# Electro-Gravity Via Geometric Chronon Field

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**THANKS TO THE PROFESSIONAL PEER REVIEW PROCESS THAT HAD SIGNIFICANTLY IMPROVED THE PAPER. INCLUDED: SMALL CORRECTION AND REMARK ON THE MAJORANA CHARGELESS FIELD**

## ABSTRACT

**Aim:** To develop a model of matter that will account for electro-gravity.
Interacting particles with non-gravitational fields, can be seen as clocks whose trajectory is not Minkowsky geodesic. A field in which a small enough clock is not geodesic, can be described by a scalar field of time whose gradient has non-zero curvature. The scalar field is either real which describes acceleration of neutral clocks made of charged matter or imaginary, which describes acceleration of clocks made of Majorana type matter. This way the scalar field adds information to space-time, which is not anticipated by the metric tensor alone. The scalar field can't be realized as a coordinate because it can be measured from a reference sub-manifold along different curves. In a "Big Bang" manifold, the field is simply an upper limit on measurable time by interacting clocks, backwards from each event to the big bang singularity as a limit only. In De Sitter / Anti De Sitter space-time, reference sub-manifolds from which such time is measured along integral curves, are described as all events in which the scalar field is zero. The solution need not be unique but the representation of the acceleration field by an anti-symmetric matrix, is unique up to SU(2) x U(1) degrees of freedom. Matter in Einstein Grossmann equation is replaced by the action of the acceleration field, i.e. by a geometric action which is not anticipated by the metric alone. This idea leads to a new formalism of matter that replaces the conventional stress-energy-momentum-tensor. The formalism will be mainly developed for classical but also for quantum physics. The result is that a positive charge manifests small attracting gravity and a stronger but small repelling acceleration field that repels even uncharged particles that measure proper time, i.e. have rest mass. Negative charge, manifests a repelling anti-gravity but also a stronger acceleration field that attracts even uncharged particles that measure proper time, i.e. have rest mass.

*Keywords: Time; general relativity; electro-gravity; reeb class; godbillon-Vey class.*

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## 1. INTRODUCTION

The motivation of this theory is to show matter is an acceleration field i.e. prohibition of inertial motion/free-fall of every small particle that can measure proper time. The scalar field is either real which describes acceleration of neutral clocks made of charged matter or imaginary, or partially imaginary with some condition which describes acceleration of clocks made of Majorana type matter.

Non-geodesic motion as a result of interaction with a field, is not a geodesic motion in a gravitational field, i.e. it is not free fall. Moreover, material fields by this interpretation prohibit geodesic motion curves of particles moving at speeds less than the speed of light and by this, reduce the measurement of proper time! The expression of such a field is thus by a field of time that is not part of the geometry of space-time but rather adds new information to the space-time manifold by expressing how material force fields bend the curve of a small enough clock. This bending is not anticipated by general relativity but can account for a complementary field to the gravitational field and it offers a new expression of matter. In this paper, the author replaces the notion of a force field by an acceleration field. In fact, the covariant description of such an acceleration field is by an anti-symmetric matrix that is multiplied by the velocity vector in order to point to a perpendicular direction i.e. to an acceleration 4-vector. A 4-vector of acceleration is only a representative of such a field, which as we shall see, leaves an SU(2)xU(1) degrees of freedom in the definition of the matrix. An acceleration can be seen as curvature of a particle's trajectory, however, we seek a formalism that will be independent of any specific trajectory. This goal can be achieved by an introduction of a very special scalar field, namely, a field of time. By the principles of General Relativity, no coordinate of time should be preferable and therefore any such scalar field should not lead to a realizable preferable coordinate of time. Also, because more than one curve can measure the same maximal proper time between a predefined reference sub-manifold and an event, a definition of maximal time may not lead to unique integral curves. The theory that will be presented is a geometric interpretation of Sam Vaknin's Chronon Theory [1] by a Godbillon-Vey [2],[3] curvature vector / Reeb vector. This vector is a part of the Godbillon-Vey 3rd order form and it belongs to the De Rham Cohomology. Locally, it offers foliations which their tangent bundle is the kernel of a 1-Form, namely the gradient of a scalar field of time.

By the principle of parsimony, even the maximum time requirement is not necessary if the scalar field that defines proper time along curves, is emergent out of a mathematical formalism. Here we should introduce an equivalence relation.

### 1.1 Avoidance of Closed Time-like Curves

In this paper, the curves along which maximal proper time is measured do not contain backward travel in time. If two events $e_1$ and $e_2$ are on a curve such that $e_2$ is in the future relative to $e_1$ then no geodesic exists between $e_2$ and $e_1$ such that $e_1$ is in the future in relation to $e_2$, otherwise the curve is excluded from the calculation of maximal time.

The equations of the presented theory may render this definition redundant.

### 1.2 Geodesic Equivalence between Events

Given a sub-manifold $M_1$ in $M$ . Events in a set of events $M_2$ are equivalent, if for each event $e_{M2}$ in $M_2$ there exists an event $e_{M1}$ in $M_1$ such that the maximal proper time $\tau_{Max}$ measured along curves that connect $e_{M2}$ to events in $M_1$ is the same for each other event in $M_2$, i.e. If the maximal proper time from event $\tilde{e}_{M2}$ to $M_1$ is to event $\tilde{e}_{M1} \in M_1$ then the curve length in proper timer is also $\tau_{Max}$. The main interest of this paper is that for each $\tau_{Max}$, there exists a class $M_2$ such that $e_{M2} \to e_{M1}$ is a surjective map. $\tau_{Max}$ is subject to acceleration fields, i.e. to fields that prevent any small test particle from moving along geodesic curves. $M_1$ may not be unique and all such $M_1$ sub-manifolds of $M$ should be dictated by a physical law, i.e. a minimum action principle.
Before we continue, we have to define the "big bang" as it embodies the most simple case of $M_1$. Please note that $M_2$ may not be a neat

geometric object as a 3D sub-manifold. It can be a countable unification of such sub-manifolds.

**Big Bang:** *The Big Bang in this paper is a presumed event or manifold of events, such that if looking backwards from any clock - "Test Particle", at an event 'e' - that measures the maximum possible time up to 'e' then such clock must have started the measurement from the big bang as a limit. The Big Bang synchronizes all possible such clocks that measure the maximal time to any event from the past.* The definition allows prior time even before such a manifold of events but requires that clocks will be synchronized on the "big bang" manifold or that the synchronization will be done by measuring the limit of time backwards to a "big bang" singularity.

The idea of a test particle measuring time and even transferring time is not new, thanks to Sam Vaknin's dissertation from 1982 in which he introduced the Chronon field [1] in an amendment to Dirac's equation. Instead of defining the maximal time backwards to a single event, the definition can be to a sub-manifold, but to which sub-manifold and what if small test particles are not allowed to move along geodesic curves due to interactions with force fields ?

The physical law that we reach will have to solve that problem too.

An earlier incomplete paper of the author about this inertial motion prohibition in material fields, can be found [4].

In general, the Euler number of the gradient of the time field may not be zero [5]. To avoid such singularities, this paper harnesses some of the results known to belong to Lars Hormander by using Distribution Theory [6]. If our time field is $\tau$, we may consider a new scalar field $P=\sqrt{\tau}\psi$ such that $\psi$ is complex. $PP^{*}=\tau\psi\psi^{*}$ avoids gradient field singularities where $\psi\psi^{*}$ becomes zero if the derivatives of $\tau$ are discontinuous.

If test particles are forced to move along non-geodesic curves, i.e. experience trajectory curvature due to an external field, a mathematical formalism of such curvature will have to be developed and will replace matter in Einstein-Grossmann's field equations, as such curvature fields become a new description of matter. It is quite known that acceleration can be seen as a curvature and therefore acceleration field is another interpretation of a curvature vector perpendicular to 4-velocity [7] though it leaves SU(2)xU(1) degrees of freedom. An acceleration field that acts on any particle, can't be expressed as a 4-vector because a 4-vector does depend on a specific trajectory and by Tzvi Scarr and Yaakov Friedman, such a field is expressible by an anti-symmetric matrix $A_{\mu\nu}=-A_{\nu\mu}$ such that if $V_{\mu}$ is the normalized 4-velocity and $V_{\mu}V^{\mu}=1$ then the 4-acceleration is actually $a_{\nu}/C^{2}=A_{\mu\nu}V^{\mu}$ such that $C$ is the speed of light.

## 2. THE CLASSICAL NON-RELATIVISTIC LIMIT – MASS AT REST IN A GRAVITATIONAL FIELD

### 2.1 Definitions

**Gravity:** *Gravity is the phenomenon that causes all forms of energy to be inertial if and only if they freely fall, including a projectile that starts upwards. All forms of energy including light appear to accelerate towards the source of gravity. Gravity is seen as a phenomenon that influences the metrics of space-time.*

**Mass Dependent Force:** *A mass dependent force is a presumed force that accelerates any massive object that does not propagate at the speed of light and the force is mass dependent. The mass dependent force does not change the metrics of space-time. i.e. clocks in the field will not tick slower than clocks far from the field unless gravity coincides.*

**An Acceleration Field / Non-Inertial Field:** *An acceleration Field is Mass Dependent Force that is not intrinsic to an object but rather appears as a property of space-time. Unlike gravity, it does not affect photons or any particle that propagates at the speed of light. The idea is that such a field affects measurement of time by prohibition of inertial motion of particles that can measure proper time. Non geodesic curves in Minkowsky space-time measure less time between events*. As was already mentioned, an acceleration field is expressible as $a_{\nu}/c^{2}=A_{\mu\nu}V^{\mu}$ and $A_{\mu\nu}=-A_{\nu\mu}$ is locally similar to the Tzvi Scarr and Yaakov Friedman matrix [7].

**Very important:** The acceleration field need not represent an interaction of a small test particle with the matter in which the field appears. As a property of space-time, it can represent an interaction with the entire space – time.

**Caution:** Although throughout the classical non-relativistic limit sections of this paper, the author uses vectors to describe trajectory dependent representative acceleration vectors, an acceleration field can't be described as a covariant field without the Scarr-Friedman [7] formalism $a_\nu / c^2 = A_{\mu\nu} V^\mu$ and $V_\mu V^\mu = 1$. As an author, it is important to me to make sure that the reader fully understands that classical limit calculations are nothing more than classical limit calculations and the author is fully aware of and does not mix non relativistic and covariant formalisms!

Motivation beyond this section: For the pedant physicist there is no point in presenting a potential intrinsic to a massive object and a non-relativistic potential energy as the classical limit of a covariant theory. To such a reader, the author will say that the purpose of this paper is to replace the conventional energy momentum tensor $T_{\mu\nu}$ – which is part of Einstein-Grossmann's field equation – by a tensor with fully geometric meaning. Recall Einstein-Grossmann's field equations in his writing convention as $\frac{8\pi K}{C^4} T_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ such that $K$ is the gravity constant, $C$, the speed of light, $R_{\mu\nu}$ the Ricci tensor, $g_{\mu\nu}$ the metric tensor, $R = g_{\mu\nu} R^{\mu\nu}$ the Ricci scalar. The replacement will be with a totally geometric tensor and thus will achieve a gravity equation which is geometric on both sides. To give a further clue, the author will say that $T_{\mu\nu}$ will be replaced by a tensor which is the result of a representative acceleration $\frac{a_\lambda}{C^2}$. $\frac{a_\lambda}{C^2}$ seems as a curvature vector of a particle's trajectory with units of 1/length but as such, it is an intrinsic property of the particle and not of a field. So eventually, we will have to derive our curvature vector from the gradient of a scalar field and not from the velocity of any specific particle. Since our new tensor is purely geometric, the constant $\frac{8\pi K}{C^4}$ will be replaced by $1$. To be more precise, the equation will be written as $\sigma = 8\pi$ or another value e.g. $\sigma = 4\pi$ and

$$\frac{8\pi K}{C^4}\left(\frac{a_\mu a_\nu + other\_terms_{\mu\nu}}{\sigma K}\right) = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

and in some special cases, where electric charge is not involved i.e. $Q = 0$, a simple equation will be valid, $-\frac{a_\lambda a^\lambda}{C^4} = -R$. In any case this implies that $\frac{a^2}{\sigma K} = \frac{a_\lambda a^\lambda}{\sigma K}$ can be construed as energy density and hopefully the reader is not annoyed by the sloppy notation $a^2$.

Using a potential field intrinsic to an object instead of correct covariant formalism and using gravitational pseudo-acceleration, the following will shed some light on the general intuition as to the expected relation between energy and acceleration fields although as a physical argument, it is not fully acceptable. We will now consider classical non-relativistic gravity and classical non-relativistic acceleration as qualitative limits that will hint at the relationship between non - inertial and non - geodesic acceleration fields and energy. Estimates will be discussed in the next section. The classical non-relativistic and non-inertial acceleration caused by material fields will be denoted by $a = (a_x, a_y, a_z)$. By the principle of equivalence, we will calculate the integral of the square acceleration of a particle at rest which is accelerated because it is prevented from inertial motion, i.e. from falling in a gravitational field of a ball of mass. We will see how this value is related to classical potential energy. The qualitative hint starts with integration. $K$ is the constant of gravity, $M$ mass, $r$ radii of a hollowed ball quite the way it is done in the electromagnetic theory

$$\iiint_{V=Volume} a^2 dV = \int_0^{r_0} \left(\frac{KM}{r^2}\right)^2 4\pi r^2 dr = K\frac{4\pi KM^2}{r_0} \quad (1)$$

Now we calculate the negative potential energy $-E_g$,

$$\int_0^M \left(\frac{Km}{r_0}\right) dm = \frac{KM^2}{2r_0} = -E_g \quad (2)$$

So from (1) and (2)

$$\frac{1}{8\pi K}\iiint_{V=Volume} a^2 dV = -E_g \quad (3)$$

(3) qualitatively implies the following relation between energy and non-inertial acceleration where $\rho C^2$ is the energy density and $\rho$ is the mass density

$$\frac{a^2}{C^4} = \frac{8\pi K\rho}{C^2} \Rightarrow Energy\_Density = \frac{1}{2}\frac{a^2}{4\pi K} = \frac{a^2}{8\pi K} \quad (4)$$

In special relativity, the square norm of a normalized by $C$, 4-velocity of a particle is constant $N^2 = u_i u^i = 1$ and also $N^2,_k = (u_i u^i),_k = \frac{d(u_i u^i)}{dx^k} = 0$ such that $u^i = \frac{dx^i}{Cd\tau}$ and the normalized by $C$, 4-acceleration is $a^i = \frac{d^2x^i}{C^2 d\tau^2}$ which is 1/length in units which is the curvature of a specific particle's trajectory. If $N^2$ was not the norm of a particle's velocity, we could think of another way to describe acceleration. More or less, that will be the subject of more advanced sections of this paper.

## 3. THE CLASSICAL NON-RELATIVISTIC LIMIT – THE ELECTROSTATIC FIELD

The following uses the standard definitions of electric and electrostatic fields.

What can we say about the density of the electrostatic field? We know it is

$$Energy\_Density = \frac{\varepsilon_0}{2}E^2 \quad (5)$$

such that $\varepsilon_0$ is the permittivity of vacuum and $E$ is the electrostatic field. (4) has a very deep meaning which is, that acceleration of neutral charge-less test particles should appear also within an electric field. That is because acceleration of all small enough rest mass due to the existence of a field, is assumed in this paper to be the reason of all energy densities including the electric field. Prohibition on geodesic Minkowsky motion can be concentrated as an acceleration field in some areas around electric charge and may not be well distributed, however, such argument is less appealing if we consider the principle of parsimony.

$$\frac{a^2}{8\pi K} \approx \frac{\varepsilon_0}{2}E^2 \Rightarrow a \approx \sqrt{4\pi K\varepsilon_0}\left|E\right| \quad (6)$$

Definition: The constant that divides the non-geodesic square acceleration is called "Electro-gravity constant". In this paper we choose as an educated guess $8\pi K$. In general we can write our guess as $\sigma = 8\pi$ so (6) becomes:

$$\frac{a^2}{\sigma K} \approx \frac{\varepsilon_0}{2}E^2 \Rightarrow a \approx \sqrt{\frac{\sigma K\varepsilon_0}{2}}\left|E\right| \quad (7)$$

Other constants yield different theories and have dramatic cosmic consequences. (6) implies a very weak acceleration i.e. mass dependent force on small enough charge-less neutral test particles, about $8.61 cm/\sec^2$ in a field of 1000000 volts over 1 millimeters distance. See Timir Datta et. al. work as an elegant way to focus field lines by metal cone and plane and to observe the effect [8], however, in this paper we shall see that there is another effect due to gravity and therefore the acceleration in (6) is not the only effect that has to be taken into account. The acceleration in (6) exposes non-inertial, non-gravitational acceleration of particles that can measure proper time. On its own, it is not an interesting acceleration but it can explain the electric interaction as repulsive when the integration of the square acceleration increases and attractive when this integration is reduced. The author believes the acceleration of charge-less particles in an electric field is from positive to negative.

