, we study the general form of the cross sections of tachyonic spin one-half particles. The tachyonic Dirac equation can be written as

$$H_s\psi = c(\alpha.p)\psi + \beta_s m_s c^2 \psi = c(\alpha.p)\psi + \beta\gamma_5 m_s c^2 \psi \quad (30)$$

or, in its abbreviated form, as

$$(i\gamma^\mu \partial_\mu - \gamma^5 m)\psi(x) = 0 \quad (31)$$

The tachyonic Lagrangian and dirac current are

$$\mathcal{L}_s = i\bar{\psi}\gamma^5\gamma^\mu(\partial_\mu\psi) - m\bar{\psi}\psi \quad (32)$$

$$J^\mu = c(\bar{\psi}\gamma^\mu\gamma^5\psi) \quad (33)$$

and the tachyonic Hamiltonian is,

$$H = H_s + H_I \quad (34)$$

Its interaction Hamiltonian will be

$$H_I = J^\mu A_\mu \quad (35)$$

Because (33) is different from the subluminal current, the cross section will be different. Actually we cannot continue because there does not yet exist a successful tachyonic field theory. Nevertheless, the tachyonic propagator is written as [28]

$$S(p) = \frac{1}{\not{p} - \gamma^5(m + i\epsilon)} = \frac{\not{p} - \gamma^5 m}{p^2 + m^2 + i\epsilon} \quad (36)$$

Therefore, for quark pair production in (e^+e^-) collisions, we have

$$e^+(p) + e^-(p') \rightarrow q^+(k) + q^-(k') \quad (37)$$

Its amplitude will be

$$\mathcal{M} = ie_q e [\bar{u}(k')\gamma_\alpha\gamma_5 v(k)]_{(q)} \frac{1}{(p+p')^2} [\bar{v}(p)\gamma^\alpha u(p')]_{(e)} \quad (38)$$

We have

$$\sum_{spin} [\bar{u}(p')\gamma^\mu v(p)][\bar{u}(p')\gamma^\nu v(p)]^* = 4(p'^\mu p^\nu + p'^\nu p^\mu - (p' \cdot p + m_e^2)g^{\mu\nu}) \quad (39)$$

The following gamma relations are useful:

$$(\gamma^5)^2 = 1 \quad \gamma^{5\dagger} = \gamma^5 \quad \gamma^5 \gamma^\mu = -\gamma^\mu \gamma^5 \quad (40)$$

By using the above gamma relation we obtain:

$$\sum_{spin} [\bar{u}(k')\gamma_\mu \gamma_5 u(k)][\bar{u}(k')\gamma_\nu \gamma_5 u(k)]^* = 4(k'^\mu k^\nu + k'^\nu k^\mu - (k' \cdot k - m_q^2)g_{\mu\nu}) \quad (41)$$

Therefore, the amplitude will be

$$\overline{\mathcal{M}}^2 = \frac{8e^2 e_q^2}{(k - k')^4} [(k' \cdot p')(k \cdot p) + (k' \cdot p)(k \cdot p') + m_e^2 p \cdot p' - m_q^2 k' \cdot k - 2m_e^2 m_q^2] \quad (42)$$

This result can be compared with subluminal electron muon scattering:

$$\overline{\mathcal{M}}^2 = \frac{8e^4}{(k - k')^4} [(k' \cdot p')(k \cdot p) + (k' \cdot p)(k \cdot p') + m_e^2 p \cdot p' + m_\mu^2 k' \cdot k + 2m_e^2 m_\mu^2] \quad (43)$$

If the quark mass is on the order of electron mass at the extreme relativistic limit, we ignore the masses of electrons and quarks, and the cross section will be similar to the electron muon scattering cross section.

$$\frac{d\sigma}{d\Omega} \Big|_{cm} = \frac{\alpha^2 e_q^2}{4(E_A + E_B)(E_C + E_D)e^2} \frac{p_f}{p_i} (1 + \cos^2 \theta) \quad (44)$$

Here, the superluminal model predicts that the total cross section is one third of the value that we obtained from the traditional QCD calculations of electron to quark annihilation, which considers the color factor. The problem can be solved by what we obtain in the next section, i.e., the fact that quarks are more massive than what traditional QCD predicts. If quark mass (up-down) is much greater than electron mass, then in the annihilation of an electron positron to quark-antiquark pair superluminal scattering, we always have $p_f = \sqrt{m_q^2 + E^2} > \sqrt{-m_e^2 + E^2} = p_i$ which increases the differential cross section in (44).

