The proof of Twin Primes Conjecture

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Abstract.

Twin Primes Conjecture statement: “There are infinitely many primes p such that $(p + 2)$ is also prime”.

Initially, to prove this conjecture, we can form two arithmetic sequences (**A** and **B**), with all the natural numbers, lesser than a number $\boldsymbol{x}$, that can be primes and being each term of sequence **B** equal to its partner of sequence **A** plus 2.
By analyzing the pairing process, in general, between all non-prime numbers of sequence **A**, with terms of sequence **B**, or vice versa, we note that some pairs of primes are always formed. This allow us to develop a non-probabilistic formula to calculate the approximate number of pairs of primes, p and $(p + 2)$, that are lesser than $\boldsymbol{x}$.
The result of this formula tends to infinite when $\boldsymbol{x}$ tends to infinite, which allow us to confirm that the Twin Primes Conjecture is true.
The prime numbers theorem by Carl Friedrich Gauss, the prime numbers theorem in arithmetic progressions and some axioms have been used to complete this investigation.

1. Prime numbers and composite numbers.

A *prime number* (or *prime*) is a natural number greater than 1 that has only two divisors, 1 and the number itself.
Examples of primes are: 2, 3, 5, 7, 11, 13, 17. The Greek mathematician Euclid proved that there are infinitely many primes, but they become more scarce as we move on the number line.
Except 2 and 3, all primes are of form $(6n + 1)$ or $(6n – 1)$ being n a natural number.
We can differentiate primes 2, 3 and 5 from the rest. The 2 is the first prime and the only one that is even, the 3 is the only one of form $(6n – 3)$ and 5 is the only one finished in 5. All other primes are odd and its final digit will be 1, 3, 7 or 9.

In contrast to primes, a *composite number* (or *composite*) is a natural number that has more than two divisors.
Examples of composites are: 4 (divisors 1, 2, 4), 6 (1, 2, 3, 6), 15 (1, 3, 5, 15), 24 (1, 2, 3, 4, 6, 8, 12, 24).

Except 1, every natural number is prime or composite. By convention, the number 1 is considered neither prime nor composite because it has only one divisor.

We can classify the set of primes (except 2, 3 and 5) in 8 groups depending of the situation of each of them with respect to multiples of 30, $(30 = 2{\cdot}3{\cdot}5)$. Being: $n = 0, 1, 2, 3, 4,…, \infty$.

$30n + 7$ $30n + 11$ $30n + 13$ $30n + 17$ $30n + 19$ $30n + 23$ $30n + 29$ $30n + 31$

These expressions represent all arithmetic progressions of module 30, $(30n + b)$, such that $\gcd(30, b) = 1$ being: $32 > b > 6$.
In them, the 8 terms b correspond to the 8 first primes greater than 5. The next prime, 37, already is the second of the group $(30n + \mathbf{7})$.
These 8 groups contain all primes (except 2, 3 and 5). They also include all composites that are multiples of primes greater than 5.
As 30 and b are coprime, they cannot contain multiples of 2 or 3 or 5.
Logically, when n increases, decreases the primes proportion and increases the composites proportion that there are in each group.

Dirichlet's theorem statement[1]: “An arithmetic progression $(an + b)$ such that $\gcd(a, b) = 1$ contains infinitely many prime numbers”.
Applying this theorem for the 8 groups of primes, we can say that each of them contains infinitely many primes.

You can also apply the prime numbers theorem in arithmetic progressions. It states[2]: “For every module a, the prime numbers tend to be distributed evenly among the different progressions $(an + b)$ such that $\gcd(a, b) = 1$”.
To verify the precision of this theorem, I used a programmable logic controller (PLC), like those that control automatic machines, having obtained the following data:
There are 50.847.531 primes lesser than 10^9, (2, 3 and 5 not included), distributed as follows:

Group $(30n + 7)$	6.356.475 primes	12,50104946 %	50.847.531 / 6.356.475 = 7,999328401
Group $(30n + 11)$	6.356.197 primes	12,50050273 %	50.847.531 / 6.356.197 = 7,999678267
Group $(30n + 13)$	6.356.062 primes	12,50023723 %	50.847.531 / 6.356.062 = 7,999848176
Group $(30n + 17)$	6.355.839 primes	12,49979866 %	50.847.531 / 6.355.839 = 8,000128858
Group $(30n + 19)$	6.354.987 primes	12,49812307 %	50.847.531 / 6.354.987 = 8,001201419
Group $(30n + 23)$	6.356.436 primes	12,50097276 %	50.847.531 / 6.356.436 = 7,999377481
Group $(30n + 29)$	6.356.346 primes	12,50079576 %	50.847.531 / 6.356.346 = 7,999490745
Group $(30n + 31)$	6.355.189 primes	12,49852033 %	50.847.531 / 6.355.189 = 8,0009471

We can see that the maximum deviation for 10^9, (between 6.354.987 and 6.355.941 average), is lesser than 0,01502 %.
I gather that, in compliance with this theorem, the maximum deviation tends to 0 % when larger numbers are analyzed.

2. Definition of Twin Primes.

The primes 2 and 3 are consecutive natural numbers so they are at the shortest possible distance. As all other primes are odd, the minimum distance is 2 because there is always an even number between two consecutive odd numbers. Examples: (5, 7), (11, 13).
We call *Twin Primes* the pair of consecutive primes that are separated only by an even number. The conjecture stated at the beginning, proposes that the number of twin prime pairs is infinite. Since it is a conjecture, it has not yet been demonstrated. In this document, and based on a different approach to the one used in mathematical research, I expose a proof to solve it.
The first pairs of twin primes are (3, 5) and (5, 7). They contain 3 and 5 that do not appear in the 8 groups of primes.
These same primes (3, 5, 7) are the only possible case of primes triplets. They cannot appear more primes triplets because in each group of three consecutive odd numbers greater than 7, one of them, is always a multiple of 3.

3. Combinations of groups of primes which generate twin prime pairs.

We will write the three combinations of groups of primes with which all pairs of twin primes greater than 7 will be formed:

$(30n_1 + 11)$ and $(30n_1 + 13)$ $(30n_2 + 17)$ and $(30n_2 + 19)$ $(30n_3 + 29)$ and $(30n_3 + 31)$

4. Example.

The concepts described can be applied to number 780 with the combination $(30n_1 + 11)$ and $(30n_1 + 13)$ serving as example for any of the three exposed combinations and for any number $\boldsymbol{x}$, even being a large number. We use the list of primes lesser than 1.000.

We will write the sequence **A** of all numbers $(30n_1 + 11)$ from 0 to 780. I highlight the primes in **bold**.
Also we will write the sequence **B** of all numbers $(30n_1 + 13)$ from 0 to 780.

A **11-41-71-101-131-**161-**191-**221-**251-281-311-**341-371-**401-431-461-491-521-**551-581-611-**641-**671-**701-**731-**761**
B **13-43-73-103-**133-**163-193-223-**253-**283-313-**343-**373-**403-**433-463-**493-**523-**553-583-**613-643-673-**703-**733-**763

In the above two sequences, the 11 twin prime pairs (finished in 1 and 3) that are lesser than 780 are underlined.
The study of sequences **A** and **B**, individually and collectively, is the basis of this demonstration.

To calculate the number of terms in each sequence **A** or **B** we must remember that these are arithmetic progressions of module 30.

$\dfrac{\boldsymbol{x}}{\mathbf{30}}$ Number of terms in each sequence **A** or **B** for a number $\boldsymbol{x}$. Obviously, it is equal to the number of pairs that are formed.
(26 terms in each sequence and 26 pairs of terms that are formed to $\boldsymbol{x}$ = 780).

To analyze, in general, the above formula and for combination $(30n_1 + 11)$ and $(30n_1 + 13)$, we have:

Number of terms = number of pairs = formula result if $\boldsymbol{x}$ is multiple of 30
Number of terms = number of pairs = integer part of result if the decimal part is lesser than 13/30
Number of terms = number of pairs = (integer part of result) + 1 if the decimal part is equal to or greater than 13/30

For combination $(30n_2 + 17)$ and $(30n_2 + 19)$:

Number of terms = number of pairs = formula result if $\boldsymbol{x}$ is multiple of 30
Number of terms = number of pairs = integer part of result if the decimal part is lesser than 19/30
Number of terms = number of pairs = (integer part of result) + 1 if the decimal part is equal to or greater than 19/30

And for combination $(30n_3 + 29)$ and $(30n_3 + 31)$:

Number of terms = number of pairs = (formula result) – 1 if $\boldsymbol{x}$ is multiple of 30
Number of terms = number of pairs = integer part of result if $\boldsymbol{x}$ is not multiple of 30

5. Applying the conjecture to small numbers.

As we have seen, the composites present in the 8 groups of primes are multiples of primes greater than 5 (primes 7, 11, 13, 17, 19,…).
The first composites that appear on them are:

$49 = \mathbf{7^2}$ $77 = \mathbf{7 \cdot 11}$ $91 = \mathbf{7 \cdot 13}$ $119 = \mathbf{7 \cdot 17}$ $121 = \mathbf{11^2}$ $133 = \mathbf{7 \cdot 19}$ $143 = \mathbf{11 \cdot 13}$ $161 = \mathbf{7 \cdot 23}$ $169 = \mathbf{13^2}$

And so on, forming products of two or more factors with primes greater than 5.

From the above, we conclude that for numbers lesser than 49, all terms of sequences **A-B** are primes and all pairs that are formed will be twin prime pairs. We will write all pairs between terms of sequences **A-B** that are lesser than 49.

(**11**, **13**) (**41**, **43**) (**17**, **19**) (**29**, **31**)

Furthermore, we note that in sequences **A-B** for number 780, used as an example, the primes predominate (17 primes with 9 composites in each sequence). This occurs on small numbers (up to $x \approx 4.500$).
Therefore, for numbers lesser than 4.500, is ensured the formation of twin prime pairs with the sequences **A** and **B** because, even in the event that all composites are paired with primes, there will always be, left over in the two sequences, some primes that will form pairs between them. Applying this reasoning to number 780 we would have:
17 – 9 = 8 twin prime pairs at least (finished in 1 and 3) (in the previous chapter we can see that are 11 pairs).