In "Electro-gravitational thrust, Dark Matter and Dark Energy" it will be shown that there is an electro-gravitational effect opposite in direction to the acceleration of an uncharged particle in an electro-static field. There is at least informal evidence that the elecro-gravitational effect shows thrust of the entire dipole towards the positive direction [9] and the author does not imply asymmetrical capacitors of 1 - 0.1 Pico-Farad with 45000 Volts. Such capacitors according to the calculations in the section "Electro-gravitational thrust, Dark Matter and

Dark Energy" in this paper, can't manifest any measurable effect of at least 1 micro Newton thrust. Here is a testimony of Hector Luis Serrano, a former NASA physicist in reply to Peter Liddicoat: *"Actually by the generally accepted definition of what constitutes high vacuum 10^-6 Torr is about in the middle. This pressure is about equal to low Earth orbit. More importantly at this pressure the 'Mean Free Path' of the molecules in the chamber is far too great to support Corona/Ion wind effects. We've tested from atmosphere to 10^-7 Torr with no change in performance either. However, I'm glad the results have you thinking. It looks simple, but trust me it's not".*

### 3.1 Serious Experimental Problem – Electron Mobility

The down side of the non-geodesic acceleration is that it is about 10 orders of magnitude smaller than the accepted and known electric field interaction. For example, negative charge suspended above the Earth will cause charge to move in the ground. This charge will have a much stronger effect than the interaction with the acceleration field as is, and will cause a shielding effect i.e. the fields will cancel out within the Earth. Even the almost ideal insulator, i.e. diamond crystals, have impurities such as Nitrogen Vacancies [10] that allow charge carriers to move in the lattice i.e. high electron mobility. In the most pure diamonds the NV impurities are about $10^{18}$ nodes per $cm^{-3}$ comparing to $1.77x10^{23}$ carbon atoms per $cm^{-3}$. The donor electrons lie deep in the band gap of 5.47ev, at about 1.7 ev.

## 4. THE NON-GEODESIC ACCELERATION FIELD

The author's strategy in developing the idea of an acceleration field that is not gravity is as follows:

1) The curvature vector of the gradient of a scalar field will be developed. If the meaning of the scalar field is that it is proper time, measured along different curves from a reference sub-manifold, then non-zero curvature vector means acceleration. The trajectory of any small enough clock that measures non-zero proper time will not be parallel to any geodesic field as it interacts with material fields.
2) A specific 4-vector can't account for an acceleration field because we need a representation that does not depend on any specific velocity 4-vector. Such an acceleration is exactly Zvi Scarr and Yaakov Friedman matrix [7]. A basic singular matrix will be developed. It is singular because 2 vectors are not enough to describe rotation and scaling in Minkowsky space-time. There are still at least two more degrees of freedom and if our curvature vector is complex then there are SU(2)xU(1) degrees of freedom.
3) We develop a non-singular acceleration matrix in which there is SU(2)xU(1) degrees of freedom.
4) We represent the gradient of our scalar field of time by means of the perpendicular foliation and show an additional SU(3) degrees of freedom. Although SU(3)xSU(2)xU(1) is the symmetry group of the Standard Model, it is shown in this paper to be a result of geometry and not of any second quantization technique.
5) We explore more scaling degrees of freedom in the definition of the time field.
6) We use the square norm – the second power of the Minkowsky norm – of the curvature field as an action operator that can replace the material action in Einstein-Grossmann-Hilbert action.
7) We show that the Euler Lagrange equations lead to a new term that expresses the divergence of the curvature field. This divergence is attributed to electric charge because an electric field is a form of energy for which the simplest explanation is a very small acceleration of even uncharged small clocks along electro-static field lines. So electric interaction is predicted as either increasing or decreasing the energy of the acceleration field.
8) The interpretation of charge is of having two unpredicted fields, an acceleration field and an opposite weaker gravitational field. Electrons are predicted to manifest attractive acceleration field and a weaker repulsive gravity. Free intergalactic electrons are thus prime candidates for Dark Matter.
9) Warp drive is mentioned as a result of very large charge separation. Implementation by radio photons as possibly behaving as oscillating pairs of virtual +e and –e charge

is only briefly discussed though even slight imbalance of such virtual charge distribution is equivalent to large amounts of ordinary separated charge and may therefore account for Roger Shawyer's EMDrive, Warp Drive, experiments by Dr. David Pares from Nebraska and for NASA's approval in the late news that EMDrive indeed involves Warp Drive effects. This idea also offers energy production promises to developing countries by Dennis Sciama inertial induction. This idea will be reminded.

10) The time field in Schwarzschild solution is discussed.
11) Ideas of how to extend this theory to quantum mechanics are offered by stochastic calculus.

It is first required to achieve a curvature field without resorting to Tzvi Scarr and Yaakov Friedman representation [7] that is required for a general acceleration field. $A_{\mu\nu} = -A_{\nu\mu}$ such that if $V_\mu$ is the 4-velocity such that $V_\mu V^\mu = 1$ then the 4-acceleration is actually $\frac{a_\nu}{c^2} = A_{\mu\nu}V^\mu$. In special relativity $V^\mu = \frac{(1, v_x / c, v_y / c, v_z / c)}{\sqrt{1 - v^2 / c^2}}$ such that $x, y, z$ are the well known three dimensional Cartesian coordinates, $v_x, v_y, v_z$ are three dimensional velocity coordinates, $c$ the speed of light. The first coordinate is $1/\sqrt{1 - v^2 / c^2}$ is the speed along the time axis.

The vector field that this paper uses is the gradient of the scalar field of time, $P,_\mu = P_\mu = \frac{dP}{dx^\mu}$ and it will replace the velocity vector of specific particles. Please note that due to intersection of different curves, $\frac{P^\nu}{\sqrt{P_\lambda P^{*\lambda}}}$ is not a velocity, however it is a unit vector and $A_{\mu\nu} \frac{P^\nu}{\sqrt{P_\lambda P^{*\lambda}}} = -\frac{U_\mu}{2}$ should emerge for some vector $U_\mu$ that replaces acceleration. The following is simply an exercise in differential geometry. Considering a scalar field $P$ and its gradient $P_\mu = \frac{dP}{dx^\mu}$ in covariant writing, such that $dx^\mu$ are the coordinates, find the second power of the curvature of the field of curves generated by $P_\mu = \frac{dP}{dx^\mu}$. It is a problem in differential geometry that can be left for the reader as an exercise. However, if the reader wants to get the answer without too much effort along with some physical interpretations, he/she should read the following.

The idea is to use a scalar field of time - that represents the maximum possible time measured by test particles - back to the big bang singularity as a limit or to a sub-manifold of events - and from this non -physical observable, to generate observable local measurements.

The square curvature of a conserving vector field is defined by an arc length parameterization $t$ along the curves it forms.

**Caution:** This $t$ may not be the time measured by any physical particle because the scalar field from which the vector field is derived may be the result of an intersection of multiple trajectories. However, a particle that follows the gradient curves will indeed measure $t$ even if its trajectory is not geodesic.

Let our time field be denoted by $P$ and let $P_\mu$ denote the derivative by coordinates $P_\mu = \frac{dP}{dx^\mu}$ or in Einstein-Grossmann's convention $P,_\mu = P_\mu$. Let $t$ be the arc length measured along the curves formed by the vector field $P_\mu$ which may not be always geodesic due to intersections. By differential geometry, we know that the second power of curvature along these curves is simply

$$Curv^2 \equiv \frac{d}{dt}\left(\frac{P_\lambda}{\sqrt{P^k P_k}}\right)\frac{d}{dt}\left(\frac{P_\mu}{\sqrt{P^k P_k}}\right)g^{\lambda\mu} \quad (8)$$

such that $g^{\lambda\mu}$ is the metric tensor. For convenience we will write $Norm \equiv \sqrt{P^k P_k}$ and $\dot{P}_\lambda \equiv \frac{d}{dt}P_\lambda$. For the arc length parameter $t$.

Here it is the main trick, $Norm$ may not be constant because $P_\lambda$ is NOT the 4-velocity of a specific particle, also but not only (see "APPENDIX – The time field in the Schwarzschild solution"), due to intersections of more than one possible particle trajectory curve.

Let $W_\lambda$ denote:

$$W_\lambda = \frac{d}{dt}\left(\frac{P_\lambda}{\sqrt{P^k P_k}}\right) = \frac{\dot{P}_\lambda}{Norm} - \frac{P_\lambda}{Norm^3} P_k \dot{P}_\nu g^{k\nu} \tag{9}$$

Obviously

$$W_\lambda P_k g^{\lambda k} = \frac{\dot{P}_\lambda P_k g^{\lambda k}}{Norm} - \frac{P_\lambda P_s g^{\lambda s}}{Norm^3} P_k \dot{P}_\nu g^{k\nu} = \frac{\dot{P}_\lambda P_k g^{\lambda k}}{Norm} - \frac{P_k \dot{P}_\nu g^{k\nu}}{Norm} = 0 \tag{10}$$

Thus

$$Curv^2 = W_\lambda W^\lambda = \frac{\dot{P}_\lambda \dot{P}_\nu g^{\lambda\nu}}{Norm^2} - \frac{P_\lambda \dot{P}_s g^{\lambda s}}{Norm^4} P_k \dot{P}_\nu g^{k\nu} = \frac{\dot{P}_\lambda \dot{P}^\lambda}{Norm^2} - \left(\frac{P_\lambda \dot{P}^\lambda}{Norm^2}\right)^2 \tag{11}$$

Following the curves formed by $P_\lambda = P,_\lambda = \frac{dP}{dx^\lambda}$, The term $\frac{dx^r}{dt} = \frac{P_\lambda}{Norm}$ is the derivative of the normalized curve or normalized "velocity", using the upper Christoffel symbols, $P_\lambda;_r \equiv \frac{d}{dx^r} P_\lambda - P_s \Gamma^s_{\lambda r}$.

**Caution**: Using normalized velocity, here has a differential geometry meaning but not a physical meaning because a physical particle will not necessarily follow the lines which are generated by the curves parallel to the gradient $P_\lambda$ unless in vacuum. $P_\lambda$ may result from an intersection of curves along which particles move but may not be parallel to any one of such curves intersecting with an event !!!

$\frac{d}{dt} P_\lambda = \left(\frac{d}{dx^r} P_\lambda - P_s \Gamma^s_{\lambda r}\right)\frac{dx^r}{dt} = (P_\lambda;_r)\frac{P^r}{Norm}$ such that $x^r$ denotes the local coordinates. If $P_\lambda$ is a conserving field then $P_\lambda;_r = P_r;_\lambda$ and thus $P_\lambda,_r P^r = \frac{1}{2} Norm^2,_\lambda$ and

$$\begin{aligned} Curv^2 &= \frac{\dot{P}_\lambda \dot{P}^\lambda}{Norm^2} - \left(\frac{P_\lambda \dot{P}^\lambda}{Norm^2}\right)^2 = \\ &\frac{1}{4}\left(\frac{Norm^2,_\lambda Norm^2,_k g^{\lambda k}}{Norm^4} - \left(\frac{Norm^2,_s P_r g^{sr}}{Norm^3}\right)^2\right) \end{aligned} \tag{12}$$

We define the Curvature Vector

$$U_m = \frac{(\mathrm{P}^\lambda \mathrm{P}_\lambda),_\mathrm{m}}{\mathrm{P}^\mathrm{i}\mathrm{P}_\mathrm{i}} - \frac{(\mathrm{P}^\lambda \mathrm{P}_\lambda),_\mu \mathrm{P}^\mu}{(\mathrm{P}^\mathrm{i}\mathrm{P}_\mathrm{i})^2} P_m = \frac{Norm^2,_\mathrm{m}}{Norm^2} - \frac{Norm^2,_\mu \mathrm{P}^\mu}{Norm^4} P_m \tag{13}$$

which from [7] and simple calculations, should have the meaning $\frac{1}{2}U_m = \frac{a_\mu}{C^2}B^\mu{}_m$ such that $a_m$ denotes a 4-acceleration field that will accelerate every particle, that can measure proper time, by an anti-symmetric matrix, and $C$ is the speed of light and $B^\mu{}_m$ is a rotation matrix, i.e. $B^\mu{}_m V^m B^\lambda{}_i V^i g_{\mu\lambda} = V_k V^k$, $V_m$ is a vector and $g_{\mu\lambda}$ is the General Relativity metric tensor. The curvature itself does not depend on any specific acceleration since it is a scalar field.

$$\boxed{Curv^2 = \frac{1}{4}U_m U^m} \tag{14}$$

Obviously $U_\mu P^\mu = 0$ and therefore like 4-acceleration that is perpendicular to 4-velocity $U_\alpha$ is perpendicular to $P_\alpha$. In its complex form (13) becomes

$$\mathrm{Z} \equiv N^2 \equiv \frac{\mathrm{P}_\mu \mathrm{P}*^\mu + \mathrm{P}*_\mu \mathrm{P}^\mu}{2} \text{ and } \mathrm{Z}_\lambda \equiv \frac{dZ}{dx^\lambda} \text{ and } \hat{U}_\lambda = \frac{Z_\lambda}{Z} - \frac{Z_k P*^k P_\lambda}{Z^2} \tag{15}$$

and by using $\frac{1}{2}(\hat{U}_k \hat{U}*^k + \hat{U}*_k \hat{U}^k)$

$$\boxed{Curv^2 = \frac{1}{4}(\frac{1}{2}(\hat{U}_k \hat{U}*^k + \hat{U}*_k \hat{U}^k))} \tag{16}$$

Obviously $U_\mu P*^\mu = 0$.

**Possible sources for an acceleration field:** An acceleration field can be represented by the Tzvi Scarr and Yaakov Friedman [7] matrix as $A_{\mu\nu}\frac{P*^\nu}{\sqrt{P_k P^k}} = \frac{U_\mu}{2}$ such that $A_{\mu\nu} = -A_{\nu\mu}$

We may write this matrix explicitly by (15) and (16) and also require an additional $1/2$ scaling and reach the following anti-symmetric singular matrix

$$A_{\mu\nu} = \frac{U_\mu P_\nu - P_\mu U_\nu}{2\sqrt{Z}} \Rightarrow A_{\mu\nu}\frac{P*^\nu}{\sqrt{Z}} = \frac{U_\mu P_\nu P*^\nu - P_\mu U_\nu P*^\nu}{2Z} = U_\mu / 2$$

So $A_k{}^\mu (A_\mu{}^\nu \frac{P*_\nu}{\sqrt{Z}})* = -\frac{(U_\lambda U*^\lambda)}{4}\frac{P_k}{\sqrt{Z}}$. It easily verifiable that $Det(A_{\mu\nu}) = 0$. What is required, however, is that $Det(A_{\mu\nu}) \neq 0$.

So we need a modified $A_{\mu\nu}$. On our path we will see symmetries that are usually achieved by second quantization and particle symmetries but with a very different geometric source.

## 5. THE STANDARD MODEL SU(3) x SU(2) x U(1) SYMMETRIES VIA THE GODBILLON-VEY CLASS

**Caution**: Please note, the following is NOT a quantum theory of the discussed acceleration field $U_\mu$. The sole purpose of the following equations, is to show the degrees of freedom in the matrix representation of the acceleration field action.

We now present the curvature quite closely to the Reinhart-Wood metric formula [2],

$$A_{\mu\nu}=\frac{U_\mu P_\nu - P_\mu U_\nu}{2\sqrt{Z}}=\frac{1}{2\sqrt{Z}}(\frac{Z_\mu P_\nu}{Z}-\frac{Z_k P^{*k}\, P_\mu P_\nu}{Z^2}-(\frac{Z_\nu P_\mu}{Z}-\frac{Z_k P^{*k}\, P_\nu P_\mu}{Z^2}))= \frac{1}{2\sqrt{Z}}(\frac{Z_\mu P_\nu}{Z}-\frac{Z_\nu P_\mu}{Z})=(\frac{P_\mu}{\sqrt{Z}}),_\nu-(\frac{P_\nu}{\sqrt{Z}}),_\mu=(\frac{P_\mu}{\sqrt{Z}}+\phi_\mu),_\nu-(\frac{P_\nu}{\sqrt{Z}}+\phi_\nu),_\mu \quad (17)$$

For some conserving field $\phi_\mu = d\phi / dx^\mu$. It is immediate that if $P_\mu$ is imaginary so is $U_\mu$ because then $A_{\mu\nu}$ is complex, as $\sqrt{Z}$ is real. The divergence of the real part is then $(U_\mu + U^*{}_\mu);^\mu / 2 = 0$ and this fact will become crucial when electro-gravity is discussed. It is easily verifiable that the exterior derivative $d\omega = d(\frac{P_\mu}{\sqrt{Z}})dx^\mu$, can be written with $\eta = \frac{U_\nu}{2}dx^\nu$ by the Godbillon-Vey observation [2][3], as $d\omega = (\eta \wedge \omega)_{ij} = (\frac{P_\mu}{\sqrt{Z}}),_\nu dx^\mu \wedge dx^\nu = \frac{U_\mu}{2}\frac{P_\nu}{\sqrt{Z}}dx^\mu \wedge dx^\nu$ with coordinates $x^\mu$. Another observation is the Godbillon-Vey class, on the foliations whose tangent bundle $T(F)$ is perpendicular to $\frac{P_\mu}{\sqrt{Z}}$. $GV(F) = \frac{U_\mu}{2}\frac{U_{\nu},_\lambda}{2}dx^\mu \wedge dx^\nu \wedge dx^\lambda = \eta \wedge d\eta \in H^3(M)$ is closed on $T(F)$. More important is the observation that $[U_\mu]$ is the Reeb cohomology Class $[\eta]$.