From our previous findings on spin one-half particles, equations (42) and (43), we can deduce one interesting fact when calculating the cross section that is always valid: only the second power of the mass appears in the cross section. Thus, if we use the superluminal Lagrangian for strong interaction calculations to find the net results, we can simply change the sign of the superluminal particle mass $m_q^2 \rightarrow -m_q^2$ that appears in the cross section and use traditional QED calculations and omit the γ^5 terms for the mass in the Dirac tachyonic equations (30) to (32). In other words, because there is no imaginary term in the cross section, we can easily use QED calculations to obtain strong interaction results.

If quarks (up and down) are extremely massive, we cannot easily apply equation (44) as an approximation. the extended result is

$$\frac{d\sigma}{d\Omega} \Big|_{cm} = \frac{\alpha^2 e_q^2}{4(E_A + E_B)(E_C + E_D)e^2} \frac{p_f}{p_i} \left(1 - \frac{m_q^2}{E^2} + \left(1 + \frac{m_q^2}{E^2}\right) \cos^2 \theta\right) \quad (45)$$

The rates of variation of the above differential cross sections with respect to the scattering angle θ , i.e, $1 + \lambda \cos^2 \theta$ are different and must affect experimental results of two jet events. At low energy $E_{cm} \leq 4.8$ Gev there is higher sphericity and less jet like behavior but it seems that λ is very small [29, 30]. At $E_{cm} = 7.4 \frac{GeV}{c^2}$ the observed jet axis indicated $\lambda = 0.45$ and $\lambda = 0.50$ but SLAC-LBL Collaboration used Monte Carlo simulation to get higher values $\lambda = 0.78$, $\lambda = 0.97$ [31, 32]. even small difference of λ from 1 creates great mass. for $\lambda = 0.78$ at $E_{cm} = 7.4$ Gev we obtain $| \langle m_q \rangle | = 1.3$ Gev. Justifying two jet events on the base of perturbative standard QCD for massive quarks contradicts asymptotic freedom, however only measurement of λ at different energies can reveal its true nature. PLUTO Collaboration obtained $\lambda = 0.76$ and $\lambda = 1.63$ at $E_{cm} = 7.7$ and upsilon resonance $E_{cm} = 9.4$ respectively [33]. The rapid change of λ at upsilon resonance indicates that λ is related to (bottom) quark mass in differential cross section resonance. However at $E_{cm} = 13$ and $E_{cm} = 17$ Gev again the value of $\lambda = 1.7$ was suggested by TASSO Collaboration [34]. Finally $\lambda \neq 1$ is observed in drell yan angular distribution too but QCD usually offers more justifications for this anomaly in drell yan process [35].

5 Wilson loop and confinement

The Wilson loop was designed to prove confinement in Yang-Mills theory [36]. Yet, it is still an open question whether the Wilson loop in Yang-Mills theory at a finite distance offers an infinite result. Here we derive the interquark potential and contrast it with the standard QCD model and proof that both Wilson integration and interquark potential and cross section diverge beyond the hadron border. We begin with the Wilson integration

$$\langle e^{-ie_q \oint A_\mu dx^\mu} \rangle = \exp[-e_q^2 \oint dx^\mu \oint dy^\nu \frac{g_{\mu\nu}}{8\pi^2 \epsilon_0 [(x-y)^2 - i\epsilon]}] \quad (46)$$

The exponent can be written as

$$\begin{aligned} & -2e_q^2 \int_{C_2} dx^\mu \int_{C_4} dy^\nu \frac{g_{\mu\nu}}{8\pi^2 \epsilon_0 [(x-y)^2 - i\epsilon]} \\ & -2e_q^2 \int_{C_1} dx^\mu \int_{C_3} dy^\nu \frac{g_{\mu\nu}}{8\pi^2 \epsilon_0 [(x-y)^2 - i\epsilon]} \\ & = -2e_q^2 \int_0^T dx^0 \int_T^0 dy^0 \frac{g_{00}}{8\pi^2 \epsilon_0 [(x^0 - y^0)^2 - r^2 - i\epsilon]} \\ & -2e_q^2 \int_0^R dx^1 \int_R^0 dy^1 \frac{g_{11}}{8\pi^2 \epsilon_0 [(x^0 - y^0)^2 - r^2 - i\epsilon]} \\ & \approx -\frac{e_q^2 T}{4\pi^2 \epsilon_0} \int_{+\infty}^{-\infty} dy^0 \frac{g_{00}}{[(x^0 - y^0)^2 - r^2 - i\epsilon]} \\ & -\frac{e_q^2 R}{c4\pi^2 \epsilon_0} \int_{+\infty}^{-\infty} dy^1 \frac{g_{11}}{[(x^0 - y^0)^2 - r^2 - i\epsilon]} \\ & = i \frac{e_q^2}{4\pi \epsilon_0 R} T - i \frac{e_q^2}{4\pi \epsilon_0 T c^2} R = -iV_s T \end{aligned} \quad (47)$$