6. Applying logical reasoning to the conjecture.

The sequences **A** and **B** are composed of terms that may be primes or composites that form pairs between them. To differentiate, I define as **free composite** the one which is not paired with another composite and having, as partner, a prime of the other sequence. Thus, the pairs between terms of sequences **A-B** will be formed by:

(Composite of sequence **A**) + (Composite of sequence **B**)	(CC pairs)
(Free composite of sequence **A** or **B**) + (Prime of sequence **B** or **A**)	(C**P**-**P**C pairs)
(Prime of sequence **A**) + (Prime of sequence **B**)	(**PP** pairs)

We will substitute the primes by a **P** and the composites by a C in the sequences **A-B** of number 780, that we use as example.

A	**P**	**P**	**P**	**P**	**P**	C	**P**	C	**P**	**P**	**P**	C	C	**P**	**P**	**P**	**P**	**P**	C	C	C	**P**	C	**P**	C	**P**
B	**P**	**P**	**P**	**P**	C	**P**	**P**	**P**	C	**P**	**P**	C	**P**	C	**P**	**P**	C	**P**	C	C	**P**	**P**	**P**	C	**P**	C

The number of twin prime pairs (P_T) that will be formed will depend on the free composites number of one sequence that are paired with primes of the another sequence. In general, we can define the following axiom:

P_T = (Number of primes of **A**) – (number of free composites of **B**) = (Number of primes of **B**) – (number of free composites of **A**)

For number 780: $P_T = 17 - 6 = 17 - 6 = 11$ twin prime pairs in the sequences **A-B**.

I consider that this axiom is perfectly valid although being very simple and "obvious". It will be used later in the proof of the conjecture.

Given this axiom, enough pairs of composites must be formed between the two sequences **A-B** because the number of free composites of sequence **A** cannot be greater than the number of primes of sequence **B**.
Conversely, the number of free composites of sequence **B** cannot be greater than the number of primes of sequence **A**.
This is particularly important for sequences **A-B** of very large numbers in which the primes proportion is much lesser than the composites proportion.
Later, this question is analyzed in more detail when algebra is applied to the sequences **A-B**.

With what we have described, we can devise a logical reasoning to support the conclusion that the twin primes conjecture is true.
Later, a general formula is developed to calculate the approximate number of twin prime pairs that are lesser than a number x.

As I have indicated, the formation of twin prime pairs is secured for small numbers (lesser than 4.500), since in corresponding sequences **A** and **B**, the primes predominate. Therefore, in these sequences we will find **PP** pairs and, if there are composites, CC and C**P**-**P**C pairs. If we verify increasingly large numbers, we note that already predominate composites and decreases the primes proportion.
Let us suppose that from a sufficiently large number, twin primes will not appear.
In this case I understand that, when increasing x, each new prime that will appear in sequence **A** will be paired with a new composite of the sequence **B**. Conversely, each new prime that appear in sequence **B** will be paired with a new composite of the sequence **A**. Let us recall that, with increasing x, will appear infinitely many primes in each of the sequences **A** and **B**.

If the conjecture would be false, these pairs with a term that is prime (prime-composite and composite-prime) would go appearing, and with no prime-prime pairs formed, in the three combinations of groups of primes that form twin primes from the number large enough that we have supposed to infinity, which is hardly acceptable. Although this reasoning does not serve as a demonstration, it allows me to deduce that the Twin Primes Conjecture is true.
Later, I will reinforce this deduction through the formula to calculate the approximate number of twin prime pairs lesser than x.

7. Studying how the pairs between terms of sequences A-B are formed.

We will analyze how the composite-composite pairs with the sequences **A** and **B** are formed. If the proportion of CC pairs is higher, there are less composites (free) that need a prime as a partner and, therefore, there will be more primes to form pairs.

The secret of Twin Primes conjecture is the number of composite-composite pairs formed with the sequences **A** and **B**.

Let us recall that in the sequences **A-B**, apart from primes, there are composites that are multiples of primes greater than 5.
For this analysis, I consider m as the natural number that is not multiple of 2 or 3 or 5 and j as natural number (including 0).
Analyzing the pairs between the terms of the sequences **A-B**, and in relation to the primes (7, 11, 13, 17, 19,…), we deduce that:

All multiples of 7 ($7m_{11}$) of sequence **A** are paired with all terms ($7m_{11} + 2$) of sequence **B**.
All multiples of 11 ($11m_{12}$) of sequence **A** are paired with all terms ($11m_{12} + 2$) of sequence **B**.
All multiples of 13 ($13m_{13}$) of sequence **A** are paired with all terms ($13m_{13} + 2$) of sequence **B**.
All multiples of 17 ($17m_{14}$) of sequence **A** are paired with all terms ($17m_{14} + 2$) of sequence **B**.

And so on, from the prime 7 to the one previous to $\sqrt{\boldsymbol{x}}$, since these primes are sufficient to define all multiples of the sequences **A-B**.
For this question, we must consider that a prime is a multiple of itself.

Similarly, we deduce that:

All terms ($7m_{21} – 2$) of sequence **A** are paired with all multiples of 7 ($7m_{21}$) of sequence **B**.
All terms ($11m_{22} – 2$) of sequence **A** are paired with all multiples of 11 ($11m_{22}$) of sequence **B**.
All terms ($13m_{23} – 2$) of sequence **A** are paired with all multiples of 13 ($13m_{23}$) of sequence **B**.
All terms ($17m_{24} – 2$) of sequence **A** are paired with all multiples of 17 ($17m_{24}$) of sequence **B**.

And so on to the prime previous to $\sqrt{\boldsymbol{x}}$.

Summarizing the above, we can define the following axiom:

All groups of multiples $7m_{11}$, $11m_{12}$, $13m_{13}$, $17m_{14}$,… (including the primes lesser than $\sqrt{\boldsymbol{x}}$ that are present) of sequence **A** are paired, group to group, with all groups of terms ($7m_{11} + 2$), ($11m_{12} + 2$), ($13m_{13} + 2$), ($17m_{14} + 2$),… of sequence **B**.
Conversely, all groups of terms ($7m_{21} – 2$), ($11m_{22} – 2$), ($13m_{23} – 2$), ($17m_{24} – 2$),… of sequence **A** are paired, group to group, with all groups of multiples $7m_{21}$, $11m_{22}$, $13m_{23}$, $17m_{24}$,… (including the primes lesser than $\sqrt{\boldsymbol{x}}$ that are present) of sequence **B**.

We will apply the above described, to number 780. It serves as an example for any number $\boldsymbol{x}$, even being a large number.
We will write the corresponding sequences **A-B**. $\sqrt{\mathbf{780}}$ = 27,93

In sequence **A** we will underline all multiples $7m_{11}$, $11m_{12}$, $13m_{13}$, $17m_{14}$, $19m_{15}$ and $23m_{16}$.
And in sequence **B** we will underline all terms ($7m_{11} + 2$), ($11m_{12} + 2$), ($13m_{13} + 2$), ($17m_{14} + 2$), ($19m_{15} + 2$) and ($23m_{16} + 2$).

A **11**-**41**-**71**-**101**-**131**-161-**191**-221-**251**-**281**-**311**-341-371-**401**-**431**-**461**-**491**-**521**-551-581-611-**641**-671-**701**-731-**761**
B **13**-**43**-**73**-**103**-133-**163**-**193**-**223**-253-**283**-**313**-343-**373**-403-**433**-**463**-493-**523**-553-583-**613**-**643**-**673**-703-**733**-763

Now, in sequence **A** we will underline all terms ($7m_{21} – 2$), ($11m_{22} – 2$), ($13m_{23} – 2$), ($17m_{24} – 2$), ($19m_{25} – 2$) and ($23m_{26} – 2$).
And in sequence **B** we will underline all multiples $7m_{21}$, $11m_{22}$, $13m_{23}$, $17m_{24}$, $19m_{25}$ and $23m_{26}$.

A **11**-**41**-**71**-**101**-**131**-161-**191**-221-**251**-**281**-**311**-341-371-**401**-**431**-**461**-**491**-**521**-551-581-611-**641**-671-**701**-731-**761**
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The terms that are not underlined form the 10 twin prime pairs (finished in 1 and 3) that there are between $\sqrt{\mathbf{780}}$ and 780.
We added the pair of primes (**11**, **13**) that have been underlined for being a multiple of 11 ($11m_{12}$), the first, and ($11m_{12} + 2$) the second.

(**41**, **43**) (**71**, **73**) (**101**, **103**) (**191**, **193**) (**281**, **283**) (**311**, **313**) (**431**, **433**) (**461**, **463**) (**521**, **523**) (**641**, **643**) (**11**, **13**)

It can be seen that all multiples $7m$, $11m$, $13m$, $17m$, $19m$, $23m$,… of a sequence **A** or **B** are paired with multiples or primes of the other, to form multiple-multiple pairs, multiple-prime pairs and prime-multiple pairs, according to the axiom defined.
Finally, the remaining prime-prime pairs are the twin prime pairs, (one of three combinations), that there are between $\sqrt{\boldsymbol{x}}$ and $\boldsymbol{x}$.

The above exposition helps us understand the relation between terms of sequence **A** and terms of sequence **B** of any number $\boldsymbol{x}$.

To numerically support the axiom exposed, I used a programmable controller to obtain data of sequences **A-B** corresponding to several numbers $\boldsymbol{x}$, (between 10^6 and 10^9), and that can be consulted from page 16.

8. Proving the conjecture.

To prove the conjecture, as a starting point, I will use the first part of the axiom from the previous chapter:
All multiples $7m_{11}$, $11m_{12}$, $13m_{13}$, $17m_{14}$,… (including the primes lesser than $\sqrt{x}$ that are present) of sequence **A** are paired, respectively, with all terms $(7m_{11} + 2)$, $(11m_{12} + 2)$, $(13m_{13} + 2)$, $(17m_{14} + 2)$,… of sequence **B**.

In this axiom, the concept of *multiple*, applied to the terms of each sequence **A** or **B**, includes all composites and the primes lesser than $\sqrt{x}$ that are present. By this definition, all terms that are lesser than $\sqrt{x}$ of each sequence **A** or **B** are *multiples*.
Simultaneously, and also in this axiom, the concept of *prime*, applied to the terms of each sequence **A** or **B**, refers only to primes greater than $\sqrt{x}$ that are present in the corresponding sequence.
According to these concepts, each term of sequences **A** or **B** will be *multiple* or *prime*. Thus, with the terms of the two sequences can be formed multiple-multiple pairs, free multiple-prime pairs, prime-free multiple pairs and prime-prime pairs.

$\dfrac{x}{30}$ Number of terms in each sequence **A** or **B** for the number x. (Page 2)

$\pi(x)$ Symbol[3], normally used, to express the number of primes lesser or equal to x.