From Reeb it follows that the projection of the curvature vector $U_\mu$ on the tangent bundle $T(F)$ of the foliations $F$ that are perpendicular to $P_\nu$, have a vanishing exterior derivative. In other words, $U_\mu(F)$ is a closed form on $F$ i.e. $U_\mu;_\nu(F) - U_\nu;_\mu(F) = 0$. . From (6) and the remark after (13) it follows that the rotor of the electric field should be zero on $F$, but that is possible only if even photons are pairs of charged particles, possibly oscillating with a total zero dipole moment as indeed informally predicted by physicists such as Hans W. Giertz [11]. This conclusion, as will be discussed, regards very important late findings by NASA. It is worth mentioning that the expansion of the holonomy of the foliations is dictated by the norm of the Reeb vector [12] which in the real case is $\sqrt{-U_\mu U^\mu}$ and therefore this value is of great cosmological interest.

Following is an offered complex form of (17),

$$\boxed{\begin{aligned}&\frac{1}{4}(A_{\mu\nu}A^{*\mu\nu}+A^*_{\mu\nu}\ A^{\mu\nu})=\frac{1}{4}((\frac{U_\mu P_\nu-P_\mu U_\nu}{2\sqrt{Z}})(\frac{U^{*\mu}\ P^{*\nu}-P^{*\mu}\ U^{*\nu}}{2\sqrt{Z}})+...)=\\&\frac{1}{16}((U^*_\mu\ U^\mu+U^*_\nu\ U^\nu)+(U_\mu U^{*\mu}+U_\nu U^{*\nu}))=\frac{1}{4}(\frac{U^*_\mu\ U^\mu+U^*_\nu\ U^\nu}{2})\end{aligned}} \tag{18}$$

which is the time field curvature action. In the real case where $A_{\mu\nu}=A^*_{\mu\nu}$ we define $F_{\mu\nu}=\frac{1}{\sqrt{2}}A_{\mu\nu}$ and the action (16) takes the form $F_{\mu\nu}F^{\mu\nu}$ which reminds of the electro-magnetic theory but without the vector potential. As we shall see, this vector potential is indeed redundant because electric charge is a geometric phenomena. From the theory of Lie Algebras, $Det(\exp(A_{\mu\nu}))=\exp(Trace(A_{\mu\nu}))=\exp(0)=1$ and therefore either $\exp(A_{\mu\nu})\in SO(4)$ or $\exp(A_{\mu\nu})\in SU(4)$ depending on whether $A_{\mu\nu}$ is real or not. We proved that $A_\mu{}^\nu=A_{\mu k}g^{k\nu}$ does rotate and scale $\frac{P^*_\nu}{\sqrt{Z}}$ into $U_\mu/2$. So by the Tzvi Scarr and Yaakov Friedman representation [7], for every test particle with rest mass and with velocity $V^\mu$ and the speed of light $C$ we have,

$$A^\mu{}_\nu\frac{V^\nu}{C}=\frac{1}{C^2}\frac{dV^\mu}{d\tau} \tag{19}$$

Just like $U_\mu$ the term $\frac{1}{C^2}\frac{dV^\mu}{d\tau}$ represents a curvature vector but of a specific particle related trajectory.

We continue with the Tzvi Scarr and Yaakov Friedman acceleration representation [7] matrix and for simplicity we restrict our discussion to the real case. $A_{\mu\nu}$ is singular and we can easily define a matrix that rotates vectors in a plane perpendicular to both $U_\mu$ and to $P_\nu$. That is the matrix

$$\boxed{B^{\alpha\beta}\equiv\frac{1}{\sqrt{2}}\varepsilon^{\mu\nu\alpha\beta}A_{\mu\nu}} \tag{20}$$

Where $\varepsilon^{\mu\nu\alpha\beta}$ is the Levi-Civita symbol. It is easily verified that

$$(A^{\alpha\beta}+B^{\alpha\beta})(A_{\alpha\beta}+B_{\alpha\beta})=A^{\alpha\beta}A_{\alpha\beta}+B^{\alpha\beta}B_{\alpha\beta}$$

and also

$$\begin{aligned}&B^{\alpha\beta}B_{\alpha\beta}=\frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}A_{\mu\nu}\varepsilon_{ij\alpha\beta}A^{ij}=\frac{1}{2}\varepsilon^{\mu\nu}{}_{ij}A_{\mu\nu}A^{ij}=\\&\frac{1}{2}(\delta_i^\mu\delta_j^\nu-\delta_j^\mu\delta_i^\nu)(A^{ij}A_{\mu\nu})=\frac{1}{2}(A^{\mu\nu}A_{\mu\nu}-A^{\nu\mu}A_{\mu\nu})=A^{\mu\nu}A_{\mu\nu}\end{aligned}$$

Therefore $(A^{\alpha\beta}+B^{\alpha\beta})(A_{\alpha\beta}+B_{\alpha\beta})=A^{\alpha\beta}A_{\alpha\beta}+B^{\alpha\beta}B_{\alpha\beta}=2A^{\mu\nu}A_{\mu\nu}$

$\delta_i^j$ is the Kronecker delta.

$$\boxed{A_{\alpha\beta}\to A_{\alpha\beta}+B_{\alpha\beta}} \tag{21}$$

(21) is the matrix we have been looking for and it also results in an immediate degree of freedom in (16). In particular, $\tilde{\gamma}_\alpha{}^\mu$ rotations in $SU(4)$ [13] that do not affect $A_{\alpha\beta}$ may be applied to $B_{\alpha\beta}$. These rotations are in $SU(2)xU(1)$, $B_{\alpha\beta} \neq \tilde{\gamma}_\alpha{}^\mu B_{\mu\nu} \tilde{\gamma}_\beta{}^\nu$ and $A_{\alpha\beta} = \tilde{\gamma}_\alpha{}^\mu A_{\mu\nu} \tilde{\gamma}_\beta{}^\nu$.

$$\boxed{A_{\alpha\beta} \rightarrow A_{\alpha\beta} + \tilde{\gamma}_\alpha{}^\mu B_{\mu\nu} \tilde{\gamma}_\beta{}^\nu} \quad (22)$$

**Dirac's Monopole:** If the reduction of $U_k$ to the foliation tangent space $T(F)$ that is the kernel of $P_k$ changes in relation to the point $0$ in the axis system reduced to $T(F)$ ,then the determinant of $A_{\alpha\beta} + \tilde{\gamma}_\alpha{}^\mu B_{\mu\nu} \tilde{\gamma}_\beta{}^\nu$ preserves the same sign.

For the real case, where $U_k = U^*{}_k$, note that $Curv^2 = \frac{1}{4}(U_k U^k) = (A^{\alpha\beta} A_{\alpha\beta} + B^{\alpha\beta} B_{\alpha\beta})/4$.

SU(3) degrees of freedom result from a way to represent $\frac{P_\mu}{\sqrt{Z}}$ with 3 scalar functions $q(1), q(2), q(3)$ according to the Frobenius (foliation) theorem. The Lie brackets of 3 vector fields on a perpendicular foliation to $\frac{P_\mu}{\sqrt{Z}}$, have to depend on the tangent bundle $T(F)$ of the foliation $F$ as follows,

$$\boxed{\begin{array}{l}
q(1)_j\, q(2)_s\, q(3)_\mu\, \varepsilon^{js\mu\nu} = \alpha P^\nu,\ \alpha \neq 0 \text{ is complex.} \\
q(1)_j = \frac{d}{dx^j} q(1), q(2)_s = \frac{d}{dx^s} q(2), q(3)_\mu = \frac{d}{dx^\mu} q(3). \\
1)\ \sigma^{ijk} q(1)_i\, q(2)_j\, q(3)_k \neq 0 \\
2)\ \sigma^{ijks} q(1)_i, q(2)_j, q(3)_k (q(1)^r q(2)_s,_r - q(1)_s,_r\, q(2)^r) = 0 \\
3)\ \sigma^{ijks} q(1)_i, q(2)_j, q(3)_k (q(1)^r q(3)_s,_r - q(1)_s,_r\, q(3)^r) = 0 \\
4)\ \sigma^{ijks} q(1)_i, q(2)_j, q(3)_k (q(2)^r q(3)_s,_r - q(2)_s,_r\, q(3)^r) = 0
\end{array}} \quad (23)$$

Conditions 2,3,4 in (23) define the gradients of $q(1)$, $q(2)$ and $q(3)$ as Holonomic .

Vectors $h(s)$ are Holonomic if their Lie brackets depend on them $[h(i), h(k)] = \sum_{j=1}^{3} c_j h(j)$ for some coefficients $c_j$. Condition 1 is a transversality condition. The Lie brackets of each two vectors must depend on the vectors that span $T(F)$. (23) has a deep meaning that our scalar field of time is a result of 3 scalar fields of space locally perpendicular to the gradient of the scalar field of time.

The source of these fields are possibly the Sam Vaknin's Chronon field [1]. This implies a simpler physics and it is possible that Dirac's equation [14] and spinors are an algebraic language that was required because concurrent physics theories do not have a complete analytic theory of space-time. The need for algebraic abstraction may not be related to physical reality but rather to the way we perceive it. This possibility justifies further extensive research and should not be dismissed.

## 5.1 Dr. Vaknin's Theory – One of Four of his Offers

The coming definitions will reflect Dr. Sam Vaknin's view, that even photons by entanglement of wave functions have rest mass. Not that a photon as observed on its own has rest mass. That is incorrect. Matter by Vaknin's theory [1] is a result of interaction of a field of time that quite reminds of Quarks in which summation results in positive propagation. This paper sees Dr. Sam Vaknin's theory as a starting point.

Dr. Sam Vaknin's possible description of time: *"Time as a wave function with observer-mediated collapse. Entanglement of all Chronons at the exact "monent" of the Big Bang. A relativistic QFT with Chronons as Field Quanta (excited states.) The integration is achieved via the quantum superpositions".*

We will now refer to a simple implementation of Sam Vaknin's approach as a quantization idea of time, as the action $\int_{\Omega^4} \frac{C^4}{\sigma K} Curv^2 \sqrt{-g} d\Omega^4$

Where $\sqrt{-g}$ is the root of the negative metric tensor determinant for the volume element, such that

$$P = (\lim_{n\to\infty} \psi(1) + \psi(2) + ... + \psi(n))\sqrt{Time\_Atom}$$

and such that:

$$\int_{\Omega^4} \psi(k)\psi^*(k)\sqrt{-g}d\Omega^4 = 1$$

And

$$0 < j < k < \infty \Rightarrow \int_{\Omega^4} \psi(j)\psi^*(k)\sqrt{-g}d\Omega^4 = 0$$

Here are more definitions we will need.

**Energy Conservation:** *In any physical system and its interaction*, *the sum of kinetic* (*visible*) *and latent* (*dark*) *energy is constant*, *gain of energy is maximal and loss of energy is minimal*. See E. E. Escultura [15].

**Information:** *Information is a mathematical representation of the state of a physical system with as few labels as possible. Labels can be numbers or any other mathematical object.*

**Energy Density:** *We define* $-\frac{C^4}{\sigma K}Curv^2$, *such that* $C$ *is the speed of light and* $K$ *is the gravity constant, as the Energy Density of space-time. If this value is defined by (14) then* $P = \tau$ *is the upper limit of measurable time from an event back to near big bang event or to a sub-manifold of events and therefore (14) is intrinsic to the space-time manifold because it is dictated by the equations of gravity and adds no information that is not included in the manifold and in the equations. As we shall see, if we choose to write* $P = \tau\psi$ *such that* $\psi$ *is a complex scalar field then if* $\psi$ *is a function of* $\tau$ *only then (16) is reduced to (14) as if* $P = \tau$. *Consider the set of events for which* $\tau$ *is constant. Since* $\tau$ *is not a coordinate, we can't expect that set to be a sub-manifold but a unification of such 3 dimensional geometric objects* $\Omega^3(\tau_0) = \Omega^3(\tau = \tau_0)$.

We consider $\tau$ as a Morse function on the space-time $M$ manifold. That is that $M \to \tau$ is locally smooth and the differential of this map is of rank 1. In such a case the Morse – Sard theorem states that the Lebesgue measure of the Critical Points of $M$ is zero [16].

**Energy:** *The following is equivalent to rest mass energy. The integration of* $-\int_{\Omega^3(\tau_0)} \frac{C^4}{\sigma K} Curv^2 d\Omega^3(\tau_0)$ *is defined as the Energy of the scalar* $\tau = \tau_0$. *This value is locally conserved for small neighborhoods in* $\Omega^3(\tau_0)$ *if* $U^m;_m = 0$ *as any local integration of the squared norm of a vector field is conserved if its divergence is zero. So if there is a possibility of* $U^m;_m \neq 0$ *local conservation of the term Energy does not hold. Photons too, by entanglement and superposition have rest mass but not as an isolated electro-magnetic wave.*

nice thought experiment – is the use of a covariant Gauss Law and the atom of space for $\sigma = 8\pi$. After we have these definitions in mind, we may want to extend Gauss Law to a covariant format and later use the idea that there is an atomic structure of space time that can't be curved by gravity. For start, a generalized Gauss law using the exterior algebra, should look like:

$$\int_{S3} E \cdot ds = \int_{S3} E^{\mu} \wedge dx^{i} \wedge dx^{j} \wedge dx^{k} = cTau\frac{Q}{\varepsilon_0} \quad (24)$$

such that $E$ is the electric field. $ds$ a surface element and $x$ denotes the coordinates, but here $E$ is a 4-vector and will have to be defined, $cTau$ is the length of the path of a charge enclosed in a 3 dimensional surface $S3$, $\varepsilon_0$ the permittivity of vacuum and $c$ is the speed of light. We also assume that in our local coordinates, the metric tensor is $g_{00} = 1, g_{11} = -1, g_{22} = -1, g_{33} = -1$ otherwise, $g_{\mu\nu} = 0$, i.e. flat geometry on the hyper-cylinder. Since space-time can be curved, we can only discuss small volume $S3$ and small proper time $Tau$ in local coordinates. Instead of considering the general acceleration field $A^{\mu}{}_{\nu}\frac{V^{\nu}}{C} = \frac{1}{C^2}\frac{dV^{\mu}}{d\tau}$ as in (18), we need a representative trajectory dependent boost 4-acceleration $a^{\mu} = \frac{dV^{\mu}}{d\tau}$ so now we can apply (6) as an exact relation as the outcome of the energy density $a^{\mu}a_{\mu}/(8\pi K)$ i.e. for the previously discussed guess $\sigma = 8\pi$.

$$\frac{a^{\mu}}{\sqrt{4\pi K \varepsilon_0}} = E^{\mu} \quad (25)$$

such that $E^{\mu}$ is a 4-vector that replaces the electrostatic field. Please notice that here $E^0 \neq 0$. Consider a 3 dimensional surface of 4 dimensional cylinder around a charge $Q$, i.e. $Surface = S_2 xI$ around a $Cylinder = B_3 xI$ such that both the radius of the sphere $S_2$ and of the ball $B_3$ and the length of the interval $I$, is $L$ and is small. We may consider $L$ to be an atom of length so by the relation we have from (18), acceleration is merely a curvature of a particle's trajectory. If there is an atom of length, say $L$, then the maximal curvature would appear in loops of radius $L$ so it would be $Curvature = \frac{1}{L}$ and by (18) and by the relation $\frac{a_{\nu}}{c^2} = A_{\mu\nu}V^{\mu}$ such that $V^{\mu}V_{\mu} = 1$, the maximal acceleration is therefore

$$\sqrt{\left|a^{\mu}a_{\mu}\right|} = \frac{C^2}{L} \quad (26)$$

where $C$ is the speed of light. By (6), this acceleration can be the underlying acceleration field in an electric field. The parameterization in polar coordinates of the surface $S_2 xI$ is $(L\cos(\phi)\cos(\varphi), L\sin(\phi)\cos(\varphi), L\sin(\varphi), l)$ where $0 < l < L$. It is $(r\cos(\phi)\cos(\varphi), r\sin(\phi)\cos(\varphi), r\sin(\varphi), 0)$ where $l = 0$ and it is $(r\cos(\phi)\cos(\varphi), r\sin(\phi)\cos(\varphi), r\sin(\varphi), L)$ where $l = L$, such that $r \in (0, L), \phi \in (0, 2\pi), \varphi \in (-\pi/2, \pi/2)$ and the normal to the surface which for $0 < l < L$ is $\vec{n} = (\cos(\phi)\cos(\varphi), \sin(\phi)\cos(\varphi), \sin(\varphi), 0)$ and at $l = L$ the normal vector is $\vec{n} = (0,0,0,1)$ and at $l = 0$ the normal is $\vec{n} = (0,0,0,-1)$.

An acceleration field $A^{\mu}{}_{\nu}$ around a physical source is expected to cause $E^{\mu}\vec{n}_{\mu}$ to cancel out on the "top" and "bottom" $l = L$ and $l = 0$ boundaries combined together, because acceleration changes sign as the time coordinate changes sign. So the integration (25) will be by (25) and by (26), even though the metric is Minkowsky,

$$4\pi L^3 \frac{C^2}{L\sqrt{4\pi K \varepsilon_0}} = \frac{QL}{\varepsilon_0} \Rightarrow L = \frac{Q}{C^2}\sqrt{\frac{K}{4\pi\varepsilon_0}} \quad (27)$$

But what about gravity which results from the energy of the acceleration field $A^{\mu}{}_{\nu}$ and what about the influence of such gravity on the metric tensor within the cylinder ?

Since we consider an atom of length $L$, no field can exist below that radius and therefore, (24) is valid even if the charge $Q$ is not small.