where

$$r = (x^1 - y^1) \quad g_{00} = 1 \quad g_{01} = g_{10} = 0 \quad g_{11} = 1 \quad (48)$$

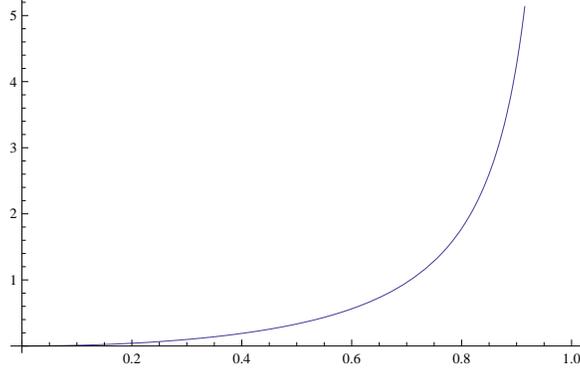


Figure 2: The graph of function $f(x) = \frac{x}{1-x^2}$. The above graph describes the interquark potential, where in approximately 1 fm (proton border), the string tension approaches infinity.

and

$$T\beta_s c = R \quad (49)$$

In addition, from Wilson-Sommerfeld quantization; we have

$$\frac{Rm_s c \beta_s}{\sqrt{\beta_s^2 - 1}} = \hbar \quad (50)$$

so we obtain

$$V_S = \frac{e_q^2}{4\pi\epsilon_0 R} \frac{\left(\frac{Rmc}{\hbar}\right)^2}{1 - \left(\frac{Rmc}{\hbar}\right)^2} \quad (51)$$

The above potential indicates a strong force potential among quarks. At the boundary of the bag or hadron, both the Wilson loop integral and interquark potential diverge because in a very small time period, the quark manages to circumvent the bag and create a completely closed loop in the integration which results in confinement. Thus, quarks must be superluminal, as it is the necessary condition for confinement. The absence of a subluminal model and potential creates a true confinement and a divergence of flux and cross section beyond the hadron surface. From equations (29) or (44) and (5), we can plot the total cross section as a function of the interquark distance

$$\sigma = \frac{\pi}{3} \frac{e^4}{16\pi^2} \frac{R^2}{\hbar^2 c^2 - m^2 c^4 R^2} \quad (52)$$

which indicates that at the center, both potential and cross section vanish contrary to the Cornell potential, which predicts a coulombic potential at a small distance.

$$V_{cornell} = -\frac{e_q^2}{4\pi\epsilon_0 R} + bR + f(R) \quad (53)$$

There are several differences between what we obtained here and what standard QCD predicts. As we know, the QCD coupling constant will predict approximately a Cornell

potential that for a small distance behaves as a coulombic potential, but our graph (Figure 2) is different and the interquark potential will never be zero unless in the center of the bag. In addition, in QCD the flux between quarks remains constant, and at a large distance is not related to the interquark distance; but in our model in around the range of the quark Compton wavelength $\bar{\lambda}_q$, both the string tension and cross section diverge completely and create confinement. If the conjecture that electromagnetic and strong interaction are the same force is true; in the superluminal model, the string tension is a function of the quarks' mass and electric charge and their distance from the center of the hadron; thus, it is different for each hadron. For a lighter quark mass, the string tension would be reduced. Yet, color factor predicts a universal string tension among all types of quarks. Because in our model, the string tension diverges completely at 1 fm, this model predicts confinement. However, in standard QCD at no distance, quarks will be confined completely and the Wilson loop fails to predict the problem of confinement although the problem of quark confinement at finite distance in Yang-Mills theory and resulting mass gap is still an open question [37].