According to the prime numbers theorem[3]: $\pi(x) \sim \dfrac{x}{\ln(x)}$ being: $\lim\limits_{x\to\infty} \dfrac{\pi(x)}{\frac{x}{\ln(x)}} = 1$ $\ln(x)$ = natural logarithm of x

A better approach for this theorem is given by the offset logarithmic integral function $\mathrm{Li}(x)$: $\pi(x) \approx \mathrm{Li}(x) = \int_2^x \dfrac{dy}{\ln(y)}$

According to these formulas, for all $x \geq 5$ is true that $\pi(x) > \sqrt{x}$. This inequality becomes larger with increasing x.

$\pi(ax)$ Symbol to express the number of primes greater than $\sqrt{x}$ in sequence **A** for the number x.

$\pi(bx)$ Symbol to express the number of primes greater than $\sqrt{x}$ in sequence **B** for the number x.

For large values of x it can be accept that: $\pi(ax) \approx \pi(bx) \approx \dfrac{\pi(x)}{8}$ being 8 the number of groups of primes (page 1).

For $x = 10^9$, the maximum error of above approximation is 0,0215 % for group $(30n + 19)$.

$\dfrac{x}{30} - \pi(ax)$ Number of multiples of sequence **A** for the number x.

$\dfrac{x}{30} - \pi(bx)$ Number of multiples of sequence **B** for the number x.

We will define as a fraction $k(ax)$ of sequence **A**, or $k(bx)$ of sequence **B**, the ratio between the number of multiples and the total number of terms in the corresponding sequence. As the primes density decreases as we move on the number line, the $k(ax)$ and $k(bx)$ values gradually increase when increasing x and tend to 1 when x tends to infinite.

$$k(ax) = \frac{\frac{x}{30} - \pi(ax)}{\frac{x}{30}} = 1 - \frac{\pi(ax)}{\frac{x}{30}}$$ For sequence **A**: $k(ax) = 1 - \dfrac{30\pi(ax)}{x}$ For sequence **B**: $k(bx) = 1 - \dfrac{30\pi(bx)}{x}$

The central question of this chapter is to develop a general formula to calculate the number of multiples that there are in terms $(7m_{11} + 2)$, $(11m_{12} + 2)$, $(13m_{13} + 2)$, $(17m_{14} + 2)$,… of sequence **B** and that, complying the origin axiom, are paired with an equal number of multiples $7m_{11}$, $11m_{12}$, $13m_{13}$, $17m_{14}$,… of sequence **A**. Known this data, it can calculate the number of free multiples of sequence **A** (and that are paired with primes of sequence **B**). Finally, the remaining primes of sequence **B** will be paired with some primes of sequence **A** to determine the twin prime pairs that are formed.

We will study the terms $(7m + 2)$, $(11m + 2)$, $(13m + 2)$, $(17m + 2)$,… of sequence **B**, in a general way.
With an analogous procedure, we can study the terms $(7m – 2)$, $(11m – 2)$, $(13m – 2)$, $(17m – 2)$,… of sequence **A** if we use the second part of the axiom referred to in the above chapter.

We will analyze how primes are distributed among terms $(7m + 2)$, $(11m + 2)$, $(13m + 2)$, $(17m + 2)$,…
For this purpose, we will see the relation between prime 7 and the 8 groups of primes, serving as example for any prime greater than 5.
We will analyze how are the groups of multiples of 7 $(7m)$ and the groups $(7j + a)$ in generally, that's $(7j + 1)$, $(7j + 2)$, $(7j + 3)$, $(7j + 4)$, $(7j + 5)$ and $(7j + 6)$. Noting the fact that it is an axiom, I gather that they will be arithmetic progressions of module 210, $(210 = 7·30)$.
In the following expressions, the 8 arithmetic progressions of module 210 correspond, respectively, with the 8 groups of primes of module 30. I highlight in **bold** the prime that identifies each of these 8 groups.
I underline the groups of terms that will appear in the three types of sequences **B** of this conjecture. Being: $n = 0, 1, 2, 3, 4,…, \infty$.

(210*n* + **7**), (210*n* + 150 + **11**), $\underline{(210n + 120 + \mathbf{13})}$, (210*n* + 60 + **17**), $\underline{(210n + 30 + \mathbf{19})}$, (210*n* + 180 + **23**), (210*n* + 90 + **29**) and $\underline{(210n + 60 + \mathbf{31})}$ are multiples of 7 (7*m*). These groups do not contain primes, except the prime 7 in the group (210*n* + **7**) for *n* = 0.

(210*n* + 120 + **7**), (210*n* + 60 + **11**), $\underline{(210n + 30 + \mathbf{13})}$, (210*n* + 180 + **17**), $\underline{(210n + 150 + \mathbf{19})}$, (210*n* + 90 + **23**), (210*n* + **29**) and $\underline{(210n + 180 + \mathbf{31})}$ are terms (7*j* + 1). In the group (210*n* + 180 + **31**) we note that 180 + 31 = 211 > 210.

(210*n* + 30 + **7**), (210*n* + 180 + **11**), $\underline{(210n + 150 + \mathbf{13})}$, (210*n* + 90 + **17**), $\underline{(210n + 60 + \mathbf{19})}$, (210*n* + **23**), (210*n* + 120 + **29**) and $\underline{(210n + 90 + \mathbf{31})}$ are terms (7*j* + 2). The three underlined groups are terms (7*m* + 2).

(210*n* + 150 + **7**), (210*n* + 90 + **11**), $\underline{(210n + 60 + \mathbf{13})}$, (210*n* + **17**), $\underline{(210n + 180 + \mathbf{19})}$, (210*n* + 120 + **23**), (210*n* + 30 + **29**) and $\underline{(210n + \mathbf{31})}$ are terms (7*j* + 3).

(210*n* + 60 + **7**), (210*n* + **11**), $\underline{(210n + 180 + \mathbf{13})}$, (210*n* + 120 + **17**), $\underline{(210n + 90 + \mathbf{19})}$, (210*n* + 30 + **23**), (210*n* + 150 + **29**) and $\underline{(210n + 120 + \mathbf{31})}$ are terms (7*j* + 4).

(210*n* + 180 + **7**), (210*n* + 120 + **11**), $\underline{(210n + 90 + \mathbf{13})}$, (210*n* + 30 + **17**), $\underline{(210n + \mathbf{19})}$, (210*n* + 150 + **23**), (210*n* + 60 + **29**) and $\underline{(210n + 30 + \mathbf{31})}$ are terms (7*j* + 5).

(210*n* + 90 + **7**), (210*n* + 30 + **11**), $\underline{(210n + \mathbf{13})}$, (210*n* + 150 + **17**), $\underline{(210n + 120 + \mathbf{19})}$, (210*n* + 60 + **23**), (210*n* + 180 + **29**) and $\underline{(210n + 150 + \mathbf{31})}$ are terms (7*j* + 6).

We can note that the groups of multiples of 7 (7*m*) of sequence **B** correspond to arithmetic progressions of module 210, (210*n* + *b*), such that gcd(210, *b*) = 7 being *b* lesser than 210, multiple of 7, and having 8 terms *b*, one of each group of primes.

Also, we can see that the groups of terms (7*j* + 1), (7*j* + 2), (7*j* + 3), (7*j* + 4), (7*j* + 5) and (7*j* + 6) of sequence **B** correspond to arithmetic progressions of module 210, (210*n* + *b*), such that gcd(210, *b*) = 1 being *b* lesser than 212, not multiple of 7, and having 48 terms *b*, 6 of each group of primes.

Finally, we can verify that the 56 terms *b*, (8 + 48), are all those that appear in the 8 groups of primes and that are lesser than 212.

Applying the above axiom for all *p* (prime greater than 5 and lesser than $\sqrt{x}$) we can confirm that the groups of multiples of *p* (*pm*) of sequence **B** correspond to arithmetic progressions of module 30*p*, (30*pn* + *b*), such that gcd(30*p*, *b*) = *p* being *b* lesser than 30*p*, multiple of *p*, and having 8 terms *b*, one for each group of primes.

Also, we can confirm that the groups of terms (*pj* + 1), (*pj* + 2), (*pj* + 3),…, (*pj* + *p* – 2) and (*pj* + *p* – 1) of sequence **B** correspond to arithmetic progressions of module 30*p*, (30*pn* + *b*), such that gcd(30*p*, *b*) = 1 being *b* lesser than (30*p* + 2), not multiple of *p*, and having 8(*p* – 1) terms *b*, (*p* – 1) of each group of primes. In this conjecture, the terms (*pj* + 2) are (*pm* + 2).

Finally, we can confirm that the 8*p* terms *b*, (8 + 8(*p* – 1)), are all those that appear in the 8 groups of primes and that are lesser than (30*p* + 2).

On the other hand, an axiom that is met in the sequences **A** or **B** is that, in each set of *p* consecutive terms, there are one of each of the following groups: *pm*, (*pj* + 1), (*pj* + 2), (*pj* + 3),…, (*pj* + *p* – 2) and (*pj* + *p* – 1) (though not necessarily in this order). Example:

13	**43**	**73**	**103**	133	**163**	**193**	Terms (30*n* + 13)
(7·1 + 6)	(7·6 + 1)	(7·10 + 3)	(7·14 + 5)	7·19	(7·23 + 2)	(7·27 + 4)	Terms 7*m* and (7*j* + *a*)

Therefore, and according to this axiom, $\frac{1}{p}\frac{x}{30}$ will be the number of multiples of *p* (*pm*) (including *p*, if it would be present) and, also, the number of terms that have each groups (*pj* + 1), (*pj* + 2), (*pj* + 3),…, (*pj* + *p* – 2) and (*pj* + *p* – 1) in each sequence **A** or **B**.

This same axiom allows us to say that these groups contain all terms of sequences **A** or **B** as follows:

1. Group *pm*: contains all multiples of *p*.
2. Groups (*pj* + 1), (*pj* + 2), (*pj* + 3),…, (*pj* + *p* – 1): contain all multiples (except those of *p*) and the primes greater than $\sqrt{x}$.

As it has been described, the groups (*pj* + 1), (*pj* + 2), (*pj* + 3),…, (*pj* + *p* – 2) and (*pj* + *p* – 1) of sequence **B** (and similarly for sequence **A**) are arithmetic progressions of module 30*p*, (30*pn* + *b*), such that gcd(30*p*, *b*) = 1.
Applying the prime numbers theorem in arithmetic progressions [2], shown on page 1, to these groups we concluded that they all will have, approximately, the same amount of primes ($\approx \frac{\pi(bx)}{p-1}$ in sequence **B**) and, as they all have the same number of terms, also they will have, approximately, the same number of multiples.
Similarly, we can apply this theorem to terms belonging to two or more groups. For example, the terms that are, at once, in groups (7*j* + *a*) and (13*j* + *c*) correspond to arithmetic progressions of module 2730, (2730 = 7·13·30). In this case, all groups of a sequence **A**

or **B** that contain these terms (72 groups that result of combining 6 a and 12 c) they will have, approximately, the same amount of primes and, as they all have the same number of terms, will also have, approximately, the same number of multiples.