We now assign the electron charge $e$ to $Q$ and (27) becomes,

$$L = \frac{e}{C^2}\sqrt{\frac{K}{4\pi\varepsilon_0}} \tag{28}$$

What is the wave length of a photon that can be emitted by the field $E^\mu$ ? We know that the energy should be $\alpha$ times smaller than the energy of the field, such that $\alpha$ is known as the "Fine Structure Constant". Therefore, the wavelength is bigger by $1/\sqrt{\alpha}$ than the expected wave length if we assume that $L$ is also the wavelength of a photon that has equal energy to the field $E^\mu$ or in its matrix form $A^\mu{}_\nu$.

We have $\lambda = \frac{L}{\sqrt{\alpha}} = \frac{e}{C^2}\sqrt{\frac{K}{4\pi\varepsilon_0\alpha}}$ assign $\alpha = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar C}$ and get,

$$\lambda = \sqrt{\frac{K\hbar}{C^3}} \tag{29}$$

Which is known as the "Planck Length".

## 6. INVARIANCE UNDER DIFFERENT FUNCTIONS OF P

Here we are about to explore another degree of freedom in the action operator of the acceleration field as shown by a representative vector field $\frac{dP}{dx^i}$ that curves.

**Caution:** Although the calculation may have a quantum meaning, it is not brought here for the purpose of developing a quantum field theory.

We revisit our acceleration field, $U_m = \frac{N^2{,}_m}{N^2} - \frac{N^2{,}_\mu P^{*\mu}}{N^4}P_m$ s.t. $N^2 \equiv (P^{*i}\, P_i + P^{*i}\, P_i)/2$ (also found as $Z$ in this paper) we can sloppily omit the comma for the sake of brevity the same way we write $\mathrm{P}_\mathrm{i}$ instead of $\mathrm{P}_{,\mathrm{i}}$ for $\frac{dP}{dx^i}$ and write $U_m = \frac{N^2{}_m}{N^2} - \frac{N^2{}_\mu P^{*\mu}}{N^4}P_m$.

Suppose that we replace $P$ by $f(P)$ such that $f$ is positive and increasing, then $f(P)_i \equiv \frac{df(P)}{dx^i} = \frac{df(p)}{dp}\frac{dP}{dx^i} = f_p(P)P_i$. Let $N^2 \equiv \mathrm{P}^\lambda \mathrm{P}_\lambda$ then $\hat{N}^2 \equiv f(P)_\lambda f(P)^\lambda = N^2 f_p(P)^2$ and $\frac{\hat{N}^2{}_k}{\hat{N}^2} = \frac{N^2{}_k}{N^2} + \frac{2f_{pp}(p)}{f_p(p)}p_k$ but also

$$\begin{aligned}\hat{U}_k &= \frac{\hat{N}^2{}_k}{\hat{N}^2} - \frac{\hat{N}^2{}_s}{\hat{N}^2}\frac{f_p(p)p^s f_p(p)p_k}{\hat{N}^2} = \\ &\frac{N^2{}_k}{N^2} + \frac{2f_{pp}(p)}{f_p(p)}p_k - \left(\frac{N^2{}_s}{N^2} + \frac{2f_{pp}(p)}{f_p(p)}p_s\right)\frac{f_p(p)p^s f_p(p)p_k}{N^2 f_p(p)^2} = \\ &\frac{N^2{}_k}{N^2} - \frac{N^2{}_\mu P^\mu}{N^4}P_k = U_k\end{aligned} \tag{30}$$

Consider quantum coupling between the wave function $\psi$ of a particle and the time field $\tau$, $PP^* = \tau^2\psi\psi^*$ as follows. Where does this coupling $P = \tau\psi$ come from ? It is has some common sense if we say that the sum of wave functions that intersect/coincide with an event, influence the time measurement from near the “big bang” singularity event or from a sub-manifold of events to that specific event. A much better choice that will be discussed in the appendix "Appendix - Event Theory and Lèvy Process" is $P=\sqrt{\tau}\psi$ and $PP^*=\tau\psi\psi^*$. In that case, $\psi\psi^*$ is a Lèvy measure and time itself becomes a stochastic process with events in which time can be measured. The appendix presents an offer of how to quantize (14).

Currently, we defer the quantization of (14) and we define the curvature vector of $P=\tau\psi$ ,

$$\hat{U}_k \equiv \left( \frac{\hat{N}^2{}_{\mathrm{k}}}{\hat{N}^2} - \frac{\hat{N}^2{}_{\mathrm{j}}(\tau\psi)^{*\mathrm{j}}}{(\hat{N}^2)^2}(\tau\psi)_k \right) \tag{31}$$

Index $k$ means derivative by coordinate $x^k$ , $\hat{N}^2=(\tau\psi)_k(\tau\psi^*)^k$ , $N^2=\tau_k\tau^k$ .

As a special case, we replace $\psi$ by a wave function that depends on $\tau$ only

$$\psi = e^{\frac{-iE\tau}{\hbar}} \text{ s.t. } \mathrm{i}=\sqrt{-1} \tag{32}$$

$E$ is the energy of a coupled particle, $\hbar$ is the Barred Planck constant, so we have

$$(\tau\psi)_k = \tau_k\psi + \tau\psi_k = \tau_k\psi(1-\frac{i\tau E}{\hbar}) \tag{33}$$

$$\hat{N}^2 = \tau_k\tau^k(1+\frac{\tau^2E^2}{\hbar^2}) = N^2(1+\frac{\tau^2E^2}{\hbar^2}) \tag{34}$$

and

$$\frac{\hat{N}^2{}_s}{\hat{N}^2} = \frac{N^2{}_s}{N^2}\frac{(1+\frac{\tau^2E^2}{\hbar^2})}{(1+\frac{\tau^2E^2}{\hbar^2})} + \frac{2\tau\tau_sE^2N^2/\hbar^2}{(1+\frac{\tau^2E^2}{\hbar^2})N^2} = \frac{N^2{}_s}{N^2} + \frac{2\tau\tau_sE^2}{(\hbar^2+\tau^2E^2)} \tag{35}$$

Now we want to calculate $\frac{\hat{N}^2{}_{\mathrm{j}}(\tau\psi)^{*\mathrm{j}}}{(\hat{N}^2)^2}(\tau\psi)_k$ so we have

$$\frac{\hat{N}^2{}_{\mathrm{j}}(\tau\psi)^{*\mathrm{j}}}{(\hat{N}^2)^2}(\tau\psi)_k =$$

$$(\frac{N^2{}_j}{N^2} + \frac{2\tau\tau_jE^2}{(\hbar^2+\tau^2E^2)})\frac{(\tau^j\psi^*(1+\frac{i\tau E}{\hbar}))}{(1+\frac{\tau^2E^2}{\hbar^2})N^2}\tau_k\psi(1-\frac{i\tau E}{\hbar})) = \tag{36}$$

$$(\frac{N^2{}_j}{N^2} + \frac{2\tau\tau_jE^2}{(\hbar^2+\tau^2E^2)})\frac{\tau^j}{N^2}\tau_k = \frac{N^2{}_j\tau^j\tau_k}{(N^2)^2} + \frac{2\tau\tau_kE^2}{(\hbar^2+\tau^2E^2)}$$

From (31), (35) and (36) we have the result

$$\hat{U}_k \equiv \left( \frac{\hat{N}^2{}_{\mathrm{k}}}{\hat{N}^2} - \frac{\hat{N}^2{}_{\mathrm{j}}(\tau\psi)^{*\mathrm{j}}}{(\hat{N}^2)^2}(\tau\psi)_k \right) =$$
$$((\frac{N^2{}_k}{N^2} + \frac{2\tau\tau_k E^2}{(\hbar^2 + \tau^2 E^2)}) - (\frac{N^2{}_j \tau^j \tau_k}{(N^2)^2} + \frac{2\tau\tau_k E^2}{(\hbar^2 + \tau^2 E^2)})) = \tag{37}$$
$$(\frac{N^2{}_k}{N^2} - \frac{N^2{}_j \tau^j \tau_k}{(N^2)^2}) = (\frac{N^2{}_k}{N^2} - \frac{N^2{}_j P^j P_k}{(N^2)^2}) = U_k$$

## 7. GENERAL RELATIVITY FOR THE DETERMINISTIC LIMIT

By General Relativity, We have to add the Hilbert action to the negative sign of the square curvature of the gradient of the time field. Negative means that the curvature operator is mostly negative. We assume $\sigma = 8\pi$ .

$$\boxed{\begin{array}{l} \mathrm{Z} = N^2 = \mathrm{P}_\mu \mathrm{P}^\mu \text{ and } U_\lambda = \frac{Z_\lambda}{Z} - \frac{Z_k P^k P_\lambda}{Z^2} \text{ and } \mathrm{L} = \frac{1}{4}\mathrm{U}^{\mathrm{k}}\mathrm{U}_{\mathrm{k}} \\ R = \mathit{Ricci} \text{ curvature.} \\ \mathit{Min\ Action} = \mathit{Min}\int_\Omega \left( \frac{1}{2}R - \frac{8\pi}{\sigma}L \right) . \sqrt{-g}\, d\Omega \end{array}} \tag{38}$$

A reader that still insists on asking on where does $\tau\psi$ come from, can understand that L can be developed also for $\tau\psi$ and remain invariant if $\psi$ is only a smooth function of $\tau$ . If $P = \tau\psi$ then $L = \frac{1}{8}(\mathrm{U}^{\mathrm{k}}\mathrm{U}^*{}_{\mathrm{k}} + \mathrm{U}^{*\mathrm{k}}\ \mathrm{U}_{\mathrm{k}})$ and an integration constraint can be

$$\int_{\Omega^3(\tau)} \psi\psi^* \sqrt{-g}\, d\Omega^3(\tau) = 1 \tag{39}$$

**Caution:** $\Omega^3(\tau)$ is not a sub-manifold because $\tau$ is not a local coordinate and thus the local submersion theorem [17], [18] does not hold. However, $\Omega^3(\tau)$ is necessarily a countable unification of three dimensional sub-manifolds - Almost Everywhere - on which $\tau$ is stationary due to dimensionality considerations.

$R$ is the Ricci curvature [19], [20] and $\sqrt{-g}$ is the determinant of the metric tensor used for the 4-volume element as in tensor densities [21].

**Important:** If instead of $P = \tau\psi$ we choose $P = \sqrt{\tau}\psi$ then $U_k$ in (37) is the same if $\psi$ depends only on $\tau$ . Moreover, instead of the constraint in (39) the following

$$\int_{\Omega^4(\tau)} \psi\psi^* \sqrt{-g}\, d\Omega^4 = 1 \tag{40}$$

leads to a theory in which $\psi$ represents an event and $\int_{\Omega^4(\tau)} PP^* \sqrt{-g}\, d\Omega^4 = \tau_{Event}$ represents the time of an event, i.e. see "Appendix – Event Theory and Lèvy Process". Now we return to (38). By Euler Lagrange,

$$
\begin{aligned}
&L = \frac{(P^\lambda Z_\lambda)^2}{Z^3} \;\; s.t.\; Z = P_\mu P^\mu \\
&\frac{\partial(L\sqrt{-g})}{\partial g^{\mu\nu}} - \frac{d}{dx^m}\frac{\partial(L\sqrt{-g})}{\partial(g^{\mu\nu},_m)} = \\
&\begin{pmatrix}
-2(\frac{(P^\lambda P_\lambda),_s P^s}{Z^3} P_\mu P_\nu P^m);_m + 2\frac{(P^\lambda P_\lambda),_s P^s}{Z^3}(\Gamma_\mu{}^i{}_m P_i P_\nu P^m + \Gamma_\upsilon{}^i{}_m P_\mu P_i P^m) + \\
+2(\frac{(P^\lambda P_\lambda),_s P^s}{Z^3} P_\mu P_\nu);_m P^m - 2\frac{(P^\lambda P_\lambda),_s P^s}{Z^3}(\Gamma_\mu{}^i{}_m P_i P_\nu P^m + \Gamma_\upsilon{}^i{}_m P_\mu P_i P^m) + \\
+2(\frac{(P^\lambda P_\lambda),_s P^s}{Z^3})Z_\mu P_\nu - 3(\frac{((P^\lambda P_\lambda),_s P^s)^2}{Z^4})P_\mu P_\nu - \frac{1}{2}\frac{(P^\lambda Z_\lambda)^2}{Z^3} g_{\mu\nu}
\end{pmatrix}\sqrt{-g} \qquad (41)\\
&= \begin{pmatrix}
-2(\frac{(P^\lambda P_\lambda),_m P^m}{Z^3} P^k);_k P_\mu P_\nu - 2\frac{(P^\lambda Z_\lambda)^2}{Z^3}\frac{P_\mu P_\nu}{Z} + 2(\frac{(P^\lambda P_\lambda),_s P^s}{Z^3})Z_\mu P_\nu + \\
-\frac{1}{2}\frac{(P^\lambda Z_\lambda)^2}{Z^3} g_{\mu\nu} - \frac{(P^\lambda Z_\lambda)^2}{Z^3}\frac{P_\mu P_\nu}{Z}
\end{pmatrix}\sqrt{-g}
\end{aligned}
$$

$$
\begin{array}{l}
L = \dfrac{Z^{\lambda} Z_{\lambda}}{Z^2} \;\; s.t. \; Z = P_{\mu} P^{\mu} \\
\dfrac{\partial(L\sqrt{-g})}{\partial g^{\mu\upsilon}} - \dfrac{d}{dx^m}\dfrac{\partial(L\sqrt{-g})}{\partial g^{\mu\upsilon}{}_{,m}} = \\
\left( \begin{array}{l}
-2(\dfrac{Z^m P_{\mu} P_{\upsilon}}{Z^2});_m + 2\dfrac{(\Gamma_{\mu}{}^{i}{}_{m} P_i P_{\nu} Z^m + \Gamma_{\upsilon}{}^{i}{}_{m} P_{\mu} P_i Z^m)}{Z^2}) + \\
+2\dfrac{(P_{\mu} P_{\upsilon});_m Z^m)}{Z^2} - 2\dfrac{(\Gamma_{\mu}{}^{i}{}_{m} P_i P_{\nu} Z^m + \Gamma_{\upsilon}{}^{i}{}_{m} P_{\mu} P_i Z^m)}{Z^2}) + \\
+\dfrac{Z_{\mu} Z_{\nu}}{Z^2} - 2\dfrac{Z_s Z^s}{Z^3} P_{\mu} P_{\nu} - \dfrac{1}{2}\dfrac{Z_m Z^m}{(P^i P_i)^2} g_{\mu\nu}
\end{array} \right) \sqrt{-g} = \\
(-2(\dfrac{Z^m}{Z^2});_m P_{\mu} P_{\nu} - 2\dfrac{Z^{\lambda} Z_{\lambda}}{Z^2}\dfrac{P_{\mu} P_{\nu}}{Z} - \dfrac{1}{2}\dfrac{Z_k Z^k}{Z^2} g_{\mu\nu} + \dfrac{Z_{\mu} Z_{\nu}}{Z^2})\sqrt{-g}
\end{array}
$$


$$
\begin{array}{l}
L = \dfrac{Z^{\lambda} Z_{\lambda}}{Z^2} \;\; s.t. \; Z = P_{\mu} P^{\mu} \\
\dfrac{\partial(L\sqrt{-g})}{\partial g^{\mu\upsilon}} - \dfrac{d}{dx^m}\dfrac{\partial(L\sqrt{-g})}{\partial g^{\mu\upsilon}{}_{,m}} = \\
\left( \begin{array}{l}
-2(\dfrac{Z^m P_{\mu} P_{\upsilon}}{Z^2});_m + 2\dfrac{(\Gamma_{\mu}{}^{i}{}_{m} P_i P_{\nu} Z^m + \Gamma_{\upsilon}{}^{i}{}_{m} P_{\mu} P_i Z^m)}{Z^2}) + \\
+2\dfrac{(P_{\mu} P_{\upsilon});_m Z^m)}{Z^2} - 2\dfrac{(\Gamma_{\mu}{}^{i}{}_{m} P_i P_{\nu} Z^m + \Gamma_{\upsilon}{}^{i}{}_{m} P_{\mu} P_i Z^m)}{Z^2}) + \\
+\dfrac{Z_{\mu} Z_{\nu}}{Z^2} - 2\dfrac{Z_s Z^s}{Z^3} P_{\mu} P_{\nu} - \dfrac{1}{2}\dfrac{Z_m Z^m}{(P^i P_i)^2} g_{\mu\nu}
\end{array} \right) \sqrt{-g} = \\
(-2(\dfrac{Z^m}{Z^2});_m P_{\mu} P_{\nu} - 2\dfrac{Z^{\lambda} Z_{\lambda}}{Z^2}\dfrac{P_{\mu} P_{\nu}}{Z} - \dfrac{1}{2}\dfrac{Z_k Z^k}{Z^2} g_{\mu\nu} + \dfrac{Z_{\mu} Z_{\nu}}{Z^2})\sqrt{-g}
\end{array}
$$