For the hydrogen atom we have a fine structure constant

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \quad (54)$$

The electron reduced Compton wavelength is $\bar{\lambda}_e$ and the electron distance from the nucleus is

$$r_n = n^2 r_o = n^2 \frac{\bar{\lambda}_e}{\alpha} = n^2 \frac{\lambda_e}{2\pi\alpha} \quad (55)$$

where r_o is the Bohr radius. The electron energy is

$$E_n = \frac{m_e c^2 \alpha^2}{2n^2} \quad (56)$$

We want to create a similar equation for quarks. The quark Compton wavelength is λ_q . We know that in nucleons (for up and down quarks) the strong interaction strength is approximately $(\frac{1}{\alpha})$ times greater than the electromagnetic strength so if in equation (51) we choose

$$\frac{V_S(R = R_o)}{V_E} = \frac{1}{\alpha} \quad (57)$$

where

$$V_E = \frac{e_q^2}{4\pi\epsilon_0 R} \quad (58)$$

then we obtain

$$R_o = \frac{1}{\sqrt{1+\alpha}} \bar{\lambda}_q = \frac{1}{\sqrt{1+\alpha}} \frac{\hbar}{m_q c} \quad (59)$$

and R_o is a true hadron boundary. if in (51) the quark moves beyond R_o we have a confinement

$$V_S(R = \bar{\lambda}_q = \frac{\hbar}{m_q c}) = \infty \quad (60)$$

in addition,

$$V_S(R = R_o) = \sqrt{\alpha + 1} m_q c^2 \left(\frac{e_q^2}{e^2} \right) \quad (61)$$

Because a strong force is actually an electromagnetic force between tachyons, we cannot define the V_E between superluminal particles, but we can say

$$V_E(R = R_o) = \alpha\sqrt{\alpha + 1}m_q c^2 \left(\frac{e_q^2}{e^2}\right) \quad (62)$$

and from (5) we have

$$E(R = R_o) = \sqrt{\alpha}m_q c^2 \approx 0.085m_q c^2 \quad (63)$$

Thus, the quark momentum is

$$p(R = R_o) = \sqrt{1 + \alpha}m_q c \quad (64)$$

and its velocity is

$$v(R = R_o) = c\sqrt{1 + \frac{1}{\alpha}} \quad (65)$$

Equation (59) indicates that the radius of the bag is approximately the Compton wavelength of the quark. In addition, (63) indicates that the energy of quarks is very small in comparison with their mass. As well, quarks are very heavy particles. There is a strange point in the above derivation. It seems that in equation (61) the V_S which is the total energy derived from the strong interaction is proportional to the quark momentum $|p|$ (equation(64)) and is much greater than the total energy of the quark (63)

$$V_S(R = R_o) = |p(R = R_o)|c\frac{e_q^2}{e^2} \gg E(R = R_o) \quad (66)$$

It is not clear whether we must consider the mass of the hadron as proportional to its strong interaction potential V_S or the quark total energy E . Yet, if we choose V_S as the main contributor of hadron mass and hadron energy for first quark generation, then we can conclude that the proton energy which creates its gravitational mass is more proportional to the quark momentum (so maybe in (27); $E_A + E_B = |p_C| + |p_D| > E_C + E_D$ which is strange) than the quark energy. Perhaps this fact is the reason for the failure of all the proposed papers on tachyonic field theory. In addition, if we suppose that the mass of the hadrons depends more on V_S , we can see that in a stable hadron like any stable system, the quarks have a very small relativistic energy (63) and the quarks' mass (up and down) is much greater than what we obtained from a non-abelian formulation of quantum chromodynamics. Actually, equation (61) indicates that quark mass is on the order of hadron mass (see (52) too). Moreover cherenkov radiation increases V_S in (51) and seems to be forbidden.

6 Discussion and conclusion

As we know, lattice field theory is the ideal tool for performing precise calculations on a low energy scale in quantum chromodynamics. On the other hand, performing exact calculations with a minimum lattice distance requires supercomputing power, while in the above, we obtained the interquark potential in a simple manner. In fact, for larger couplings, smaller spacing and more powerful computers are needed which means computers never manage to solve QCD at singular values. A theory should be perfect. In other words, intuitively, if a theory is formulated correctly using appropriate and correct assumptions and formulas, that theory should never need the calculational power of such strong machines. If a theory

requires such high-caliber equipment, beyond the scope of paper and pencil, this only means that the formulation of the theory is incorrect and inappropriate. Our formula should provide maximum information but minimum error regarding our system. In other words, if we consider a tachyonic model of strong interactions instead of the traditional non-abelian theory, we obtain the ability to calculate and depict all quark-quark interactions such as interquark potentials with maximum precision. This is similar to the case in quantum electrodynamics where all types of cross sections can be calculated in principle without the help of a computer; thus, we can perform similar calculations for strong interactions.