As described, I gather that, of the $\frac{1}{7}\frac{x}{30}$ terms ($7m + 2$) that there are in sequence **B**, $\approx \frac{\pi(bx)}{6}$ will be primes.
All other terms are multiples (of primes greater than 5, except the prime 7).

In general, I gather that, of the $\frac{1}{p}\frac{x}{30}$ terms ($pm + 2$) that there are in sequence **B**, $\approx \frac{\pi(bx)}{p-1}$ will be primes.
All other terms are multiples (of primes greater than 5, except the prime p).

We will define as a fraction $k(7x)$ of sequence **B** the ratio between the number of multiples that there are in the group of terms ($7m + 2$) and the total number of these.
Applying the above for all p (prime greater than 5 and lesser than $\sqrt{x}$) we will define as a fraction $k(px)$ of sequence **B** the ratio between the number of multiples that there are in the group of terms ($pm + 2$) and the total number of these.

We can see the similarity between $k(bx)$ and factors $k(7x)$, $k(11x)$, $k(13x)$, $k(17x)$,…, $k(px)$,… so their formulas will be similar.
I will use ≈ instead of = due to the imprecision in the number of primes that there are in each group.
Using the same procedure as for obtaining $k(bx)$:

$$\boldsymbol{k}(px) \approx \frac{\frac{1}{p}\frac{x}{30} - \frac{\pi(bx)}{p-1}}{\frac{1}{p}\frac{x}{30}} = 1 - \frac{\frac{\pi(bx)}{p-1}}{\frac{1}{p}\frac{x}{30}} = 1 - \frac{30p\pi(bx)}{(p-1)x} \qquad \boldsymbol{k}(px) \approx 1 - \frac{30\pi(bx)}{x}\frac{p}{p-1}$$

For the prime 7: $\boldsymbol{k}(7x) \approx 1 - \frac{35\pi(bx)}{x}$ For the prime 11: $\boldsymbol{k}(11x) \approx 1 - \frac{33\pi(bx)}{x}$ For the prime 31: $\boldsymbol{k}(31x) \approx 1 - \frac{31\pi(bx)}{x}$

And so on to the prime previous to $\sqrt{x}$.

If we order these factors from lowest to highest value: $k(7x) < k(11x) < k(13x) < k(17x) <\ldots < k(997x) <\ldots < k(bx)$

In the formula to obtain $k(px)$ we have that: $\lim_{p\to\infty} \frac{p}{p-1} = 1$ so we can write: $\lim_{p\to\infty} k(px) = 1 - \frac{30\pi(bx)}{x} = k(bx)$

We can unify all factors $k(7x)$, $k(11x)$, $k(13x)$,…, $k(px)$,… into one, which we will call $k(jx)$, and that will group all of them together.

Applying the above, we will define as a fraction $k(jx)$ of sequence **B** the ratio between the number of multiples that there are in the set of all terms ($7m_{11} + 2$), ($11m_{12} + 2$), ($13m_{13} + 2$), ($17m_{14} + 2$),… and the total number of these.
Logically, the $k(jx)$ value is determined by the $k(px)$ values corresponding to each primes from 7 to the one previous to $\sqrt{x}$.

Summarizing the exposed: a fraction $k(jx)$ of terms ($7m_{11} + 2$), ($11m_{12} + 2$), ($13m_{13} + 2$), ($17m_{14} + 2$),… of sequence **B** will be multiples and that, complying the origin axiom, will be paired with an equal fraction of multiples $7m_{11}$, $11m_{12}$, $13m_{13}$, $17m_{14}$,… of sequence **A**.

To put it simply and in general:
A fraction $k(jx)$ of multiples of sequence **A** will have, as partner, a multiple of sequence **B**.

Recalling the axiom on page 3, and the formulas on page 5, we can record:

1. Number of multiple-multiple pairs = $k(jx)$ (Number of multiples of sequence **A**)
2. Number of free multiples in sequence **A** = ($1 - k(jx)$) (Number of multiples of sequence **A**)
3. $P_T(x)$ = actual number of twin prime pairs greater than $\sqrt{x}$
   $P_T(x)$ = (Number of primes greater than $\sqrt{x}$ of sequence **B**) – (Number of free multiples of sequence **A**)

Expressed algebraically: $\mathbf{P_T}(x) = \pi(bx) - (1 - k(jx))\left(\frac{x}{30} - \pi(ax)\right)$

Let us suppose that from a sufficiently large number, do not appear any twin prime pairs. In this case, for all x values greater than the square of this number, it would be met that $P_T(x) = 0$ since, obviously, $P_T(x)$ cannot have negative values.
We can define a factor, which I will call $k(0x)$ and that, replacing $k(jx)$ in the above formula, it results in $P_T(x) = 0$.
As a concept, $k(0x)$ would be the minimum value of $k(jx)$ for which the conjecture would be false.

$$0 = \pi(bx) - (1 - k(0x))\left(\frac{x}{30} - \pi(ax)\right) \qquad \pi(bx) = (1 - k(0x))\left(\frac{x}{30} - \pi(ax)\right)$$

Solving: $$\boldsymbol{k(0x)} = 1 - \frac{30\pi(bx)}{x - 30\pi(ax)}$$

For the conjecture to be true, $k(jx)$ must be greater than $k(0x)$ for any $\boldsymbol{x}$ value.
Let us recall that the $k(jx)$ value is determined by values of the factors $k(7x)$, $k(11x)$, $k(13x)$, $k(17x)$,…, $k(px)$,…
To analyze the relation between the factors $k(jx)$ and $k(0x)$, first, let us compare $k(0x)$ with the general factor $k(px)$.

$$\boldsymbol{k(0x)} = 1 - \frac{30\pi(bx)}{x - 30\pi(ax)} = 1 - \frac{30\pi(bx)}{x}\frac{x}{x - 30\pi(ax)} = 1 - \frac{30\pi(bx)}{x}\frac{1}{1 - \frac{30\pi(ax)}{x}}$$

$$\boldsymbol{k(px)} \approx 1 - \frac{30\pi(bx)}{x}\frac{p}{p-1} = 1 - \frac{30\pi(bx)}{x}\frac{1}{1-\frac{1}{p}}$$

To compare $k(0x)$ with $k(px)$, simply compare $\frac{30\pi(ax)}{x}$ with $\frac{1}{p}$ which are the terms that differentiate the two formulas.

Let us recall, page 5, the prime numbers theorem: $\boldsymbol{\pi(x)} \sim \frac{x}{\ln(x)}$ being $\boldsymbol{\pi(x)}$ the number of primes lesser or equal to $\boldsymbol{x}$.

As I have indicated, it can be accepted that: $\boldsymbol{\pi(ax)} \approx \frac{\pi(x)}{8}$ being 8 the number of groups of primes.

Substituting $\boldsymbol{\pi(x)}$ by its corresponding formula: $\boldsymbol{\pi(ax)} \sim \frac{x}{8\ln(x)}$

The approximation of this formula does not affect the final result of the comparison between $k(0x)$ and $k(px)$ that we are analyzing.

Compare	$\frac{30\pi(ax)}{x}$	with	$\frac{1}{p}$	Substituting $\boldsymbol{\pi(ax)}$ by its corresponding formula
Compare	$\frac{30x}{8x\ln(x)}$	with	$\frac{1}{p}$	
Compare	$\frac{3{,}75}{\ln(x)}$	with	$\frac{3{,}75}{3{,}75p}$	
Compare	$\ln(x)$	with	$3{,}75p$	Applying the natural logarithm concept
Compare	x	with	$e^{3{,}75p}$	For powers of 10: ln10 = 2,302585 3,75 / 2,302585 = 1,6286 ≈ 1,63
Compare	x	with	$10^{1{,}63p}$	

Comparison result: $k(0x)$ will be lesser than $k(px)$ if $x < 10^{1{,}63p}$ $k(0x)$ will be greater than $k(px)$ if $x > 10^{1{,}63p}$

In the following expressions, the exponents values are approximate. This does not affect the comparison result.

1. For the prime 7: $k(0x) < k(7x)$ if $x < 10^{11{,}4}$ $k(0x) > k(7x)$ if $x > 10^{11{,}4}$ $\approx 4 \cdot 10^4$ primes lesser than $10^{5{,}7}$
2. For the prime 11: $k(0x) < k(11x)$ if $x < 10^{18}$ $k(0x) > k(11x)$ if $x > 10^{18}$ $\approx 5{,}08 \cdot 10^7$ primes lesser than 10^9
3. For the prime 31: $k(0x) < k(31x)$ if $x < 10^{50}$ $k(0x) > k(31x)$ if $x > 10^{50}$ $\approx 1{,}76 \cdot 10^{23}$ primes lesser than 10^{25}
4. For the prime 997: $k(0x) < k(997x)$ if $x < 10^{1620}$ $k(0x) > k(997x)$ if $x > 10^{1620}$ $\approx 5{,}36 \cdot 10^{806}$ primes lesser than 10^{810}

By analyzing these data we can see that, for numbers lesser than $10^{11{,}4}$, $k(0x)$ is lesser than all factors $k(px)$ and, therefore, also will be lesser than $k(jx)$ which allows us to ensure that twin prime pairs will appear, at least until $10^{5{,}7}$.
For the $\boldsymbol{x}$ values greater than $10^{11{,}4}$, we can see that $k(0x)$ overcomes gradually the factors $k(px)$ ($k(7x)$, $k(11x)$, $k(13x)$, $k(17x)$,…, $k(997x)$,…).
Looking in detail, we can note that if the p value, for which the comparison is applied, increases in geometric progression, the $\boldsymbol{x}$ value from which $k(0x)$ exceeds to $k(px)$ increases exponentially. Because of this, also increases exponentially (or slightly higher) the number of primes lesser than $\sqrt{x}$ and whose factors $k(px)$ will determine the $k(jx)$ value.
Logically, if increases the number of primes lesser than $\sqrt{x}$, decreases the "relative weight" of each factor $k(px)$ in relation to the $k(jx)$ value. Thus, although from $10^{11{,}4}$ $k(7x)$ is lesser than $k(0x)$, the percentage of terms ($7m + 2$) which are not in upper groups will decrease and the factor $k(7x)$ will lose gradually influence on the $k(jx)$ value.
The same can be applied to the factors $k(11x)$, $k(13x)$, $k(17x)$,… that will lose gradually influence on the $k(jx)$ value with increasing $\boldsymbol{x}$.