(42)

$$Z = P_\mu P^\mu \ and\ U_\lambda = \frac{Z_\lambda}{Z} - \frac{Z_k P^k P_\lambda}{Z^2} \ and\ L = U^\kappa U_k$$

$$\frac{\partial(L\sqrt{-g})}{\partial g^{\mu\upsilon}} - \frac{d}{dx^m}\frac{\partial(L\sqrt{-g})}{\partial g^{\mu\upsilon},_m} =$$

$$\left(\begin{array}{l} +2(\frac{(P^\lambda P_\lambda),_m P^m}{Z^3}P^k);_k P_\mu P_\nu + \\ 2\frac{(P^\lambda Z_\lambda)^2}{Z^3}\frac{P_\mu P_\nu}{Z} - 2(\frac{(P^\lambda P_\lambda),_s P^s}{Z^3})Z_\mu P_\nu + \\ +\frac{1}{2}\frac{(P^\lambda Z_\lambda)^2}{Z^3}g_{\mu\nu} + \frac{(P^\lambda Z_\lambda)^2}{Z^3}\frac{P_\mu P_\nu}{Z} + \\ (-2(\frac{Z^m}{Z^2});_m P_\mu P_\nu - 2\frac{Z^\lambda Z_\lambda}{Z^2}\frac{P_\mu P_\nu}{Z} - \frac{1}{2}\frac{Z_k Z^k}{Z^2}g_{\mu\nu} + \frac{Z_\mu Z_\nu}{Z^2}) \end{array}\right).\sqrt{-g} =$$

$$\left(\begin{array}{l} +2((\frac{(P^\lambda P_\lambda),_m P^m}{Z^3}P^k);_k - 2(\frac{Z^m}{Z^2});_m)P_\mu P_\nu + \\ +2\frac{(P^\lambda Z_\lambda)^2}{Z^3}\frac{P_\mu P_\nu}{Z} - 2\frac{Z^\lambda Z_\lambda}{Z^2}\frac{P_\mu P_\nu}{Z} + \\ +\frac{1}{2}\frac{(P^\lambda Z_\lambda)^2}{Z^3}g_{\mu\nu} - \frac{1}{2}\frac{Z_k Z^k}{Z^2}g_{\mu\nu} + \\ +\frac{Z_\mu Z_\nu}{Z^2} - 2(\frac{(P^\lambda P_\lambda),_s P^s}{Z^3})Z_\mu P_\nu + \frac{(P^\lambda Z_\lambda)^2}{Z^3}\frac{P_\mu P_\nu}{Z} \end{array}\right).\sqrt{-g} =$$

$$\left(\begin{array}{l} +2((\frac{(P^\lambda P_\lambda),_m P^m}{Z^3}P^k);_k - 2(\frac{Z^m}{Z^2});_m)P_\mu P_\nu + \\ +2\frac{(P^\lambda Z_\lambda)^2}{Z^3}\frac{P_\mu P_\nu}{Z} - 2\frac{Z^\lambda Z_\lambda}{Z^2}\frac{P_\mu P_\nu}{Z} + \\ +U_\mu U_\nu - \frac{1}{2}U_k U^k g_{\mu\nu} \end{array}\right).\sqrt{-g} = \tag{43}$$

$$(U_\mu U_\nu - \frac{1}{2}U_k U^k g_{\mu\nu} - 2U^k;_k \frac{P_\mu P_\nu}{Z})\sqrt{-g}$$

$$L = \frac{(Z^s P_s)^2}{Z^3} \quad s.t.\ Z = P^\lambda P_\lambda \ and\ Z_m = (P^\lambda P_\lambda),_m$$

$$\frac{\partial(L\sqrt{-g})}{\partial P_\mu} - \frac{d}{dx^\nu}\frac{\partial(L\sqrt{-g})}{\partial P_\mu,_\nu} =$$

$$\left( \begin{array}{l} -4(\frac{(Z_s P^s)}{Z^3} P^\mu P^\nu);_\nu + 4\frac{(Z_s P^s)}{Z^3}\Gamma_i{}^\mu{}_\nu P^i P^\nu + \\ +4\frac{(Z_s P^s)}{Z^3} P^\mu;_\nu P^\nu - 4\frac{(Z_s P^s)}{Z^3}\Gamma_i{}^\mu{}_k P^i P^k + \\ +2\frac{Z_m P^m Z^\mu}{Z^3} - 6\frac{(Z_m P^m)^2}{Z^4} P^\mu \end{array} \right)\sqrt{-g} =$$

$$(-4(\frac{(Z_s P^s)P^\nu}{Z^3});_\nu P^\mu + 2\frac{Z_m P^m Z^\mu}{Z^3} - 6\frac{(Z_m P^m)^2}{Z^4} P^\mu)\sqrt{-g} \quad (44)$$

$$L = \frac{Z^s Z_s}{Z^2} \quad s.t.\ Z = P^\lambda P_\lambda \ and\ Z_m = (P^\lambda P_\lambda),_m$$

$$\frac{\partial(L\sqrt{-g})}{\partial P_\mu} - \frac{d}{dx^\nu}\frac{\partial(L\sqrt{-g})}{\partial P_\mu,_\nu} =$$

$$\left( \begin{array}{l} -4(\frac{P^\mu Z^\nu}{Z^2});_\nu + \frac{4}{Z^2}\Gamma_i{}^\mu{}_k P^i Z^k + \\ +\frac{4}{Z^2} P^\mu;_\nu Z^\nu - \frac{4}{Z^2}\Gamma_i{}^\mu{}_k P^i Z^k + \\ -4\frac{Z_m Z^m}{Z^3} P^\mu \sqrt{-g} \end{array} \right)\sqrt{-g} = \quad (45)$$

$$(-4(\frac{Z^\nu}{Z^2});_\nu - 4\frac{Z_m Z^m}{Z^3})P^\mu \sqrt{-g}$$

From (38), (43), (44) and (45), we get two tensor equations of gravity, assuming $\sigma = 8\pi$, where the metric variation equations (38) and (43) yield,

$$
\begin{array}{|l|}
\hline
Z = N^2 = P_\mu P^\mu \ ,\ U_\lambda = \frac{Z_\lambda}{Z} - \frac{Z_k P^k P_\lambda}{Z^2}\ , L = \frac{1}{4} U_i U^i \ \text{ and } Z = P^k P_k \\
\frac{8\pi}{\sigma}\frac{1}{4}\left( \begin{array}{l} +2((\frac{(P^\lambda P_\lambda),_m P^m}{Z^3} P^k);_k - 2(\frac{Z^m}{Z^2});_m ) P_\mu P_\nu + \\ +2\frac{(P^\lambda Z_\lambda)^2}{Z^3}\frac{P_\mu P_\nu}{Z} - 2\frac{Z^\lambda Z_\lambda}{Z^2}\frac{P_\mu P_\nu}{Z} + \\ + U_\mu U_\nu - \frac{1}{2} U_k U^k g_{\mu\nu} \end{array} \right). = \\
\frac{8\pi}{\sigma}\frac{1}{4}(U_\mu U_\nu - \frac{1}{2} U_k U^k g_{\mu\nu} - 2U^k;_k \frac{P_\mu P_\nu}{Z}) = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \\
\text{s.t. } \ R = R_{\mu\upsilon} g^{\mu\upsilon} \\
\text{s.t. } \ R_{kj} = (\Gamma_{jk}{}^P),_p - (\Gamma_{pk}{}^P),_j + \Gamma_{p\mu}{}^P \Gamma_{jk}{}^\mu - \Gamma_{pj}{}^\mu \Gamma_{k\mu}{}^p \\
\hline
\end{array}
\quad (46)
$$

$R_{\mu\nu}$ is the Ricci tensor.

In general by (7) (46) can be written as

$$\frac{1}{4}\frac{8\pi}{\sigma}(U_\mu U_\nu - \frac{1}{2} U_k U^k g_{\mu\nu} - 2U^k;_k \frac{P_\mu P_\nu}{Z}) = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

We can also see that the ordinary local conservation laws are modified if $U^k;_k \neq 0$ unless the local average around charge $(-2\overline{U}^k;_k \frac{\overline{P}_\mu \overline{P}_\nu}{\overline{Z}});^\nu = 0$ which is expected due to symmetry around charge.

**Different charges will fall at the same speed**, for a detailed proof of conservation see “APPENDIX - Conservation, Why Do Different Charges Fall At The Same Speed”.

**Majorana - like field:** *The term* $-2U^k;_k \frac{P_\mu P_\nu}{Z}$ *in (46) can be generalized to:*

$-2((U^k;_k + U^{*k};_k)/2)\frac{(P_\mu P^*_\nu + P^*_\mu P_\nu)/2}{Z}$ *and can be zero under the following condition:*

$$4(A_{\mu\nu};^\mu \frac{P^{*\nu}}{\sqrt{Z}} + A^*_{\mu\nu};^\mu \frac{P^\nu}{\sqrt{Z}}) = U_\mu U^{*\mu} + U^*_\mu U^\mu \Rightarrow U^k;_k + U^{*k};_k = 0$$

*The complimentary matrix* $B^{\mu\nu} = \frac{1}{\sqrt{2}} A_{\alpha\beta}\varepsilon^{\alpha\beta\mu\nu}$ *can be transformed to a real matrix due to the SU(2) x U(1) degrees of freedom and also be imaginary.*

**Important:** The force represented by $U_\mu$ may greatly differ from what conventional physics defines as mutual interaction forces. This is because such a force is a property of space-time (although not anticipated from the metric alone) and as such, it can be construed as an interaction with space-time itself, i.e. all matter. Please note that $U_\mu$ is a curvature vector which means time can't be measured along geodesic curves as these curves are prohibited in the field (17),(18).

Denoting $Z_L \equiv \frac{dZ}{dx^L}$, from the vanishing of the divergence of Einstein-Grossmann's tensor we have:

$$
\begin{aligned}
&(U^\mu U^\nu - \frac{1}{2}U_k U^k g^{\mu\nu} - 2U^k;_k \frac{P^\mu P^\nu}{Z});_\nu = \\
&U^\mu U^\nu;_\nu + U^\nu U^\mu;_\nu \\
&-\frac{1}{2}(U_k;_\nu U_s g^{ks} g^{\mu\nu} + U_s;_\nu U_k g^{ks} g^{\mu\nu}) - (2U^k;_k \frac{P^\mu P^\nu}{Z});_\nu = \\
&U^\mu U^k;_k + U^k U^\mu;_k - U^k U_k;^\mu - (2U^k;_k \frac{P^\mu P^\nu}{Z});_\nu = \\
&U^\mu U^k;_k + \\
&U^k((\frac{Z^\mu}{Z});_k - (\frac{Z_k}{Z});^\mu + (\frac{Z_L P^L}{Z^2}P^\mu);_k - (\frac{Z_L P^L}{Z^2}P_k);^\mu) - (2U^k;_k \frac{P^\mu P^\nu}{Z});_\nu = \\
&U^\mu U^k;_k + U^k((\frac{Z_L P^L}{Z^2}),_k P^\mu - (\frac{Z_L P^L}{Z^2});^\mu P_k) - (2U^k;_k \frac{P^\mu P^\nu}{Z});_\nu = \\
&U^\mu U^k;_k + U^k(\frac{Z_L P^L}{Z^2}),_k P^\mu - (2U^k;_k \frac{P^\mu P^\nu}{Z});_\nu = 0 = 4(R^{\mu k} - \frac{1}{2}Rg^{\mu k});_k
\end{aligned}
\qquad (47)
$$

The term $U^k(\frac{Z_L P^L}{Z^2}),_k P^\mu$ is due to the fact that $U^k = \frac{Z^k}{Z} - \frac{Z_L P^L}{Z^2}P^k$ and $U^k P_k = 0$ from which $U^k(-(\frac{Z_L P^L}{Z^2});^\mu P_k) = 0$. The bottom line is that it is possible that $-(2U^k;_k \frac{P^\mu P^\nu}{Z});_\nu \neq 0$, though by local averaging, due to symmetry, it is expected that $-(2\overline{U}^k;_k \frac{\overline{P}^\mu \overline{P}^\nu}{\overline{Z}}{}^k);_\nu = 0$. We will later refer to $-2U^k;_k \frac{P^\mu P^\nu}{Z}$ as "electro-gravity".

We can now explore the relation between local parallel translation in loops and non-zero divergence $U^k;_k \neq 0$. Contraction of (46) by $\frac{P^\mu P^\nu}{Z}$ yields

$$
\begin{aligned}
&\frac{1}{4}\frac{8\pi}{\sigma}(U_\mu U_\nu - \frac{1}{2}U_k U^k g_{\mu\nu} - 2U^k;_k \frac{P_\mu P_\nu}{Z})\frac{P^\mu P^\nu}{Z} = (R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu})\frac{P^\mu P^\nu}{Z} \Rightarrow \\
&\frac{1}{4}\frac{8\pi}{\sigma}(-\frac{1}{2}U_k U^k - 2U^k;_k) = R_{\mu\nu}\frac{P^\mu P^\nu}{Z} - \frac{1}{2}R
\end{aligned}
\qquad (48)
$$

Contraction of (46) by $g^{\mu\nu}$ yields

$$\frac{1}{4}\frac{8\pi}{\sigma}(-\frac{1}{2}U_{\mathrm{k}}U^{k}-U^{k};_{k})=-\frac{1}{2}R \tag{49}$$

So from both equations we have,

$$-\frac{1}{4}\frac{8\pi}{\sigma}U^{k};_{k}=R_{\mu\nu}\frac{P^{\mu}P^{\nu}}{Z} \tag{50}$$

If the divergence $U^{k};_{k}$ is related to electric charge then this result implies that even photons generate pairs of oscillating charge. By the definition of the Ricci tensor

$$R_{\mu\nu}=R^{L}{}_{\mu L\nu}\Rightarrow R_{\mu\nu}\frac{P^{\mu}P^{\nu}}{Z}=R^{L}{}_{\mu L\nu}\frac{P^{\mu}P^{\nu}}{Z}= \\ (P^{L};_{L};_{\nu}-P^{L};_{\nu};_{L})\frac{P^{\nu}}{Z} \tag{51}$$

This is because by the definition of the Riemann curvature tensor, the anti-symmetric derivation of a vector field $V^{L}$ yields $V^{L};_{j};_{k}-V^{L};_{k};_{j}=R^{L}{}_{\mu j\mathrm{k}}V^{\mu}$ and so

$(V^{L};_{j};_{k}-V^{L};_{k};_{j})V^{k}=R^{L}{}_{\mu j\mathrm{k}}V^{\mu}V^{k}$ and therefore $(V^{L};_{L};_{k}-V^{L};_{k};_{L})V^{k}=R^{L}{}_{\mu L\mathrm{k}}V^{\mu}V^{k}=R_{\mu\mathrm{k}}V^{\mu}V^{k}$

$$\boxed{\begin{aligned}&-\frac{1}{4}\frac{8\pi}{\sigma}U^{k};_{k}=R_{\mu\nu}\frac{P^{\mu}P^{\nu}}{Z}\Rightarrow\\&-\frac{1}{4}\frac{8\pi}{\sigma}U^{k};_{k}=(P^{L};_{L};_{\nu}-P^{L};_{\nu};_{L})\frac{P^{\nu}}{Z}\end{aligned}} \tag{52}$$

**Definition**: We will call the latter "**Charge Holonomy Equation**".

Or in its complex form,

$$\boxed{-\frac{1}{8}\frac{8\pi}{\sigma}(U^{k};_{k}+U^{*k};_{k})=\mathrm{Re}((P^{L};_{L};_{\nu}-P^{L};_{\nu};_{L})\frac{P^{*\nu}}{Z})} \tag{53}$$

And if $\sigma=8\pi$ we have $-\frac{1}{4}U^{k};_{k}=(P^{L};_{L};_{\nu}-P^{L};_{\nu};_{L})\frac{P^{\nu}}{Z}$ in the real case and $-\frac{1}{8}(U^{k};_{k}+U^{*k};_{k})=\mathrm{Re}((P^{L};_{L};_{\nu}-P^{L};_{\nu};_{L})\frac{P^{*\nu}}{Z})$ in the complex case.

For $(P^L;_L;_\nu - P^L;_\nu;_L)\frac{P^{*\nu}}{Z}$ not to be zero, it requires that locally, parallel translation along different paths that connect two events, will yield different results. Obviously because $P$ is a scalar field, in flat geometry $(P^k;_L;_\nu - P^k;_\nu;_L) = 0$ and in curved geometry we can have $(P^k;_L;_\nu - P^k;_\nu;_L) \neq 0$.

By Lee C. Loveridge [22], another illuminating equation which is simple if $\sigma = 8\pi$ is,

$$R(3) = \frac{8\pi}{\sigma}\left(\frac{1}{4}U^\lambda U_\lambda + U^k;_k\right) \tag{54}$$

or in the complex case $R(3) = \frac{8\pi}{\sigma}\frac{1}{2}\left(\frac{1}{4}(U^\lambda U^*_\lambda + U^{*\lambda} U_\lambda) + U^k;_k + U^{*k};_k\right)$

Where $R(3)$ is a three dimensions scalar curvature in the space perpendicular to $P_\mu = P,_\mu$ or in the complex case, to $(P_\mu + P^*_\mu)/2$. (54) can be viewed as a non linear version of Proportional to square error – Differential case of PID control which is well known to physicists who also have background in engineering. $\frac{1}{4}U^\lambda U_\lambda$ is the square proportional term and $U^k;_k$ is the differential term. The three dimensional scalar curvature $R(3)$ is merely an output of such a control system. Note that both $R(3)$ and $\frac{1}{4}U^\lambda U_\lambda$ are curvature terms where $U_\lambda$ describes the curvature of the gradient of the time field i.e. the curvature of $P_\mu$.