The following discussion is not related to the subject of this research paper on quantum chromodynamics, yet bears mentioning. Some scientists such as Einstein considered quantum mechanics to be an incomplete and raw picture of physics because it did not provide the exact and maximum information that its predecessor, classical mechanics, could provide [38, 39]. In another example, we would point out the large discrepancy in gravitational theory-between what our theory, i.e., general relativity predicts, and what our data indicates, e.g., in rotation galaxy curves and the cosmological constant problem-which seeks to fix the error by introducing hypothetical yet unobserved phenomena such as dark mass and dark energy.

If what we presented in this paper is valid, an important point regarding strong interaction is the fact that there is no gluon-gluon interaction term and no non-abelian behavior. Thus, strong interaction is a linear theory in principle and the superluminal motion of quarks creates strange specifications of strong force. Therefore, among the three interactions, only gravitation seemingly has a non-abelian behavior and graviton-graviton interaction. Actually, in any theory, a non-abelian characteristic creates a singularity in the theory for which we cannot perform calculations at the singular point. The singularity in QCD is the low energy region of interaction. If we consider the superluminal model, we are able to predict all phenomena at the singular point. A similar point in Einstein's field equations is that they are nonlinear partial differential equations too. Non-linear partial differential equations have non-exact solutions. Furthermore, the theory has singularity in the Schwarzschild radius and we hope the theory of quantum gravity provides a solution at this scale. Thus, we expect that a theory of quantum gravity must probably offer both exact solutions to the field equations and enable us to either eliminate singularity completely or have the ability to obtain results at any precision in singular points. Here, another question arises. If any inappropriate formulation of the system creates a singularity, is the singularity created by the Schwarzschild radius the result of an inappropriate formulation of gravitational theory? If we consider quantum mechanics as an inappropriate formulation of reality, is the uncertainty principle its singularity? Must all singular points be, in principle, calculable at any given precision? In addition, is there a singularity in physics or are all of the singularities the results of our inappropriate formulations?

Theory	Incalculable singularity
QCD SU(3)	Low energy limit
General Relativity	Schwarzschild radius
Quantum mechanics	Uncertainty principle

At this point, we consider other facts regarding the immature and raw formulation of tachyonic dynamics. Unlike the process in the Higgs mechanism that creates real mass from the positive unstable potential term in the Lagrangian, tachyonic condensation is not observed in hadrons and hadrons are stable composites. Unlike reasonable predictions [40, 41],

no Cherenkov radiation is observed in hadrons either. Quarks do not obey the Pauli exclusion principle, which is a fact that is not predicted in the tachyonic field theory of spin one-half particles. Generally speaking, superluminal particles do not obey the traditional laws of quantum mechanics. We do not know why the electromagnetic field among superluminal particles is always attractive and why the net charge of hadrons must be an integer. Tachyonic field theory must explain why we do not have a single tachyon. Quantum field theory for superluminal particles needs significant review. It seems that applying Feynman-Stueckelberg interpretation or Feinberg reinterpretation principle contradict experimental results $\lambda < 1$ of two jet event. An appropriate tachyonic field theory must explain the observed phenomena in hadrons not neutrinos.