On the other hand, taking as an example the prime 997, we can note that, when $k(0x)$ exceeds $k(997x)$, there are already ≈ $5{,}36 \cdot 10^{806}$ primes whose factors $k(px)$ (which will be greater than $k(0x)$) added to factors $k(7x)$ to $k(997x)$ (165 factors that will be lesser than $k(0x)$) will determine the $k(jx)$ value. Note the large difference between 165 and ≈ $5{,}36 \cdot 10^{806}$.

These data allow us to intuit that $k(jx)$ will be greater than $k(0x)$ for any $\boldsymbol{x}$ value.

After these positive data, we continue developing the formula to calculate the approximate value of $k(jx)$.
Let us compare $k(bx)$ with $k(jx)$. Let us recall the definitions relating to these two factors.

$k(bx)$ = ratio between the number of multiples and the total number of terms of sequence **B**.

Sequence **B**	$\frac{x}{30}$ terms	$\pi(bx)$ primes	$\frac{x}{30} - \pi(bx)$ multiples	$k(bx) = 1 - \frac{30\pi(bx)}{x}$
Terms of sequence **B**	1/7 are multiples of 7,	1/11 are multiples of 11,	1/13 are multiples of 13,	1/17 are multiples of 17,…

And so on to the prime previous to $\sqrt{x}$.

$k(jx)$ = ratio between the number of multiples that there are in the set of all terms $(7m_{11} + 2)$, $(11m_{12} + 2)$, $(13m_{13} + 2)$, $(17m_{14} + 2)$,… of sequence **B** and the total number of these. Its value is determined by the values of the factors $k(7x)$, $k(11x)$, $k(13x)$, $k(17x)$,…

As described when we applied the prime numbers theorem in arithmetic progressions, the actual number of primes that there are in each groups $(7m_{11} + 2)$, $(11m_{12} + 2)$, $(13m_{13} + 2)$, $(17m_{14} + 2)$,… will be, approximately, equal to the average value indicated.

Group $(7m + 2)$	$\frac{1}{7}\frac{x}{30}$ terms	$\approx \frac{\pi(bx)}{6}$ primes	$\approx \left(\frac{1}{7}\frac{x}{30} - \frac{\pi(bx)}{6}\right)$ multiples	$k(7x) \approx 1 - \frac{30\pi(bx)}{x}\frac{7}{6}$
Terms $(7m + 2)$	No multiples of 7,	1/11 are multiples of 11,	1/13 are multiples of 13,	1/17 are multiples of 17,…
Group $(11m + 2)$	$\frac{1}{11}\frac{x}{30}$ terms	$\approx \frac{\pi(bx)}{10}$ primes	$\approx \left(\frac{1}{11}\frac{x}{30} - \frac{\pi(bx)}{10}\right)$ multiples	$k(11x) \approx 1 - \frac{30\pi(bx)}{x}\frac{11}{10}$
Terms $(11m + 2)$	1/7 are multiples of 7,	no multiples of 11,	1/13 are multiples of 13,	1/17 are multiples of 17,…
Group $(13m + 2)$	$\frac{1}{13}\frac{x}{30}$ terms	$\approx \frac{\pi(bx)}{12}$ primes	$\approx \left(\frac{1}{13}\frac{x}{30} - \frac{\pi(bx)}{12}\right)$ multiples	$k(13x) \approx 1 - \frac{30\pi(bx)}{x}\frac{13}{12}$
Terms $(13m + 2)$	1/7 are multiples of 7,	1/11 are multiples of 11,	no multiples of 13,	1/17 are multiples of 17,…
Group $(17m + 2)$	$\frac{1}{17}\frac{x}{30}$ terms	$\approx \frac{\pi(bx)}{16}$ primes	$\approx \left(\frac{1}{17}\frac{x}{30} - \frac{\pi(bx)}{16}\right)$ multiples	$k(17x) \approx 1 - \frac{30\pi(bx)}{x}\frac{17}{16}$
Terms $(17m + 2)$	1/7 are multiples of 7,	1/11 are multiples of 11,	1/13 are multiples of 13,	no multiples of 17,…

And so on to the prime previous to $\sqrt{x}$.

It can be noted that, in compliance to the prime numbers theorem in arithmetic progressions, the groups $(7m_{11} + 2)$, $(11m_{12} + 2)$, $(13m_{13} + 2)$, $(17m_{14} + 2)$,… behave with some regularity, mathematically defined, for the number of terms, the number of primes and the number of multiples that contain, and that is maintained regardless of $\boldsymbol{x}$ value.

Continuing the study of these terms we can see some data, obtained with a programmable controller, that refers to the group $(30n + 19)$ (chosen as example) and the numbers 10^6, 10^7, 10^8 and 10^9.
Although for this analysis, any sequence of primes can be chosen, I will do it in ascending order (7, 11, 13, 17, 19, 23,…, 307).
They are the following data, and are numbered as follows:

1. Total number of terms $(7m + 2)$, $(11m + 2)$, $(13m + 2)$, $(17m + 2)$,…
2. Multiples that there are in the group $(7m + 2)$: they are all included.
3. Multiples that there are in the group $(11m + 2)$: not included those who are also $(7m + 2)$.
4. Multiples that there are in the group $(13m + 2)$: not included those who are also $(7m + 2)$ or $(11m + 2)$.
5. Multiples that there are in the group $(17m + 2)$: not included those who are also $(7m + 2)$ or $(11m + 2)$ or $(13m + 2)$.

And so on until the group $(307m + 2)$. This data can be consulted from page 16.
The percentages indicated are relative to the total number of terms $(7m + 2)$, $(11m + 2)$, $(13m + 2)$, $(17m + 2)$,…

	10^6		10^7		10^8		10^9	
Terms ($7m + 2$), ($11m + 2$),…	23.546		250.283		2.613.261		26.977.923	
Multiples ($7m + 2$) and %	3.110	13,21 %	33.738	13,48 %	356.180	13,63 %	3.702.682	13,72 %
Multiples ($11m + 2$) and %	1.796	7,63 %	19.062	7,62 %	199.690	7,64 %	2.067.716	7,66 %
Multiples ($13m + 2$) and %	1.387	5,89 %	14.764	5,90 %	154.739	5,92 %	1.600.794	5,93 %
Multiples ($17m + 2$) and %	1.008	4,28 %	10.553	4,22 %	110.124	4,21 %	1.137.526	4,21 %
Total multiples groups 7 to 307	14.989	63,66 %	156.968	62,72 %	1.642.597	62,86 %	17.013.983	63,07 %

These new data continue to confirm that the groups ($7m_{11} + 2$), ($11m_{12} + 2$), ($13m_{13} + 2$), ($17m_{14} + 2$),… behave in a uniform manner, because the percentage of multiples that supply each is almost constant when $\boldsymbol{x}$ increases.

The regularity of these groups allows us to intuit that the approximate value of $k(j\boldsymbol{x})$ can be obtained by a general formula.

Considering the data of each group, and to develop the formula of $k(j\boldsymbol{x})$, we can think about adding, on one hand, the number of terms of all of them, on the other hand, the number of primes and finally the number of multiples and making the final calculations with the total of these sums. This method is not correct, since each term can be in several groups so they would be counted several times what would give us an unreliable result.
To resolve this question in a theoretical manner, but more accurate, each term ($7m_{11} + 2$), ($11m_{12} + 2$), ($13m_{13} + 2$), ($17m_{14} + 2$),… should be analyzed individually and applying inclusion-exclusion principle, to define which are multiples and those who are primes.
After several attempts, I have found that this analytical method is quite complex, so that in the end, I rejected it.
I hope that any mathematician interested in this topic may resolve this question in a rigorous way.
Given the difficulty of the mathematical analysis, I opted for an indirect method to obtain the formula for $k(j\boldsymbol{x})$.

Gathering information from the Internet of the latest demonstrations of mathematical conjectures, I have read that it has been accepted the use of computers to perform some of calculations or to verify the conjectures up to a certain number.
Given this information, I considered that I can use a programmable logic controller (PLC) to help me get the formula for $k(j\boldsymbol{x})$. To this purpose, I have developed the programs that the controller needs to perform this work.
I will begin by analyzing the exposed data from which it can be deduced:

1. The concepts of $k(j\boldsymbol{x})$ and $k(b\boldsymbol{x})$ are similar so, in principle, their formulas will use the same variables.
2. The parameters (number of terms, number of primes and number of multiples) involved in $k(j\boldsymbol{x})$ follow a certain "pattern".
3. The $k(j\boldsymbol{x})$ and $k(b\boldsymbol{x})$ values, and also those of $\boldsymbol{\pi(ax)}$ and $\boldsymbol{\pi(bx)}$, gradually increase with increasing $\boldsymbol{x}$.
4. The $k(j\boldsymbol{x})$ value is lesser than the $k(b\boldsymbol{x})$ value but will tend to equalize, in an asymptotically way, when $\boldsymbol{x}$ tends to infinite.

Here are some values, obtained by the controller, concerning to $k(b\boldsymbol{x})$, $k(j\boldsymbol{x})$ and the group ($30n + 19$), (consult from page 16).

1. To 10^6 — $k(b\boldsymbol{x}) = 0{,}706897069$ — $k(j\boldsymbol{x}) = 0{,}700798437$ — $k(j\boldsymbol{x}) / k(b\boldsymbol{x}) = 0{,}991372673$
2. To 10^7 — $k(b\boldsymbol{x}) = 0{,}751125751$ — $k(j\boldsymbol{x}) = 0{,}747054334$ — $k(j\boldsymbol{x}) / k(b\boldsymbol{x}) = 0{,}99457958$
3. To 10^8 — $k(b\boldsymbol{x}) = 0{,}783999078$ — $k(j\boldsymbol{x}) = 0{,}780690103$ — $k(j\boldsymbol{x}) / k(b\boldsymbol{x}) = 0{,}995779363$
4. To 10^9 — $k(b\boldsymbol{x}) = 0{,}809362808$ — $k(j\boldsymbol{x}) = 0{,}806782605$ — $k(j\boldsymbol{x}) / k(b\boldsymbol{x}) = 0{,}996812056$

By analyzing these data, it can be seen that, as $\boldsymbol{x}$ increases, the $k(j\boldsymbol{x})$ value tends more rapidly to the $k(b\boldsymbol{x})$ value that the $k(b\boldsymbol{x})$ value with respect to 1.
Expressed numerically: To 10^6: $(1 – 0{,}706897069) / (0{,}706897069 – 0{,}700798437) = 48{,}06$
To 10^9: $(1 – 0{,}809362808) / (0{,}809362808 – 0{,}806782605) = 73{,}88$

Then, based on the formulas for $k(b\boldsymbol{x})$ and $k(0\boldsymbol{x})$, I will propose a formula for $k(j\boldsymbol{x})$ with a constant. To calculate its value, I will use the programmable controller.