From (44),(45) we have,

$$\frac{d}{dx^\mu}\left(\frac{\partial}{\partial P_\mu} - \frac{d}{dx^\nu}\frac{\partial}{\partial P_{\mu},_\nu}\right)(U_k U^k\sqrt{-g}) = W^\mu;_\mu\sqrt{-g} = 0$$

We recall, $W^\mu = \left(\frac{\partial}{\partial P_\mu} - \frac{d}{dx^\nu}\frac{\partial}{\partial P_{\mu},_\nu}\right)(U_k U^k\sqrt{-g})$

$$W^{\mu} =$$

$$(-4(\frac{Z^{\nu}}{Z^{2}});_{\nu} -4\frac{Z_{\mathrm{m}}Z^{m}}{Z^{3}})\mathrm{P}^{\mu} + 4(\frac{(Z_{s}P^{s})P^{\nu}}{Z^{3}});_{\nu} P^{\mu} - 2\frac{Z_{\mathrm{m}}P^{m}Z^{\mu}}{Z^{3}} + 6\frac{(Z_{m}P^{m})^{2}}{Z^{4}}P^{\mu} =$$

$$-4(\frac{Z^{\nu}}{Z^{2}});_{\nu} \mathrm{P}^{\mu} - 4\frac{Z_{\mathrm{m}}Z^{m}}{Z^{3}}\mathrm{P}^{\mu} +$$

$$+4(\frac{(Z_{s}P^{s})P^{\nu}}{Z^{3}});_{\nu} P^{\mu} + 4\frac{(Z_{m}P^{m})^{2}}{Z^{4}}P^{\mu}$$

$$-2\frac{Z_{\mathrm{m}}P^{m}}{Z^{2}}(\frac{Z^{\mu}}{Z} - \frac{Z_{m}P^{m}P^{\mu}}{Z^{2}}) =$$

$$-4((\frac{U^{k}}{Z});_{k} + \frac{U^{k}U_{k}}{Z})P^{\mu} - 2\frac{Z_{\mathrm{m}}P^{m}}{Z^{2}}U^{\mu} = 0$$

$$\boxed{W^{\mu};_{\mu} = \left(-4U^{\nu};_{\nu}\frac{P^{\mu}}{Z} - 2\frac{(Z_{m}P^{m})}{Z^{2}}U^{\mu}\right);_{\mu} = 0} \tag{55}$$

A simpler solution to zero Euler Lagrange equations, is

$$(\frac{\partial}{\partial P_{\mu}} - \frac{d}{dx^{\nu}}\frac{\partial}{\partial P_{\mu},_{\nu}})(U_{k}U^{k}\sqrt{-g}) = 0 \tag{56}$$

Which results in a special case, “Zero Charge” as charged particles are related to non-zero divergences and either $U^{\mu}U_{\mu} = 0$ or $Z_{m}P^{m} = 0$ .

$$(U^{\nu});_{\nu} = 0 \tag{57}$$

and (46) becomes,

$$\frac{8\pi}{\sigma}\frac{1}{4}(U_{\mu}U_{\nu} - \frac{1}{2}U_{\mathrm{k}}U^{k}g_{\mu\nu}) = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \tag{58}$$

The reader can either refer to the following calculation or skip it.

$$(-4(\frac{Z^{\nu}}{Z^{2}});_{\nu}-4\frac{Z_{\mathrm{m}}Z^{m}}{Z^{3}})\mathrm{P}^{\mu}+4(\frac{(Z_{s}P^{s})P^{\nu}}{Z^{3}});_{\nu}P^{\mu}$$
$$-2\frac{Z_{\mathrm{m}}P^{m}Z^{\mu}}{Z^{3}}+6\frac{(Z_{m}P^{m})^{2}}{Z^{4}}P^{\mu}=$$
$$-4(\frac{Z^{\nu}}{Z^{2}});_{\nu}\mathrm{P}^{\mu}-4\frac{Z_{\mathrm{m}}Z^{m}}{Z^{3}}\mathrm{P}^{\mu}+$$
$$+4(\frac{(Z_{s}P^{s})P^{\nu}}{Z^{3}});_{\nu}P^{\mu}+4\frac{(Z_{m}P^{m})^{2}}{Z^{4}}P^{\mu} \tag{59}$$
$$-2\frac{Z_{\mathrm{m}}P^{m}}{Z^{2}}(\frac{Z^{\mu}}{Z}-\frac{Z_{m}P^{m}P^{\mu}}{Z^{2}})=$$
$$-4((\frac{U^{k}}{Z});_{k}+\frac{U^{k}U_{k}}{Z})P^{\mu}-2\frac{Z_{\mathrm{m}}P^{m}}{Z^{2}}U^{\mu}=0$$

Recall that $U^{k}P_{k}=0$, multiplication by $\frac{-P_{\mu}}{4}$ and contraction yields,

$$((\frac{Z^{\nu}}{Z^{2}});_{\nu}-(\frac{(Z_{s}P^{s})P^{\nu}}{Z^{3}});_{\nu})Z+\frac{Z_{\mathrm{m}}Z^{m}}{Z^{2}}-\frac{(Z_{m}P^{m})^{2}}{Z^{3}}=0 \tag{60}$$

$$(\frac{Z^{\nu}}{Z^{2}});_{\nu}-(\frac{(Z_{s}P^{s})P^{\nu}}{Z^{3}});_{\nu}+\frac{1}{Z}(\frac{Z_{\mathrm{m}}Z^{m}}{Z^{2}}-\frac{(Z_{m}P^{m})^{2}}{Z^{3}})=0 \tag{61}$$

and as a result of (61), the following term from (46) vanishes,

$$-2(U^{k});_{k}\frac{P^{\mu}P^{\nu}}{Z}=-2(\frac{U^{k}}{Z});_{k}P^{\mu}P^{\nu}-2U^{k}U_{k}\frac{P^{\mu}P^{\nu}}{Z}=$$
$$2\left((\frac{Z^{m}}{Z^{2}});_{m}-(\frac{(\mathrm{P}^{\lambda}\mathrm{P}_{\lambda}),_{\mathrm{m}}P^{m}}{Z^{3}}P^{k});_{k}+\frac{1}{Z}(\frac{Z^{\lambda}Z_{\lambda}}{Z^{2}}-\frac{(Z_{\mathrm{s}}P^{s})^{2}}{Z^{3}})\right)P^{\mu}P^{\nu}=0 \tag{62}$$

which yields a simpler equation (58). Recall that $U^{\nu}=\frac{Z^{\nu}}{Z}-\frac{(Z_{s}P^{s})P^{\nu}}{Z^{2}}$,

And that $\frac{Z_{\nu}}{Z}U^{\nu}=U_{\nu}U^{\nu}$

$$(\frac{Z^{\nu}}{Z^{2}});_{\nu}-(\frac{(Z_{s}P^{s})P^{\nu}}{Z^{3}});_{\nu}+\frac{1}{Z}(\frac{Z_{\mathrm{m}}Z^{m}}{Z^{2}}-\frac{(Z_{m}P^{m})^{2}}{Z^{3}})=$$
$$(\frac{U^{\nu}}{Z});_{\nu}+\frac{1}{Z}(U_{\mathrm{m}}U^{m})=\frac{1}{Z}(U^{\nu});_{\nu}-\frac{1}{Z^{2}}U^{\nu}Z_{\nu}+\frac{1}{Z}(U_{\mathrm{m}}U^{m})= \tag{63}$$
$$\frac{1}{Z}(U^{\nu});_{\nu}=0$$

which proves (57).

**Question to the reader:** If $U_\mu U_\nu - \frac{1}{2}U_k U^k g_{\mu\nu} - 2U^k;_k \frac{P_\mu P_\nu}{Z}$ describes the electro-magnetic energy momentum tensor, where is the torsion tensor $F_{\mu\nu} = -F_{\nu\mu}$ that is so basic to the electro-magnetic theory?

**Answer:** $U_\mu$ is not any electro-magnetic field. It is a property of space-time as $P = \tau$ is dictated by the equations of gravity. $U_\mu$, however, offers a way to describe an anti-symmetric tensor which is a singular Lie Algebra matrix as an acceleration field via Tzvi Scarr and Yaakov Friedman representation [7], $\frac{U_\nu}{2} = A_{\nu\mu} \frac{P^{*\mu}}{\sqrt{(P^{*\lambda} P_\lambda + P^\lambda P^*_\lambda)/2}}$ such that $A_{\mu\nu} = -A_{\nu\mu}$. See "Appendix – Acceleration field representation". The field is actually represented by $A_{\mu\nu}$ but it is not an electromagnetic field, rather, it is the underlying mechanism that results in what we call, the electric field. In the complex formalism either, $\frac{U^*_\nu}{2} = (A_{\nu\mu} \frac{P^{*\mu}}{\sqrt{(P^{*\lambda} P_\lambda + P^\lambda P^*_\lambda)/2}})^*$ or $\frac{U_\nu}{2} = A_{\nu\mu} \frac{P^{*\mu}}{\sqrt{(P^{*\lambda} P_\lambda + P^\lambda P^*_\lambda)/2}}$. Increasing or decreasing $\frac{C^4}{\sigma K}\frac{1}{8}(U^*_\nu U^\nu + U_\nu U^{*\nu})$ results in change of Energy density and in the phenomena we call Electro-Magnetism. $A_{\mu\nu}$ represents a non-inertial acceleration of every particle that can measure proper time and not of photons as single particles.

**Inertia Tensor:** *We define inertia tensor as* $\frac{8\pi}{\sigma}\frac{1}{4}\left(U_\mu U_\nu - \frac{1}{2}U_k U^k g_{\mu\nu}\right)$ *this reflects what we know as energy momentum tensor.*

Question: why is the tensor $-\frac{2}{4}U^k;_k \frac{P_\mu P_\nu}{\sqrt{P^k P_k}}$ not included in the Inertia Tensor ?

Answer: Because the term $\frac{P_\mu P_\nu}{\sqrt{P^k P_k}}$ is not a material field.

**Electro gravity tensor:** *We define the electro-gravity tensor as* $-\frac{2}{4}U^k;_k \frac{P_\mu P_\nu}{\sqrt{P^k P_k}}$.

As we shall see, $-\frac{1}{2}U^k;_k$ is equivalent to electric charge density $-\frac{1}{2}U^k;_k \approx -\sqrt{\frac{4\pi K}{\varepsilon_0}}\frac{\rho}{C^2}$ or by (7) $-\frac{1}{2}U^k;_k \approx -\sqrt{\frac{\sigma K}{2\varepsilon_0}}\frac{\rho}{C^2}$ if $\sigma \neq 8\pi$.

But the sign could be also plus. Charge conservation yields the following:

From, $(U_\mu U_\nu - \frac{1}{2}U_k U^k g_{\mu\nu} - 2U^k;_k \frac{P_\mu P_\nu}{Z})g^{\mu\nu} = -U_k U^k - 2U^k;_k$ it follows that $\frac{1}{4}\int_{\Omega}(-U_{k}U^{k}-2U^{k};_{k})\sqrt{-g}d\Omega=\frac{1}{4}\int_{\Omega}-U_{k}U^{k}\sqrt{-g}d\Omega=\int_{\Omega}-R\sqrt{-g}d\Omega$.

**Construction and Destruction:** *We define construction or destruction as local appearance and disappearance of non zero* $-\frac{1}{2}U^{k};_{k}$ *neighborhoods of space-time as a function of* $\tau$ *. This definition alludes to the well known terms of construction and destruction brackets in Quantum Mechanics.*

## 8. RESULTS - ELECTRO-GRAVITATIONAL THRUST, DARK MATTER AND DARK ENERGY

**Dark Matter:** Dark Matter will be defined as additional Gravity not due to the Inertia Tensor. It is meant that the cause of such gravity is not inertial mass that resists non-inertial acceleration. It also emanates from the acceleration field as expressed by the Scarr-Friedman matrix [7], $A_{\mu\nu}=-A_{\nu\mu}$. This field prohibits Minkowsky geodesic motion of rest mass.

**Dark Energy:** Dark Energy will be defined as negative Gravity not due to the Inertia Tensor. It is meant that the cause of such gravity is not inertial mass that resists non-inertial acceleration. Also in this case electro-gravity is not the only cause. The acceleration field $A_{\mu\nu}=-A_{\nu\mu}$ that prohibits Minkowsky geodesic motion of rest mass must also be taken into account. The following will describe a technology that can take energy from space-time apparently by Sciama Inertial Induction [23] and is closely related to Alcubierre Warp Drive [24]. Electro-gravity follows from (6), (46) and (55). For several reasons we may assume the weak acceleration of uncharged particles mentioned in (6) is from positive to negative charge see also [8], consider the general relativity equation

$\boxed{\frac{1}{4}(U_{\mu}U_{\nu}-\frac{1}{2}U_{k}U^{k}g_{\mu\nu}-2U^{k};_{k}\frac{P_{\mu}P_{\nu}}{Z})=G_{\mu\nu}=R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}}$ such that the Ricci tensor is $R_{kj}=(\Gamma_{jk}{}^{P}),_{p}-(\Gamma_{pk}{}^{P}),_{j}+\Gamma_{p\mu}{}^{P}\Gamma_{jk}{}^{\mu}-\Gamma_{pj}{}^{\mu}\Gamma_{k\mu}{}^{p}$

$G_{\mu\nu}$ is the Einstein-Grossmann's tensor. From (4) in a weak gravitational background field and $\sigma=8\pi$,

$$\frac{1}{2}U_{m}=\frac{1}{2}(\frac{(P^{\lambda}P_{\lambda}),_{m}}{P^{i}P_{i}}-\frac{(P^{\lambda}P_{\lambda}),_{\mu}P^{\mu}}{(P^{i}P_{i})^{2}}P_{m})\approx\frac{a}{C^{2}}=\sqrt{4\pi K\varepsilon_{0}}\frac{E_{m}}{C^{2}} \tag{64}$$

$$\frac{a}{C^{2}}=\sqrt{\frac{\sigma K\varepsilon_{0}}{2}}\frac{E_{m}}{C^{2}}$$

$C$ is the speed of light, $a$ is the non-relativistic weak acceleration of an uncharged particle, $\varepsilon_0$ is the permittivity constant in vacuum, $K$ is the gravitational constant and $E$ is a static non-relativistic electric field in weak gravity, assuming that by correct choice of coordinates,

$$E_{m}=(E_{0}=0,E_{1},E_{2},E_{3}) \tag{65}$$

and also

$$E^{k};_{0}=0 \tag{66}$$

The reader is requested to notice that (70) is only a non-relativistic and non-covariant limit! From electro-magnetism

$$E^{k};_{k}=\frac{\rho}{\varepsilon_{0}} \tag{67}$$

Such that $\rho$ is the charge density

$$\frac{8\pi}{\sigma}\frac{1}{4}(U_\mu U_\nu - \frac{1}{2}U_k U^k g_{\mu\nu})$$
$$\approx 8\pi K \frac{\varepsilon_0}{2}\frac{1}{C^4}(E_\mu E_\nu - \frac{1}{2}E_k E^k g_{\mu\nu}) \quad (68)$$

Or by (7) if $\sigma \neq 8\pi$ (68) becomes

$$-\frac{1}{4}2U^k;_k \frac{P_\mu P_\nu}{Z} \approx \sqrt{\frac{\sigma K \varepsilon_0}{2}}\frac{E^k;_k}{C^2}\frac{P_\mu P_\nu}{Z} \quad (69)$$

From the electro-magnetic theory $E^k;_k = \frac{\rho}{\varepsilon_0}$ such that $\rho$ is the charge density and so for $t$ Schwarzschild coordinate time,

$$\sqrt{\frac{\sigma K}{2\varepsilon_0}}\frac{\rho}{C^2} \approx G_{tt} = G_{00} \quad (70)$$

So $\frac{1}{C^4}\sqrt{\frac{\sigma K}{2\varepsilon_0}}\rho C^2 \approx G_{tt} = G_{00}$ such that

$\frac{1}{\sigma}\sqrt{\frac{\sigma}{2\varepsilon_0 K}}\rho$ behaves like mass density and therefore we can define an electro-gravitational virtual mass as dependent on charge $Q$:

$$\boxed{M_{Virtual} = \frac{1}{\sigma}\sqrt{\frac{\sigma}{2\varepsilon_0 K}}Q = \frac{Q}{\sqrt{2\sigma\varepsilon_0 K}}} \quad (71)$$

We will calculate $Virtual\_Mass = \frac{\pm Q}{\sqrt{2\sigma\varepsilon_0 K}}$ for $\pm 20$ Coulombs by assuming $\sigma = 8\pi$,

$$\frac{\pm 1\,Coulomb}{\sqrt{16\pi\varepsilon_0 K}} = \pm 5.80233169106035888812808337934 17\times 10^9\,Kg.$$

Multiplied by 20 we have

$$\frac{\pm 20\,Coulombs}{\sqrt{16\pi\varepsilon_0 K}} = \pm 1.16046633821207177762561667586 83\times 10^{11}\,Kg.$$

Within 1 cubic meter the effect would be a feasible electro-gravitational field because Newton's gravitational acceleration as a rough approximation yields,

$$\frac{K \cdot Virtual\_Mass}{radius^2} =$$
$$K \cdot 1.16046633\ 8212071777\ 6256166758\ 683 \times 10^{11}\,Kg\,/1^2 =,$$
$$7.74477595\ 0343958808\ 9631666010\ 105\ \frac{\text{Meter}}{\text{Second}^2}$$

Consider parallel metal plates of 10cm x 10cm with a gap of 2cm and with low relative permittivity thin slab of 10 grams in the middle, such that the voltage of 37 Kilovolts is applied to the plates. The net classical non-relativistic gravitational force on the slab without regarding the non-geodesic acceleration field, is less than one third of a micro-Newton. That is a dauntingly small force which is very difficult to measure.