In spite of the troublesome nature of tachyonic field theory, the superluminal model offers a united model for strong interaction, but the SU(3) model does not have this advantage. In other words in standard QCD, we have different models to justify a specific result and usually each model offers appropriate predictions for a specific spectrum of experimental results; and in the range of energies where we can simultaneously use other models, we usually face contradictory results which means that the foundations of our models are inappropriate. For example, we know that the QCD summation rule is based on non abelian theory. In addition, we know that the MIT bag model is created on the assumptions of both the subluminality of Dirac current and the confinement of this current. Yet, on the other hand, the predicted results and parameters, such as vacuum energy and pressure of hadrons from these two theories, are contradictory to each other. In addition, due to the fact that rigid boundary condition can lead to spurious quark motions, the MIT bag model is not Lorentz invariant[42]. Other models such as Skyrminion model and Chiral bag model are not ideal models. We offer different models at different energies and seek to close the gap between them, and we are faced with contradiction. The reason is that no subluminal model can create true confinement and an appropriate model of strong interaction. Thus, we logically face several errors in the results derived from these two different methods designed to solve the system. There are other contradictions in the SU(3) subluminal model. One of the greatest questions about the SU(3) model is that if this model is the true model of confinement, why has no glue-ball yet been observed? However, we have achieved energies in the range of a hypothetical glue ball [43, 44]. Thus, although the SU(3) model offers some approximations of true strong interactions and mimics the superluminal model, there is no strong experimental evidence to confirm the color concept.

As another sign in favor of the superluminal model for quarks, we can consider the great difference between the mass of vector mesons and of pseudoscalar mesons. For instance $\pi^+(u\bar{d}) = 140$, $\rho^+(u\bar{d}) = 775$, $K^+(u\bar{s}) = 493$, $K^{*+}(u\bar{s}) = 892 \frac{MeV}{c^2}$. In the subluminal model, spin interaction (fine structure splitting) can be considered a perturbation (order of α^2) to the principal Hamiltonian (because electron speed is of order $\frac{v}{c} = \alpha$). Yet the fact that the mass of vector mesons is much greater than that of the pseudoscalar mesons with the same quark contents can be justified only by the superluminal motion of quarks and its effect on the quarks' spin-spin interaction. In fact spin-spin coupling energy is of order of strong interaction and greater than electromagnetic potential. Logically, the Thomas precession and subluminal motion of electric charges cannot account for such a great Hamiltonian and the energy difference due to the different quark's spin state in vector and pseudoscalar mesons. Note that for great quark masses which can be concluded from angular distribution of two jet event, Chromomagnetic Mass Splitting fails to predict meson mass gap.

On the other hand for next generation of quarks, the ratio of mass gap to meson mass

among pseudoscalar and vector meson is very smaller (for instance $B_s^0(s\bar{b}) = 5366$, $B_s^{*0}(s\bar{b}) = 5415$, $\frac{\Delta E}{E} = 0.01$). Maybe this means that the speed of next generation of quarks in the bag is considerably different from first generation. Does jump in the angular distribution at upsilon resonance indicates that bottom quark has a tiny mass in contrast to upsilon mass? If that guess is true heavy mesons made of next generation quarks have more energetic quarks (thus less stable system apart from effects of CKM matrix) and V_S is not the main indicator of their mass. It seems that what in non abelian model is called QCD binding energy or gluon binding energy is actually Wilson loop potential V_S in the superluminal model and what QCD considers as the mass of quarks is actually relativistic energy of quarks E in the superluminal model (equation (5)).

There is a similar question about meson mass; if quark mass up and down are negligible why we must observe different masses among chiral partners? For example $J^{PC} = 1^{--}$ has mass of $m_\rho=770$ Mev while 1^{++} has mass of $m_{a_1}=1260$ Mev [45]. In the superluminal model a_1 has angular momentum of 1 while ρ has angular momentum of 0 and this fact necessarily must affect mass of hadrons composed of tachyonic quarks. Is chiral symmetry breaking a good excuse for explaining huge hadron mass composed of up and down quark?

Besides intuition, the key, finally, for determining if really strong interaction is based on the superluminal model or a non-abelian model must be determined from experiment. The fact that first generation quark mass is imaginary and is much greater than what the SU(3) model predicts (and maybe the mass of the hadron depending on the momentum of the quarks and not their energy) must be detectable in all types of scatterings, decays and cross section formulations (provided we develop a successful interacting tachyonic field theory). The SU(3) model predicts that up and down quark masses are in the range of electron mass while the superluminal model predicts that the mass of up and down quarks is on the order of hadron mass. If further experiments on cross sections prove that SU(3) is inappropriate model for strong interaction, it would seem that we must review other concepts such as the strong CP problem, axions, and other concepts that grand unified theories predict such as proton mass decay. up to this time, no direct evidence for axion and proton mass decay has been observed and possible detections rely on interpreting astronomical observations[46, 47, 48].

If someday there are experimental proofs that quarks are tachyons; I would proffer this quote by George Bernard Shaw "Science never solves a problem without creating ten more".

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