Formula of $k(b\boldsymbol{x})$: $$\boldsymbol{k(bx) = 1 - \frac{30\pi(bx)}{x}}$$ Formula of $k(0\boldsymbol{x})$: $$\boldsymbol{k(0x) = 1 - \frac{30\pi(bx)}{x - 30\pi(ax)}}$$

Proposed formula for $k(j\boldsymbol{x})$: $$\boldsymbol{k(jx) = 1 - \frac{30\pi(bx)}{x - c(jx)\pi(ax)}}$$

Being: $\boldsymbol{x}$ = Number for which the conjecture is applied and that defines the sequences **A-B**.
$\boldsymbol{\pi(ax)}$ = Number of primes greater than $\sqrt{\boldsymbol{x}}$ in sequence **A** for $\boldsymbol{x}$.
$\boldsymbol{\pi(bx)}$ = Number of primes greater than $\sqrt{\boldsymbol{x}}$ in sequence **B** for $\boldsymbol{x}$.
$\boldsymbol{k(jx)}$ = Factor in study. The data from the PLC allow us to calculate its value for various numbers $\boldsymbol{x}$.
$\boldsymbol{c(jx)}$ = Constant that can be calculated if we know the values of $\boldsymbol{\pi(ax)}$, $\boldsymbol{\pi(bx)}$ and $\boldsymbol{k(jx)}$ for each number $\boldsymbol{x}$.

Let us recall that $k(j\boldsymbol{x})$ is lesser than $k(b\boldsymbol{x})$ so, comparing their corresponding formulas, it follows that $c(j\boldsymbol{x})$ would have a minimum value of 0. Also let us remember that, as a concept, $k(0\boldsymbol{x})$ would be the minimum value of $k(j\boldsymbol{x})$ for which the conjecture is false. According to this statement, and comparing their corresponding formulas, it follows that $c(j\boldsymbol{x})$ would have a maximum value of 30.

The program, which works in the programmable controller, is described below, in a simplified way:

1. It stores the 3.398 primes that are lesser than 31.622. With them, we can analyze the sequences **A-B** until number 10^9.
2. It divides each term of each sequence **A** or **B**, by the primes lesser than $\sqrt{x}$, to define which are multiples and those who are primes.
3. In the same process, it determines the terms $(7m - 2)$, $(11m - 2)$,… of sequence **A** and the terms $(7m + 2)$, $(11m + 2)$,… of the **B**.
4. 8 counters are scheduled (4 in each sequence) to count the following data:
   5. Number of multiples that there are in each sequence **A** or **B** (it includes all composites and the primes who are lesser than $\sqrt{x}$).
   6. Number of primes that there are in each sequence **A** or **B** (only which are greater than $\sqrt{x}$).
   7. Number of multiples and number of primes that there are in the terms $(7m - 2)$, $(11m - 2)$,… of sequence **A** (as 5 and 6).
   8. Number of multiples and number of primes that there are in the terms $(7m + 2)$, $(11m + 2)$,… of sequence **B** (as 5 and 6).
9. With the final data data of these counters, and using a calculator, the values of $k(ax)$, $k(bx)$, $k(jx)$, $c(jx)$,… can be obtained.

Then, I indicate the calculated values of $c(jx)$ related to some numbers x, (between 10^6 and 10^9), and their corresponding groups of primes. The details of these calculations can be consulted in the numerical data presented from page 16.

	$(30n + 11)$	$(30n + 13)$	$(30n + 17)$	$(30n + 19)$	$(30n + 29)$	$(30n + 31)$
10^6	2,251	2,25	2,082	2,084	2,7	2,7
10^7	1,746	1,746	1,937	1,938	2,214	2,214
10^8	2,125	2,125	2,095	2,095	2,184	2,184
10^9	2,136	2,136	2,101	2,101	2,134	2,134
$268.435.456 = 2^{28}$	2,147	2,147	2,131	2,131	2,194	2,194

The following average values of $c(jx)$ are calculated using actual data from Wikipedia.
For more details, consult the numerical data presented from page 22.

10^{10}	≈ 2,095	10^{12}	≈ 2,058	10^{14}	≈ 2,029	10^{16}	≈ 2,005
10^{11}	≈ 2,075	10^{13}	≈ 2,042	10^{15}	≈ 2,016	10^{18}	≈ 1,987

Consulting the numeric calculations presented from page 16 to 22, we can note that the axiom which has been used as a starting point at beginning of this chapter is met:

1. The number of multiples $7m_{11}$, $11m_{12}$,… of sequence **A** is equal to the number of terms $(7m_{11} + 2)$, $(11m_{12} + 2)$,… of sequence **B**.
2. The number of terms $(7m_{21} - 2)$, $(11m_{22} - 2)$,… of sequence **A** is equal to the number of multiples $7m_{21}$, $11m_{22}$,… of sequence **B**.
3. The number of multiples that there are in the terms $(7m_{21} - 2)$, $(11m_{22} - 2)$,… of sequence **A** is equal to the multiples in the terms $(7m_{11} + 2)$, $(11m_{12} + 2)$,… of sequence **B**, being the number of multiple-multiple pairs that are formed with the two sequences.

Let's review the above data:

1. Lowest number analyzed: 10^6.
2. Highest number analyzed with the programmable controller: 10^9.
3. Highest number analyzed with data from Wikipedia: 10^{18}.
4. Highest $c(jx)$ value: 2,7 for the number 10^6 in the combination $(30n + 29)$ and $(30n + 31)$.
5. Lowest $c(jx)$ value with the programmable controller: 1,746 for the number 10^7 in the combination $(30n + 11)$ and $(30n + 13)$.
6. Lowest $c(jx)$ value with data from Wikipedia: 1,987 for the number 10^{18} (average value) (we take 1,987 as minimum value).
7. Maximum number of terms analyzed by programmable controller in a sequence **A** or **B**: 33.333.333 for the number 10^9.

In the analyzed numbers with PLC, 10^9 is 10^3 times greater than 10^6. Using data from Wikipedia, 10^{18} is 10^{12} times greater than 10^6.
It can be seen that, although there is a great difference between the values of the analyzed numbers, the $c(jx)$ values vary little (from 2,7 to 1,987). We also note that the average value of $c(jx)$ tends to decrease slightly when increasing x.
Finally, it can be intuited that, for large values of x, the average value of $c(jx)$ tends to an approximate value to 2.
I believe that this data is sufficiently representative to be applied in the proposed formula for $k(jx)$.

Given the above, we can define an approximate average value for $c(jx)$: $c(jx) \approx 2,2$ (for large numbers: $c(jx) \approx 2$)

With this average value of $c(jx)$, the final formula of $k(jx)$ can be written: $$\boldsymbol{k(jx) \approx 1 - \frac{30\pi(bx)}{x - 2{,}2\pi(ax)}}$$

I consider that this formula is valid to prove the conjecture although it has not been obtained through mathematical analysis.
Also, I consider that it can be applied to large numbers because the regularity in the characteristics of terms $(7m_{11} + 2)$, $(11m_{12} + 2)$, $(13m_{13} + 2)$, $(17m_{14} + 2)$,… is maintained, and I intuit that with more precision, with increasing x.
Likewise, I believe that this formula and the formula that can be obtained through a rigorous analytical method can be considered equivalent in purpose of validity to prove the conjecture although the respective numerical results may differ slightly.

Let us analyze the deviation that can affect the average value defined for $c(jx)$. As already described, $k(jx)$ is always lesser than $k(bx)$ so that, comparing their corresponding formulas, it follows that $c(jx)$ would have a minimum value greater than 0.

We can see that the maximum deviation in decreasing is from 2,2 to 0 (or close to 0). I understand that, by symmetry, the maximum deviation in increasing will be similar so that, in principle, the $c(jx)$ value would always be lesser than 4,4 and greater than 0.
On the other hand, and as I have indicated, $c(jx)$ would have a maximum value of 30. Considering as valid the final formula proposed for $k(jx)$, considering that will be equivalent to analytical formula and comparing 30 with the calculated values of $c(jx)$, (between 2,7 and 1,987), it can be accepted that $c(jx) < 30$ will always be met.

At this point, let's make a summary of the exposed questions:

1. All multiples $7m_{11}$, $11m_{12}$, $13m_{13}$,… of sequence **A** are paired with all terms $(7m_{11} + 2)$, $(11m_{12} + 2)$, $(13m_{13} + 2)$,… of sequence **B**.
2. The groups $(7m_{11} + 2)$,… follow a "pattern" for the number of terms, number of primes and number of multiples that contain.
3. We define as $k(jx)$ the fraction of terms $(7m_{11} + 2)$, $(11m_{12} + 2)$, $(13m_{13} + 2)$,… of sequence **B** that are multiples.
4. The analysis of paragraph 2 allows us to intuit that the approximate value of $k(jx)$ can be obtained by a general formula.
5. Proposed formula for $k(jx)$: $\boldsymbol{k(jx) = 1 - \dfrac{30\pi(bx)}{x - c(jx)\pi(ax)}}$. In the exposed calculations, the $c(jx)$ value has resulted to be lesser than 3.
6. Final formula for $k(jx)$: $\boldsymbol{k(jx) \approx 1 - \dfrac{30\pi(bx)}{x - 2{,}2\pi(ax)}}$. I consider that will be equivalent to the formula obtained by mathematical analysis.
7. Considering valid the above formula and considering the calculated values of $c(jx)$, (< 3), it can be accepted that $c(jx) < 30$.
8. Applying $c(jx) < 30$ in the proposed formula for $k(jx)$: $\boldsymbol{k(jx) > 1 - \dfrac{30\pi(bx)}{x - 30\pi(ax)} = k(0x)}$
9. Finally, for any x value: $\boldsymbol{k(jx) > k(0x)}$. This statement must be rigorously demonstrated in the analytical formula.