The calculations rule out any measurable vacuum thrust of Pico-Farad or less, asymmetrical capacitors even with 50000 volts supply, simply because the net effect depends on the total amount of separated charge which is far from sufficient in standard Biefeld Brown capacitors [25].

**Hypothetical use of sub-luminal photons:** It is yet needed to be demonstrated that sub-luminal photons do exist and that if very close to the speed of light can interact with electric currents and/or fields as if they are high density electric charge distributions. After (17) there is a discussion on the vanishing of the Reeb class based on [2][3] from which the restriction of $\mathrm{U}_{\mu}$ to the foliation $\mathrm{F}$ must have a zero rotor i.e. $\mathrm{U}_{\mu};_{\nu}(F) - \mathrm{U}_{\nu};_{\mu}(F) = 0$. This results in electric field which is always a result of electric charge, i.e. non vanishing divergence. The idea is that even photons can appear as a pair of oscillating negative and positive charge. Slight imbalance in charge distribution in photons can result in alleged effects such as EMDrive [26]. A slight 1% imbalance in +e and –e charge of say $10^{24}$ photons is equivalent to over $\pm 1600$ separated Coulombs and therefore if Hans Giertz assumption [11] holds, using photons for warp drive effects is the most feasible technological solution.

The $\frac{K \cdot Virtual\_Mass}{radius^2}$ shows a gravitational acceleration of over 126 gees between $\pm 1600$ coulombs separated by a gap of 2 meters. In such a case, if a photon behaves like a dynamic oscillating or rotating dipole, these dipoles will be of different dipole moments aligned or anti-aligned with relativistic electric fields. The result is equivalent to true charge separation and can have a great technological potential for the development of a feasible warp drive thrust.

**Use of plasma:** Another idea is to use ionized plasma. Let us see what we can do with one gram of ionized hydrogen. The number of atoms by Avogadro's number is $\mathrm{n} = 6.02214129 \times 10^{23}$. The charge of the electron is $\mathrm{e} = 1.602176565 \times 10^{-19}$ Coloumbs so $Q = \pm 9.64853364595686885 \times 10^{4}$ Coloumbs $K = 6.67384 \times 10^{-11} \mathrm{m}^{3}\mathrm{kg}^{-1}\mathrm{sec}^{-1}$ and $\varepsilon_0 = 8.8541878176.. \times 10^{-12}$ F/m so 1 gram of hydrogen reaches a virtual mass of $Virtual\_Mass \approx \pm 5.598399254619768891042208444814 \times 10^{14} Kg$. That is far less than the mass of the Earth $M_{Earth} = 5.97219 \times 10^{24} Kg$ but the distance between two clouds of positive and negative ionized hydrogen can be much less than the average Earth radius and therefore a field that overcomes the Earth gravitational field is feasible.

Dark Matter and Dark Energy follow immediately from negatively ionized gas in the galaxy and positively ionized gas outside or on the outskirts if we assume energy density $\frac{a_{\lambda} a^{\lambda}}{8\pi K}$ which means our electro-gravity constant is $Const = 8\pi K$. In other theories, e.g. $Const = K$, the center of the galaxy should be positive. Negative charge as in the first case $Const = 8\pi K$, will only behave as Dark Matter more at distance because close to stars it is expected to cause induced dipoles within the star just as a small negatively charged ball above the Earth will polarize the ground. We now consider the classical non-covariant limit of the summation of two effects, the non-inertial acceleration and electro-gravity. Let $Q$ be the charge of a ball at radius $r$, then by (7) the observed non-relativistic classical acceleration $a$ of an uncharged particle without any induced diploes is

$$a \approx \frac{-KQ}{r^2\sqrt{2\sigma K\varepsilon_0}} + \sqrt{\frac{\sigma K \varepsilon_0}{2}}\frac{Q}{4\pi\varepsilon_0 r^2} = \sqrt{\frac{\sigma K}{2\varepsilon_0}}\frac{Q}{r^2}\left(\frac{1}{\sigma} - \frac{1}{4\pi}\right) = g_{Electro\_gravity} - a$$

If $\sigma = 8\pi$ then $a \approx \sqrt{\frac{\sigma K}{2\varepsilon_0}}\frac{Q}{r^2}\left(\frac{1}{8\pi} - \frac{1}{4\pi}\right) = g_{Electro\_gravity} - a$

A friend, Mr. Yossi Avni suggested that two marble balls will be suspended on a balance above charged balls, left minus and right plus, and that by the checking if the balanced is tipped or not, we can decide $\sigma = 4\pi$ for unbiased balance and different value else, e.g. $\sigma = 8\pi$. There is a problem, however, that charge in the ground below the balls will polarize the ground.

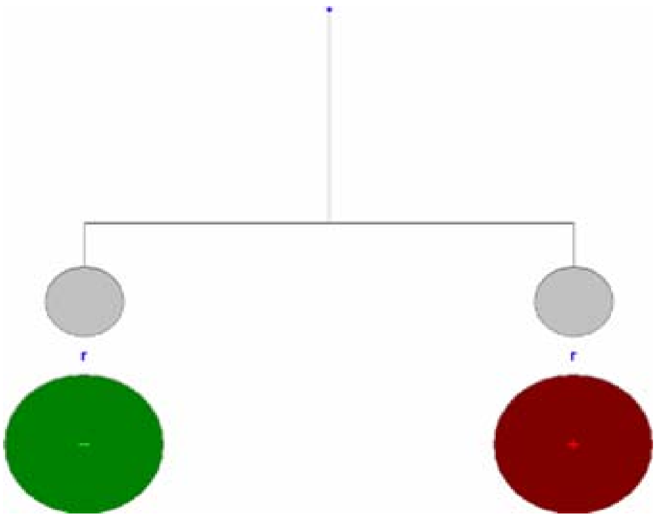

**Fig. 1. Avni suggestion, the left lower ball is negatively charged and the right is positively charged. The balls above are marble or even of lower dielectric constant**

This theory means that a positive charge manifests attracting gravity but has a repelling acceleration field that acts even on uncharged particles that can measure proper time, i.e. have rest mass. The curvature $U^k U_k < 0$ and for positive charge $U^k;_k < 0$.

Negative charge manifests a repelling anti-gravity but has an acceleration field that attracts even uncharged particles and acts on particles that can measure time, i.e. have rest mass.

The following table describes the relation between $\sigma$ and the Dark Matter and Dark Energy.

**Table 1. The relation between constants and Dark Matter**

| $\sigma$ and classical non-relativistic acceleration | Cause for dark matter | Cause for dark energy | Dark matter causes |
|---|---|---|---|
| $\sigma < 4\pi \Rightarrow$ $\sqrt{\frac{\sigma K}{2\varepsilon_0}\frac{Q}{r^2}(\frac{1}{\sigma}-\frac{1}{4\pi})} > 0$ | Positive charge | Negative charge | Caused by gravity |
| $\sigma = 4\pi \Rightarrow$ $\sqrt{\frac{\sigma K}{2\varepsilon_0}\frac{Q}{r^2}(\frac{1}{\sigma}-\frac{1}{4\pi})} = 0$ | Negative Charge | Positive charge | Caused only by an Alcubierre Warp Drive [24] by induced dipoles. Positive side faces the negative galaxy center. |
| $\sigma > 4\pi \Rightarrow$ $\sqrt{\frac{\sigma K}{2\varepsilon_0}\frac{Q}{r^2}(\frac{1}{\sigma}-\frac{1}{4\pi})} < 0$ | Negative Charge | Positive charge | Caused by the acceleration field on particles with rest mass |

## 9. CONCLUSION

An upper limit on measurable time from each event backwards to the "big bang" singularity as a limit or from a manifold of events as in de Sitter or anti - de Sitter, may exist only as a limit and is not a practical physical observable in the usual sense. Since more than one curve on which such time can be virtually measured, intersects the same event - as is the case in material fields which prohibit inertial motion, i.e. prohibit free fall - such time can't be realized as a coordinate. Nevertheless using such time as a scalar field, enables to describe matter as acceleration fields and it allows new physics to emerge as a replacement of the stress-energy-momentum tensor. The punch line is electro-gravity as a neat explanation of the Dark Matter effect and the advent of Sciama's Inertial Induction, which becomes realizable by separation of high electric charge. This paper totally rules out any measurable Biefeld Brown effect in vacuum on Pico-Farad or less, Ionocrafts due to insufficient amount of electric charge [25]. The electro-gravitational effect is due to field divergence and not directly due to intensity or gradient of the square norm. Inertial motion prohibition by material fields, e.g. intense electrostatic field, can be measured as a very small mass dependent force on neutral particles that have rest mass and thus can measure proper time. Such acceleration should be measured in very low capacitance capacitors in order to avoid electro-gravitational effect. The non-gravitational acceleration should be from the positive to the negative charge. The electro-gravitational effect is opposite in direction, requires large amounts of separated charge carriers and acts on the entire negative to positive dipole.

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Thanks to Mr. Yossi Avni who approved my research in 2003 into minimum cost diffeomorphism as part of a pattern matching algorithm. This early research helped in acquiring special knowledge in geometry that was later used in the presented theory.

Also a historical justice with the philosopher Rabbi Joseph Albo, Circa 1380-1444 must be made. In his book of principles, essay 18 appears to be the first known historical account of what Measurable Time – In Hebrew "Zman Meshoar" and Immeasurable Time "Zman Bilti Meshoar" are. His Idea of the immeasurable time as a limit [27], is the very reason for an 11 years of research and for this paper.

## COMPETING INTERESTS

Author has declared that no competing interests exist.

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   Follow up 2 with constants update: http://vixra.org/pdf/1407.0114v6.pdf
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# APPENDIX

## APPENDIX – The time field in the Schwarzschild solution

**Motivation:** To make the reader familiar with the idea of maximal proper time and to calculate the background scalar time field of the Schwarzschild solution.

We would like to calculate $\left(\frac{(P^{\lambda}P_{\lambda}),_m(P^sP_s),_k g^{mk}}{(P^iP_i)^2} - \frac{((P^{\lambda}P_{\lambda}),_m P^m)^2}{(P^iP_i)^3}\right)$ in Schwarzschild coordinates for a freely falling particle. This theory predicts that where there is no matter, the result must be zero. The result also must be zero along any geodesic curve but in the middle of a hollowed ball of mass the gradient of the absolute maximum proper time from "Big Bang" event or from a sub-manifold of events, derivatives by space must be zero due to symmetry which means the curves come from different directions to the same event at the center. Close to the edges, gravitational lenses due to granularity of matter become crucial. The speed $U$ of a falling particle as measured by an observer in the gravitational field is

$$V^2 = \frac{U^2}{C^2} = \frac{R}{r} = \frac{2GM}{rC^2} \tag{72}$$

Where $R$ is the Schwarzschild radius. If speed $V$ is normalized in relation to the speed of light then $V = \frac{U}{C}$. For a far observer, the deltas are denoted by $dt', dr'$ and,

$$\dot{r}^2 = (\frac{dr}{dt})^2 = V^2(1-\frac{R}{r}) \tag{73}$$

because $dr = dr'/\sqrt{1-R/r}$ and $dt = dt'\sqrt{1-R/r}$ .

$$P = \int_0^t \sqrt{(1-\frac{R}{r}) - \frac{(dr/dt)^2}{(1-\frac{R}{r})}}\,dt = \int_0^t \sqrt{(1-\frac{R}{r}) - \frac{\frac{R}{r}(1-\frac{R}{r})^2}{(1-\frac{R}{r})}}\,dt = \int_0^t \sqrt{(1-\frac{R}{r})^2}\,dt =$$
$$\int_0^t (1-\frac{R}{r})dt$$

Which results in,

$$P_t = \frac{dP}{dt} = (1-\frac{R}{r}) \tag{74}$$

Please note, here $t$ is not a tensor index and it denotes derivative by $t$ !!!
On the other hand

$$P=\int_{\infty}^{r}\sqrt{(1-\frac{R}{r})\frac{1}{\dot{r}^2}-\frac{1}{(1-\frac{R}{r})}}dr=\int_{\infty}^{r}\sqrt{\frac{(1-\frac{R}{r})\frac{r}{R}}{(1-\frac{R}{r})^2}-\frac{1}{(1-\frac{R}{r})}}dr=\int_{\infty}^{r}\sqrt{\frac{\frac{r-R}{R}}{\frac{r-R}{r}}}dr=$$

$$\int_{\infty}^{r}\sqrt{\frac{r}{R}}dr$$

Which results in

$$P_r=\frac{dP}{dr}=\sqrt{\frac{r}{R}} \tag{75}$$

Please note, here $r$ is not a tensor index and it denotes derivative by $r$ !!!
For the square norms of derivatives we use the inverse of the metric tensor,

So we have $(1-\frac{R}{r})\to\frac{1}{(1-\frac{R}{r})}$ and $\frac{1}{(1-\frac{R}{r})}\to(1-\frac{R}{r})$

So we can write

$$N^2=P^{\lambda}P_{\lambda}=(1-\frac{R}{r})P_r^{\,2}-\frac{1}{1-\frac{R}{r}}P_t^{\,2}=(1-\frac{R}{r})(\frac{r}{R}-1)=\frac{r}{R}+\frac{R}{r}-2$$

$$N^2=\frac{r}{R}+\frac{R}{r}-2 \tag{76}$$

$N^2{}_{\lambda}=\frac{dN^2}{dx^{\lambda}}$ And we can calculate

$$\frac{N^2{}_{\lambda}N^{2\lambda}}{(N^2)^2}=\frac{(1-\frac{R}{r})^2(\frac{1}{R}-\frac{R}{r^2})^2}{(\frac{r}{R}+\frac{R}{r}-2)^2} \tag{77}$$

We continue to calculate

$$N^2{}_tP_t=(1-\frac{R}{r})^2(\frac{1}{R}-\frac{R}{r^2})\sqrt{\frac{R}{r}}\ \text{ and }\ \frac{N^2{}_tP_t}{(1-\frac{R}{r})}=(1-\frac{R}{r})(\frac{1}{R}-\frac{R}{r^2})\sqrt{\frac{R}{r}} \tag{78}$$

Please note, here $t$ is not a tensor index and it denotes derivative by $t$ !!!

$$(1-\frac{R}{r})N^2{}_rP_r=(1-\frac{R}{r})(\frac{1}{R}-\frac{R}{r^2})\sqrt{\frac{r}{R}} \tag{79}$$

Please note, here $r$ is not a tensor index and it denotes derivative by $r$ !!!

$$N^2{}_\lambda P^\lambda = (1-\frac{R}{r})(\frac{1}{R}-\frac{R}{r^2})(\sqrt{\frac{r}{R}}-\sqrt{\frac{R}{r}})$$ And

$$(N^2{}_\lambda P^\lambda)^2 = (1-\frac{R}{r})^2(\frac{1}{R}-\frac{R}{r^2})^2(\frac{r}{R}+\frac{R}{r}-2) \tag{80}$$

So

$$\frac{(N^2{}_\lambda P^\lambda)^2}{(N^2)^3} = \frac{(1-\frac{R}{r})^2(\frac{1}{R}-\frac{R}{r^2})^2}{(\frac{r}{R}+\frac{R}{r}-2)^2} \tag{81}$$

And finally, from (77) and (81) we have,

$$\left(\frac{(\mathrm{P}^\lambda \mathrm{P}_\lambda),_{\mathrm{m}}(\mathrm{P}^s\mathrm{P}_s),_{\mathrm{k}}\, g^{mk}}{(\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})^2}-\frac{((\mathrm{P}^\lambda \mathrm{P}_\lambda),_{\mathrm{m}}\mathrm{P}^m)^2}{(\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})^3}\right)=$$
$$\frac{N^2{}_\lambda N^{2\lambda}}{(N^2)^2}-\frac{(N^2{}_\lambda P^\lambda)^2}{(N^2)^3}= \tag{82}$$
$$\frac{(1-\frac{R}{r})^2(\frac{1}{R}-\frac{R}{r^2})^2}{(\frac{r}{R}+\frac{R}{r}-2)^2}-\frac{(1-\frac{R}{r})^2(\frac{1}{R}-\frac{R}{r^2})^2}{(\frac{r}{R}+\frac{R}{r}-2)^2}=0$$

which shows that indeed the gradient of time measured, by a falling particle until it hits an event in the gravitational field, has zero curvature as expected.

**APPENDIX – Event Theory and Lèvy Process**

**Motivation**: To present one of several possible quantization offers of (38) without using Stochastic Calculus. To provide an alternative integration constraint that leads to a new possible theory – a particle is a non-zero curvature vector $U_\mu$ and a family of event functions, $\psi$. An interesting alternative to (39) is that $\psi$ is not a particle wave function but an event function, i.e. a collision with a particle in a 4 dimensional space-time,

$$\int_{\Omega^4(\tau)} \psi\psi^* \sqrt{-g}\, d\Omega^4 = 1 \tag{83}$$

And instead of $P=\tau\psi$ we choose $P=\sqrt{\tau}\psi$. The reader can verify that the same curvature vector in (37) is reached by the assignment $P=\sqrt{\tau}\psi$ if $\psi$ depends only on $\tau$.