Let us recall, in page 7, the formula to calculate the number of twin prime pairs that there are between $\sqrt{x}$ and x in the sequences **A-B**.

$$\mathbf{P_T}(x) = \boldsymbol{\pi(bx) - (1 - k(jx))\left(\frac{x}{30} - \pi(ax)\right)} \qquad \text{Substituting } k(jx) \text{ for its formula:} \quad \boldsymbol{k(jx) = 1 - \frac{30\pi(bx)}{x - c(jx)\pi(ax)}}$$

$$\mathbf{P_T}(x) = \boldsymbol{\pi(bx) - \frac{30\pi(bx)}{x - c(jx)\pi(ax)}\left(\frac{x}{30} - \pi(ax)\right) = \pi(bx) - \frac{x\pi(bx) - 30\pi(ax)\pi(bx)}{x - c(jx)\pi(ax)} = \frac{x\pi(bx) - c(jx)\pi(ax)\pi(bx) - x\pi(bx) + 30\pi(ax)\pi(bx)}{x - c(jx)\pi(ax)}}$$

$$\mathbf{P_T}(x) = \boldsymbol{\frac{(30 - c(jx))\pi(ax)\pi(bx)}{x - c(jx)\pi(ax)}}$$

In this formula, we can replace $c(jx)$ by its already defined values:

$c(jx) \approx 2{,}2$ $\qquad \mathbf{P_T}(x) \approx \boldsymbol{\dfrac{(30 - 2{,}2)\pi(ax)\pi(bx)}{x - 2{,}2\pi(ax)}}$ $\qquad \mathbf{P_T}(x) \approx \boldsymbol{\dfrac{27{,}8\pi(ax)\pi(bx)}{x - 2{,}2\pi(ax)}}$

$c(jx) < 30$ $\qquad \mathbf{P_T}(x) > \boldsymbol{\dfrac{(30 - 30)\pi(ax)\pi(bx)}{x - 30\pi(ax)}}$ $\qquad \mathbf{P_T}(x) > \mathbf{0}$

This final expression indicates that $P_T(x)$ is always greater than 0 and considering that by its nature, (prime pairs), cannot be a fractional number (must be greater than 0, cannot have a value between 0 and 1) I gather that $P_T(x)$ will be a natural number equal to or greater than 1. Similarly, I conclude that the $P_T(x)$ value will increase when increasing x because also increase $\boldsymbol{\pi(ax)}$ and $\boldsymbol{\pi(bx)}$.
We can record:

$\mathbf{P_T}(x) \geq \mathbf{1}$ $\qquad$ $P_T(x)$ will be a natural number and will gradually increase when increasing x

The final expression indicates that the number of twin prime pairs that there are between $\sqrt{x}$ and x is always equal to or greater than 1.

Let us suppose n as a sufficiently large number. As has been exposed, there will always be, at least, a pair of twin primes between n and n^2 and, therefore, greater than n. This indicates us that we will not find a pair of twin primes that is the highest and the last, so that, when x tends to infinite, it will also tend to infinite the number of twin prime pairs lesser than x.

Citing the number $6^2 = 36$, we can note that there are 3 twin prime pairs between 6 and 36, (**11**, **13**), (**17**, **19**) and (**29**, **31**), (one for each combination of groups of primes). For larger numbers, we can verify that, as x increases, so does the number of twin prime pairs between $\sqrt{x}$ and x, (8.134 twin prime pairs for 10^6 and 3.424.019 twin prime pairs for 10^9).

With everything described, it can be confirmed that: **The Twin Primes Conjecture is true.**

9. Final formula.

Considering that the conjecture has already been demonstrated, a formula can be defined to calculate the approximate number of twin prime pairs lesser than a number $\boldsymbol{x}$.
According to the previous chapter, the number of twin prime pairs, formed with the sequences **A-B**, greater than $\sqrt{\boldsymbol{x}}$ and lesser than $\boldsymbol{x}$ is:

$$\mathbf{P_T}(x) \approx \frac{\mathbf{27{,}8}\boldsymbol{\pi}(ax)\boldsymbol{\pi}(bx)}{\boldsymbol{x} - \mathbf{2{,}2}\boldsymbol{\pi}(ax)}$$

If no precision in the final formula is required, and for large values of $\boldsymbol{x}$, the following can be considered:

1. On page 5 I have indicated that: $\boldsymbol{\pi}(ax) \approx \boldsymbol{\pi}(bx) \approx \frac{\boldsymbol{\pi}(x)}{8}$ being $\boldsymbol{\pi}(x)$ the number of primes lesser than or equal to $\boldsymbol{x}$.
2. The term $\mathbf{2{,}2}\boldsymbol{\pi}(ax)$ can be neglected because it is be very small compared to $\boldsymbol{x}$, (1,4 % of $\boldsymbol{x}$ for 10^9), (0,68 % of $\boldsymbol{x}$ for 10^{18}).
3. By applying the above, the value of the denominator will increase, so, to compensate, I will put in the numerator 28 instead of 27,8.
4. The exposed data allows us to intuit that, as $\boldsymbol{x}$ is larger, the average value of $c(jx)$ will decrease being lesser than 2,2.
5. The number of twin prime pairs lesser than $\sqrt{\boldsymbol{x}}$ is very small compared to the total number of pairs lesser than $\boldsymbol{x}$.
   Example: there are 1.870.585.220 twin prime pairs lesser that 10^{12} of which 8.169, (0,000437 %), are lesser than 10^6.

With this in mind, the above formula can be slightly modified to make it more simple.
As a final concept, I consider that the numeric result of the obtained formula will be the approximate number of twin prime pairs which are formed with the sequences **A** and **B** and that are lesser than a number $\boldsymbol{x}$.

$$\mathbf{P_T}(x) \approx \frac{\mathbf{28}\,\frac{\boldsymbol{\pi}(x)}{8}\,\frac{\boldsymbol{\pi}(x)}{8}}{\boldsymbol{x}} \qquad \mathbf{P_T}(x) \approx \frac{\mathbf{7}}{\mathbf{16}}\frac{\boldsymbol{\pi}^2(x)}{\boldsymbol{x}}$$

Let us recall, page 2, that there are 3 combinations of groups of primes that form twin prime pairs (3 sets of sequences **A-B**).
Being $\mathbf{G_T}(x)$ the actual number of twin prime pairs lesser than $\boldsymbol{x}$, we have:

$$\mathbf{G_T}(x) \approx \frac{\mathbf{21}}{\mathbf{16}}\frac{\boldsymbol{\pi}^2(x)}{\boldsymbol{x}} \qquad \mathbf{G_T}(x) \approx \mathbf{1{,}3125}\,\frac{\boldsymbol{\pi}^2(x)}{\boldsymbol{x}}$$

We take the actual values of $\boldsymbol{\pi}(x)$ and $\mathbf{G_T}(x)$ from Wikipedia to check the precision of the above formula.

	$\boldsymbol{\pi}(x)$	$\mathbf{G_T}(x)$	Formula result	Difference
1. To 10^6	78.498	8.169	8.087	– 1,004 %
2. To 10^8	5.761.455	440.312	435.676	– 1,053 %
3. To 10^{10}	455.052.511	27.412.679	27.178.303	– 0,855 %
4. To 10^{12}	37.607.912.018	1.870.585.220	1.856.340.998	– 0,761 %
5. To 10^{14}	3.204.941.750.802	135.780.321.665	134.815.427.591	– 0,711 %
6. To 10^{16}	279.238.341.033.925	10.304.195.697.298	10.234.094.207.318	– 0,68 %
7. To 10^{18}	24.739.954.287.740.860	808.675.888.577.436	803.335.756.334.353	– 0,66 %

We can improve the precision "adjusting" the last formula: $\mathbf{G_T}(x) \approx \mathbf{1{,}32}\,\frac{\boldsymbol{\pi}^2(x)}{\boldsymbol{x}}$

Final formula, being: $\mathbf{G_T}(x)$ = Actual number of twin prime pairs lesser than $\boldsymbol{x}$.
$\boldsymbol{x}$ = Number greater than 30.
$\boldsymbol{\pi}(x)$ = Number of primes lesser than or equal to $\boldsymbol{x}$.

To express the final formula as an $\boldsymbol{x}$ function, we will use the prime numbers theorem[3], (page 5): $\boldsymbol{\pi}(x) \sim \frac{\boldsymbol{x}}{\mathbf{ln}(x)}$

Substituting $\boldsymbol{\pi}(x)$, and simplifying, we obtain a second formula for $\mathbf{G_T}(x)$: $\mathbf{G_T}(x) \sim \mathbf{1{,}32}\,\frac{\boldsymbol{x}}{\mathbf{ln}^2(x)}$

The sign ~ indicates that this formula has an asymptotic behavior, giving results lesser than actual values when applied to small numbers (– 15 % for 10^6), but this difference gradually decreases as we analyze larger numbers (– 5 % for 10^{18}).

A better approach for this theorem is given by the offset logarithmic integral function[3] $\mathbf{Li}(x)$: $\boldsymbol{\pi}(x) \approx \mathbf{Li}(x) = \int_2^x \frac{dy}{\mathbf{ln}(y)}$

Substituting $\boldsymbol{\pi}(x)$ again, in the formula of $\mathbf{G_T}(x)$: $\mathbf{G_T}(x) \approx \mathbf{1{,}32}\int_2^x \frac{dy}{\mathbf{ln}^2(y)}$
This third formula is the most precise.

10. Comparison with the research on this conjecture.

The research[4] to solve this conjecture is focused on demonstrating that there are infinitely many pairs of primes that are at a distance equal to or lesser than a constant. For the twin primes, this constant would be equal to 2 (in this case, distance = constant).
In April 2013, the Chinese-born mathematician, Yitang Zhang of the University of New Hampshire, presented an article to a mathematics journal which shows, for the first time, that the maximum value of the constant referred to is 70 million.
Terence Tao, of the University of California, proposed the Polymath8 project so that the mathematicians, based on the work of Zhang, could progressively reduce this value. James Maynard, of the University of Montreal, using the original approach of Zhang but with an independent work, has made the constant value be lesser than 600.
In April, 2014 it has managed to reach 246 and it seems that could be reduced to 12 or even to 6.
Although the constant value continues to decrease, and in the opinion of the participants in the mathematical Polymath8 project, it is unlikely that, from the presented researches, a demonstration of the twin primes conjecture can be reached.

Let us recall the approach in which this proof is based.