Then from (40) we get a new random variable $\tau_{Event}$

$$\tau_{Event} = \int_{\Omega^4(\tau)} \tau\psi\psi^* \sqrt{-g}\, d\Omega^4 = \int_{\Omega^4(\tau)} PP^* \sqrt{-g}\, d\Omega^4 \tag{84}$$

and adding this constraint to the complex form of (38) we have the following variations system,

$$
\begin{aligned}
&P = \sqrt{\tau}\psi \\
&\mathrm{Z} = N^2 = \frac{\mathrm{P}_\mu \mathrm{P}*^\mu + \mathrm{P}*_\mu \mathrm{P}^\mu}{2} \text{ and } U_\lambda = \frac{Z_\lambda}{Z} - \frac{Z_k P*^k P_\lambda}{Z^2} \text{ and} \\
&\mathrm{L} = \frac{1}{4}(\frac{1}{2}(\mathrm{U}^\mathrm{k}\mathrm{U}*_\mathrm{k} + \mathrm{U}*^\mathrm{k} \mathrm{U}_\mathrm{k})) \\
&R = Ricci \text{ curvature,} \\
&Min\ Action = Min\int_\Omega \left(\frac{1}{2}R - L + \lambda PP*\right).\sqrt{-g}d\Omega \\
&\text{and} \\
&\int_\Omega PP*.\sqrt{-g}d\Omega = \tau_{Event} \\
&\lambda = \mathrm{Const}.
\end{aligned}
\tag{85}
$$

An alternative is by Sam Vaknin's approach as events as the collapse of the chronon field wave function $\psi$,

$$
\begin{aligned}
&L = \frac{1}{4}(\frac{1}{2}U_k U*^k + U*_k Uk)) \\
&Min\ Action = \int_\Omega \left(\frac{1}{2}R - L\right)\sqrt{-g}d\Omega \\
&P = (\lim_{n\to\infty}\psi(1) + \psi(2) + ... + \psi(n))\sqrt{Time\_Atom} \\
&\int_{\Omega^4} \psi(k)\psi*(k)\sqrt{-g}d\Omega^4 = 1 \\
&0 < j < k < \infty \Rightarrow \int_{\Omega^4} \psi(j)\psi*(k)\sqrt{-g}d\Omega^4 = 0
\end{aligned}
\tag{86}
$$

(85) is worthy of further research. To prove that (85) is consistent with quantum mechanics, e.g. Quantum Field Theory and that no other idea of how to quantize (38) is required, a family of solutions to (85) with complex functions $\psi_\tau$ should exist such that $\tau_{Event}$ will be increasing and will describe detection events of the same particle. Because we introduce a scalar field of time $\tau$, and use a probability function $\psi\psi*$, if the time $s$ is a Lèvy process [28] then we also need the following to hold,

$$
s - t = \int_\Omega (\psi(s)\psi*(s) - \psi(t)\psi*(t))\tau\sqrt{-g}d\Omega = \int_\Omega \psi(s-t)\psi*(s-t)\tau\sqrt{-g}d\Omega \tag{87}
$$

s.t. $s > t$. So actually a particle is a family of parameterized Lèvy measures $\psi(s)\psi*(s)$ such that $s$ itself denotes time and such that $\tau$ denotes the scalar field of time and such that the curvature vector field $U_j = \frac{Z,_j}{Z} - \frac{Z,_k P*^k P_j}{Z^2}$ is not zero, s.t. $P = \sqrt{\tau}\psi$ and $P_j = \frac{dP}{dx^j}$ and $x^j$ are local coordinates. This is one of the ways of the ways to try to quantize the theory. Showing that this offer works and agrees with Quantum Field Theory should be a subject of international research.

The philosophy here is that particles should emerge from geometry, stochastic processes and from stochastic calculus and not from algebra and is therefore against the concurrent physics mainstream.

**APPENDIX - Conservation, Why Do Different Charges Fall At The Same Speed**

**Theorem**: Conservation law of the real curvature field

From the vanishing of the divergence of Einstein tensor and (46) in the paper, we have to prove the following:

$$\boxed{\frac{1}{4}\left(U_\mu U_\nu - \frac{1}{2}U_k U^k g_{\mu\nu} - 2U^k;_k \frac{P_\mu P_\nu}{Z}\right);^\mu = G_{\mu\nu};^\mu = (R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu});^\mu = 0} \quad (88)$$

**Proof:**

From the zero variation by the time field (55) in the paper

$$\boxed{W^\mu;_\mu = \left(-4U^\nu;_\nu \frac{P^\mu}{Z} - 2\frac{(Z_m P^m)}{Z^2}U^\mu\right);_\mu = 0} \quad (89)$$

$$-\left(2U^\nu;_\nu \frac{P^\mu}{Z}\right);_\mu = \left(\frac{(Z_m P^m)}{Z^2}U^\mu\right);_\mu \quad (90)$$

$$\left(-2U^k;_k \frac{P^\mu P^\nu}{Z}\right);_\mu = \left(\frac{(Z_m P^m)}{Z^2}U^\mu\right);_\mu P^\nu - \left(2U^k;_k \frac{P^\mu}{Z}\right)P^\nu;_\mu =$$
$$\left(\frac{(Z_m P^m)}{Z^2}U^\mu\right);_\mu P^\nu - U^k;_k \frac{Z^\nu}{Z}$$
$$t \equiv Z_m P^m \quad (91)$$
$$\left(\frac{t}{Z^2}U^\mu\right);_\mu P^\nu - U^k;_k \frac{Z^\nu}{Z} = (\frac{t}{Z^2});_\mu U^\mu P^\nu + \frac{t}{Z^2}U^\mu;_\mu P^\nu - U^k;_k \frac{Z^\nu}{Z} =$$
$$-U^\mu;_\mu U^\nu + (\frac{t}{Z^2});_\mu U^\mu P^\nu$$

So

$$(-2U^{k};_{k}\frac{P^{\mu}P^{\nu}}{Z});_{\mu}=-U^{\mu};_{\mu}U^{\nu}+(\frac{t}{Z^{2}});_{\mu}U^{\mu}P^{\nu} \tag{92}$$

$$\begin{aligned}&\left(U^{\mu}U^{\nu}-\frac{1}{2}U_{\mathrm{k}}U^{k}g^{\mu\nu}-2U^{k};_{k}\frac{P^{\mu}P^{\nu}}{Z}\right);^{\mu}=\\&U^{\mu};_{\mu}U^{\nu}+U^{\mu}U^{\nu};_{\mu}-\frac{1}{2}(U_{\mathrm{k}};_{\mu}U_{s}+U_{\mathrm{k}}U_{s};_{\mu})g^{ks}g^{\mu\nu}-U^{\mu};_{\mu}U^{\nu}+(\frac{t}{Z^{2}});_{\mu}U^{\mu}P^{\nu}=\\&U^{\mu}U^{\nu};_{\mu}-\frac{1}{2}(U^{s}U_{s});^{\nu}+(\frac{t}{Z^{2}});_{\mu}U^{\mu}P^{\nu}=0\end{aligned} \tag{93}$$

Notice that

$$\begin{aligned}&U^{\mu}U^{\nu};_{\mu}-\frac{1}{2}U^{s}U_{s};^{\nu}=\\&U^{\mu}\left((\frac{Z_{k}}{Z});_{\mu}-(\frac{t}{Z^{2}});_{\mu}P_{k}-(\frac{t}{Z^{2}})P_{k};_{\mu}\right)g^{k\nu}-\\&U^{s}\left((\frac{Z_{s}}{Z});_{k}-(\frac{t}{Z^{2}});_{k}P_{s}-(\frac{t}{Z^{2}})P_{s};_{k}\right)g^{k\nu}=\\&-U^{\mu}(\frac{t}{Z^{2}});_{\mu}P^{\nu}\end{aligned} \tag{94}$$

Notice that $-(\frac{t}{Z^{2}});_{k}P_{s}U^{s}=0$ and therefore:

$$U^{\mu}U^{\nu};_{\mu}-\frac{1}{2}(U^{s}U_{s});^{\nu}+(\frac{t}{Z^{2}});_{\mu}U^{\mu}P^{\nu}=-U^{\mu}(\frac{t}{Z^{2}});_{\mu}P^{\nu}+(\frac{t}{Z^{2}});_{\mu}U^{\mu}P^{\nu}=0 \tag{95}$$

And we are done.

**APPENDIX - Majorana – Like fields**

By (16) in the paper, we can write

$$A_{\mu\nu}=\frac{P_{\nu}U_{\mu}}{2\sqrt{Z}}-\frac{P_{\mu}U_{\nu}}{2\sqrt{Z}}=\frac{P_{\nu}Z_{\mu}}{2Z^{3/2}}-\frac{P_{\mu}Z_{\nu}}{2Z^{3/2}} \tag{96}$$

$$2A_{\mu\nu}\frac{P*^{\nu}}{\sqrt{Z}}=2(\frac{P_{\nu}Z_{\mu}}{2Z^{3/2}}-\frac{P_{\mu}Z_{\nu}}{2Z^{3/2}})\frac{P*^{\nu}}{\sqrt{Z}}=U_{\mu} \qquad (97)$$

$$2U_{\mu};^{\mu}=4(A_{\mu\nu}\frac{P*^{\nu}}{\sqrt{Z}});^{\mu}=4(A_{\mu\nu};^{\mu}\frac{P*^{\nu}}{\sqrt{Z}}+A_{\mu\nu}(\frac{P*^{\nu};^{\mu}}{\sqrt{Z}}-\frac{P*^{\nu}Z^{\mu}}{2Z^{3/2}}))=$$
$$4(A_{\mu\nu};^{\mu}\frac{P*^{\nu}}{\sqrt{Z}}-A_{\mu\nu}\frac{P*^{\nu}Z^{\mu}}{2Z^{3/2}})=4(A_{\mu\nu};^{\mu}\frac{P*^{\nu}}{\sqrt{Z}}-(\frac{P_{\nu}Z_{\mu}}{2Z^{3/2}}-\frac{P_{\mu}Z_{\nu}}{2Z^{3/2}})\frac{P*^{\nu}Z^{\mu}}{2Z^{3/2}})=$$
$$4(A_{\mu\nu};^{\mu}\frac{P*^{\nu}}{\sqrt{Z}}-(\frac{P_{\nu}P*^{\nu}Z_{\mu}Z^{\mu}}{4Z^{3}}-\frac{P_{\mu}Z^{\mu}P*^{\nu}Z_{\nu}}{4Z^{3}}))= \qquad (98)$$
$$4(A_{\mu\nu};^{\mu}\frac{P*^{\nu}}{\sqrt{Z}}-(\frac{Z_{\mu}Z^{\mu}}{4Z^{2}}-\frac{P_{\mu}Z^{\mu}P*^{\nu}Z_{\nu}}{4Z^{3}}))=4A_{\mu\nu};^{\mu}\frac{P*^{\nu}}{\sqrt{Z}}-U_{\mu}U*^{\mu}$$

Zero charge can be written either as $4A_{\mu\nu};^{\mu}\frac{P*^{\nu}}{\sqrt{Z}}=U_{\mu}U*^{\mu}$

Or more formal as,

$$\boxed{4(A_{\mu\nu};^{\mu}\frac{P*^{\nu}}{\sqrt{Z}}+A*_{\mu\nu};^{\mu}\frac{P^{\nu}}{\sqrt{Z}})=U_{\mu}U*^{\mu}+U*_{\mu}U^{\mu}} \qquad (99)$$

An interesting case is if the curvature vector $U_{\mu}$ is a null vector.

Then we have $4(A_{\mu\nu};^{\mu}\frac{P*^{\nu}}{\sqrt{Z}}+A*_{\mu\nu};^{\mu}\frac{P^{\nu}}{\sqrt{Z}})=0$.

**Majorana fields revisited**

Suppose $q$ is a real geodesic scalar, i.e. its time-like gradient is parallel to geodesic curves. Then we can write $Y=q^{\alpha}q_{\alpha}$ and it is easy to show that $Y^{\mu}=aq^{\mu}$ for some scalar function $a$.

$$2(A_{\mu\nu}\frac{q^{\nu}}{\sqrt{Y}});^{\mu}=2A_{\mu\nu};^{\mu}\frac{q^{\nu}}{\sqrt{Y}}+2A_{\mu\nu}(\frac{q^{\nu};^{\mu}}{\sqrt{Y}}-\frac{q^{\nu}Y^{\mu}}{2Y^{3/2}})=$$
$$2A_{\mu\nu};^{\mu}\frac{q^{\nu}}{\sqrt{Y}}+2A_{\mu\nu}(\frac{q^{\nu};^{\mu}}{\sqrt{Y}}-\frac{aq^{\nu}q^{\mu}}{2Y^{3/2}})=2A_{\mu\nu};^{\mu}\frac{q^{\nu}}{\sqrt{Y}}=\tilde{U}^{\mu};_{\mu} \qquad (100)$$

Such that $\tilde{U}^{\mu}$ is the acceleration vector derived from $\frac{q^{\nu}}{\sqrt{Y}}$.

If $A_{\mu\nu};^{\mu}$ is purely imaginary then, $2(A_{\mu\nu};^{\mu}\frac{q^{\nu}}{\sqrt{Y}} + A*_{\mu\nu};^{\mu}\frac{q^{\nu}}{\sqrt{Y}}) = \tilde{U}^{\mu};_{\mu} + \tilde{U}*^{\mu};_{\mu} = 0$

**APPENDIX - Alternative description of the SU(3) symmetry**

The SU(2) x U(1) symmetry is well covered in DOI: 10.9734/PSIJ/2015/18291 and its corrigendum, both published in Physical Science International Journal. The SU(3) is briefly described so we will focus on the SU(3) symmetry. The best way to do so is to return to the Godbillon-Vey class and to the Reeb class. There are several ways to show the SU(3) symmetry, from a theorem by Frobenius, either by replacing $P_{\mu}$ with an expression that depends on 3 foliation holonomic gradient fields or by adding these fields in the rotor calculation of (17) and (18). For now we will adopt the second approach. We add three normalized gradients of scalars that change along the F foliation, This can always be done by integration along a grid that is defined in the foliation $F(P_k)$, We define the scalars $V1$, $V2$, $V3$ as integration along curves in the foliation and $V1_{\nu} \equiv \frac{dV1}{dx^{\nu}}$ gradient by the coordinates $x^{\nu}$.

$$\hat{A}_{\mu\nu} = \begin{pmatrix} (\frac{P_{\nu}}{\sqrt{P*_{\lambda} P^{\lambda}}} + \frac{V1_{\nu}}{\sqrt{V1*_{\lambda} V1^{\lambda}}} + \frac{V2_{\nu}}{\sqrt{V2*_{\lambda} V2^{\lambda}}} + \frac{V3_{\nu}}{\sqrt{V3*_{\lambda} V3^{\lambda}}}),_{\mu} - \\ (\frac{P_{\mu}}{\sqrt{P*_{\lambda} P^{\lambda}}} + \frac{V1_{\mu}}{\sqrt{V1*_{\lambda} V1^{\lambda}}} + \frac{V2_{\mu}}{\sqrt{V2*_{\lambda} V2^{\lambda}}} + \frac{V3_{\mu}}{\sqrt{V3*_{\lambda} V3^{\lambda}}}),_{\nu} \end{pmatrix}$$

Obviously we sloppily wrote the norms because we should write
$Z = \sqrt{(P*_{\lambda} P^{\lambda} + P_{\lambda} P*^{\lambda})/2}, Z(V1) = \sqrt{(V1*_{\lambda} V1^{\lambda} + V1_{\lambda} V1*^{\lambda})/2}$ etc.

Such that the three vectors are gradients of scalars that vary only along the Godbillon-Vey foliation $F(P_k)$, $V1_{\mu}, V2_{\mu}, V3_{\mu} \in T(F(P_k))$ so $V1_{\eta} P^{\eta} = V2_{\eta} P^{\eta} = V3_{\eta} P^{\eta} = 0$. It is sufficient that the norms of the gradients of such fields are <u>stationary</u> along $P_{\mu}$. Then we have
$Z(V1) \equiv \sqrt{(V1*_{\lambda} V1^{\lambda} + V1_{\lambda} V1*^{\lambda})/2}$... and we write as in the paper $Z(V1)_{\mu} = Z(V1),_{\mu}$ so
$Z(V1)_{\mu} P*^{\mu} = Z(V2)_{\mu} P*^{\mu} = Z(V3)_{\mu} P*^{\mu} = 0$ and obviously

$V1_{\mu} P^{*\mu} = V2_{\mu} P^{*\mu} = V3_{\mu} P^{*\mu} = 0$ since the gradients are along curves in the foliation, then locally using the Scarr-Friedman law for homogeneous acceleration, we get

$$\hat{A}_{\mu\nu} \frac{P^{*\nu}}{\sqrt{Z}} = \left( (\frac{P_{\nu}}{\sqrt{Z}})_{,\mu} - (\frac{P_{\mu}}{\sqrt{Z}})_{,\nu} \right) \frac{P^{*\nu}}{\sqrt{Z}} = A_{\mu\nu} \frac{P^{*\nu}}{\sqrt{Z}} = \frac{U_{\mu}}{2}$$ half the curvature vector.

This is a type of invariance that can be expressed by the 3x4 matrix

$(\frac{V1_{\mu}}{\sqrt{V1_{\lambda} V1^{\lambda}}}, \frac{V2_{\mu}}{\sqrt{V2_{\lambda} V2^{\lambda}}}, \frac{V3_{\mu}}{\sqrt{V3_{\lambda} V3^{\lambda}}})$ so if the three columns change with $P$ such that the norms are invariant along $P_{\lambda}$ and they are in $T(F(P_{\lambda}))$ then equation (46) in the paper remains invariant. The 3x3 matrix determinant in the basis $V1_{\mu}, V2_{\mu}, V3_{\mu}$ can't change sign.