1. We define the three combinations of arithmetic progressions with which all pairs of twin primes greater than 7 will be formed:
   $(30n_1 + 11)$ and $(30n_1 + 13)$ $(30n_2 + 17)$ and $(30n_2 + 19)$ $(30n_3 + 29)$ and $(30n_3 + 31)$
2. We study how are paired the composites of a progression with composites or primes of the other.
3. Through this study, we note that some pairs where the two terms are primes are always formed.
4. These primes form the pairs of twin primes whose approximate number can be calculated by a general formula.
5. The result of this formula tends to infinite when n tends to infinite.

We note that, to solve this conjecture, mathematical research and this work use different approaches.

11. Comparison with the Hardy-Littlewood Conjecture.

The Hardy-Littlewood conjecture[5] establishes a law of distribution of twin primes lesser than a number $\boldsymbol{x}$.
We can see that is similar to the prime numbers theorem which determines the number of primes lesser than or equal to $\boldsymbol{x}$.

This conjecture states: "The number of twin prime pairs lesser than $\boldsymbol{x}$ is asymptotically equal to: $\boldsymbol{\pi_2(x) \approx 2C_2 \int_2^x \frac{dy}{\ln^2(y)}}$ ".
Being $\boldsymbol{\pi_2(x)}$ the pairs number and $\mathbf{C_2}$ the twin primes constant defined as the following product of Euler:

$\mathbf{C_2} = \prod_{p\geq 3} \frac{p(p-2)}{(p-1)^2} = 0{,}66016118158\ldots$ for all primes greater than 2.

Comparing the formula of the twin primes conjecture: $\mathbf{G_T}(x) \approx \mathbf{1{,}32}\int_2^x \frac{dy}{\ln^2(y)}$ with the formula of the Hardy-Littlewood conjecture

expressed replacing $\mathbf{C_2}$ by its value: $\boldsymbol{\pi_2(x)} \approx \mathbf{1{,}32032}\int_2^x \frac{dy}{\ln^2(y)}$ is apparent that they are almost equal.

Let us recall that the formula of $\mathbf{G_T}(x)$ is obtained from: $\mathbf{P_T}(x) \approx \frac{27{,}8\pi(ax)\pi(bx)}{x - 2{,}2\pi(ax)}$ that has been obtained from the study of the terms $(7m_{11} + 2),(11m_{12} + 2),\ldots$ of sequence **B**, and that is the number of twin prime pairs formed with the sequences **A-B**.

12. Comparison with Goldbach's Conjecture.

Goldbach's Conjecture statement[6]: "Every even integer greater than 2 can be expressed as the sum of two primes".

Goldbach's conjecture and the twin primes conjecture are similar in that both can be studied by combining two groups of primes to form pairs that add an even number, in the first, or pairs of twin primes in the second.
The demonstrations that I have developed for these two conjectures are similar.

According to the demonstration, the number of prime pairs that add an even number $\boldsymbol{x}$ (power of 2) is: $\mathbf{G}(x) \approx \frac{21}{32}\frac{\pi^2(x)}{x}$

For the even number $\boldsymbol{x}$ that is multiple of 10: $\mathbf{G_{10}}(x) \approx \frac{7}{8}\frac{\pi^2(x)}{x}$

As we have seen, the number of twin prime pairs lesser than $\boldsymbol{x}$ is: $\mathbf{G_T}(x) \approx \frac{21}{16}\frac{\pi^2(x)}{x}$

Comparing the above formulas we can verify that, for an even number $\boldsymbol{x}$ that is a power of 2, the number of pairs of primes that meet Goldbach's conjecture is, approximately, ½ of the number of twin prime pairs lesser than $\boldsymbol{x}$.
Also, we can verify that, for an even number $\boldsymbol{x}$ that is a multiple of 10, the number of pairs of primes that meet Goldbach's conjecture is, approximately, 2/3 of the number of twin prime pairs lesser than $\boldsymbol{x}$.

As numerical support, and using the programmable controller, the following data has been obtained:

To 268.435.456 = 2^{28}	525.109	prime pairs that add 2^{28}, being both primes greater than 2^{14}.
	1.055.991	twin prime pairs that are greater than 2^{14} and lesser than 2^{28}.
To 10^9	2.273.918	prime pairs that add 10^9, being both primes greater than $10^{4,5}$.
	3.424.019	twin prime pairs that are greater than $10^{4,5}$ and lesser than 10^9.

13. Studying prime pairs with separations greater than 2.

The same formula of the twin primes can be used to calculate the pairs number of *cousin* primes that have the form p, $(p + 4)$ and which are lesser than a number $\boldsymbol{x}$. The three combinations of groups of primes that form pairs of *cousin* primes are:

$(30n_1 + 7)$ and $(30n_1 + 11)$, $(30n_2 + 13)$ and $(30n_2 + 17)$, $(30n_3 + 19)$ and $(30n_3 + 23)$

The twin primes and the cousin primes are always consecutive primes.

You can also apply the same formula to prime pairs with difference between 6 and 30, if the condition that they are always consecutive primes is not required. For example, to: p, $(p + 8)$ and p, $(p + 16)$

Under the same condition, and for the following cases, we can also use the same formula but the actual number of prime pairs that are formed will be higher because 14, 22, 26 and 28 are multiples, respectively, of 7, 11, 13 and 7.

p, $(p + 14)$ p, $(p + 22)$ p, $(p + 26)$ p, $(p + 28)$

In these 4 cases, the fraction of terms $(7m_{11} + a)$, $(11m_{12} + a)$, $(13m_{13} + a)$, $(17m_{14} + a)$,… that are multiples, will be higher.
For $a = 14$ and for $a = 28$, all terms $(7m_{11} + 14)$ and $(7m_{11} + 28)$ are multiples of 7.
For $a = 22$, all terms $(11m_{12} + 22)$ are multiples of 11.
For $a = 26$, all terms $(13m_{13} + 26)$ are multiples of 13.

The other prime pairs with differences between 6 and 30 have more than 3 combinations of groups of primes. If the condition that they are always consecutive primes is not required, we will have the following formulas to calculate the number of prime pairs lesser than $\boldsymbol{x}$:

$\mathbf{G_{M6}}(\boldsymbol{x}) \approx \dfrac{\mathbf{21}}{\mathbf{8}}\dfrac{\boldsymbol{\pi}^{\mathbf{2}}(\boldsymbol{x})}{\boldsymbol{x}}$ For p, $(p + 6)$ p, $(p + 12)$ p, $(p + 18)$ p, $(p + 24)$ 6 combinations for each case

$\mathbf{G_{M10}}(\boldsymbol{x}) \approx \dfrac{\mathbf{7}}{\mathbf{4}}\dfrac{\boldsymbol{\pi}^{\mathbf{2}}(\boldsymbol{x})}{\boldsymbol{x}}$ For p, $(p + 10)$ p, $(p + 20)$ 4 combinations for each case

$\mathbf{G_{M30}}(\boldsymbol{x}) \approx \dfrac{\mathbf{7}}{\mathbf{2}}\dfrac{\boldsymbol{\pi}^{\mathbf{2}}(\boldsymbol{x})}{\boldsymbol{x}}$ For p, $(p + 30)$ 8 combinations

Consider now Polignac's Conjecture [7].

Statement: "For every natural number k there are infinitely many pairs of primes whose difference is $2k$".

In the statement is not specified the condition that the primes p and $(p + 2k)$ always are consecutive primes.
Assuming that this condition is not necessary, we can calculate the minimum number of prime pairs p and $(p + 2k)$ lesser than $\boldsymbol{x}$.
In this case, the difference between the terms of sequence **A** and the terms of sequence **B** is equal to $2k$ so I gather that we can apply the formula of the twin primes to calculate the number of prime pairs between $2k$ and $\boldsymbol{x}$.

$$\mathbf{G_k}(\boldsymbol{x}) \approx \frac{\mathbf{21}}{\mathbf{16}}\frac{\boldsymbol{\pi}^{\mathbf{2}}(\boldsymbol{x})}{\boldsymbol{x}} - \frac{\mathbf{21}}{\mathbf{16}}\frac{\boldsymbol{\pi}^{\mathbf{2}}(\mathbf{2}\boldsymbol{k})}{\mathbf{2}\boldsymbol{k}}$$

The second part of the above expression will be a constant. Focusing on the first part, we note, again, that with increasing $\boldsymbol{x}$, also increases the number of prime pairs p and $(p + 2k)$ lesser than $\boldsymbol{x}$. Therefore, we will not find a pair of primes p and $(p + 2k)$ that is the highest and the last, which allow us to conclude that Polignac's Conjecture is true.

I consider valid this reasoning if the condition that the primes p and $(p + 2k)$ always be consecutive primes is not required.

Getting data using a programmable controller

Let us recall: Multiples: include all composites and the primes lesser than $\sqrt{x}$.
Primes: only those that are greater than $\sqrt{x}$.

Sequence A

1. The four data highlighted in **bold** are those obtained by the programmable controller.
2. The sum of the number of multiples $7m$, $11m$,… and the number of primes is the total number of terms of the sequence. It must match with the formula result: $\frac{x}{30}$ (page 2).
3. The sum of the number of multiples and the number of primes of form ($7m - 2$), ($11m - 2$),… is the total number of these terms. It must match with the number of multiples $7m$, $11m$,… of sequence **B** (page 4).
4. I used a calculator to obtain the following information:
5. $P_T(x)$ = Number of twin prime pairs greater than $\sqrt{x}$ and lesser than x. It must match with the $P_T(x)$ of sequence **B**.
   $P_T(x)$ = (Number of primes of sequence **A**) – (Number of primes of form ($7m - 2$), ($11m - 2$),… of sequence **A**)
6. k_{ax} = Number of multiples $7m$, $11m$,… divided by the total number of terms of sequence **A**.
7. k_{jx} = Number of multiples that there are in the terms ($7m - 2$), ($11m - 2$),… divided by the total number of these.

   Proposed formula for k_{jx}: $\boldsymbol{k(jx) = 1 - \frac{30\pi(ax)}{x - c(jx)\pi(bx)}}$ (page 10).
8. c_{jx} = Constant of proposed formula for k_{jx}. Solving: $\boldsymbol{c(jx) = \frac{x - \frac{30\pi(ax)}{1 - k(jx)}}{\pi(bx)}}$
9. k_{0x} = Minimum value of k_{jx} for which the conjecture is false: $\boldsymbol{k(0x) = 1 - \frac{30\pi(ax)}{x - 30\pi(bx)}}$ (pages 7 and 8).

Sequence B

1. The four data highlighted in **bold** are those obtained by the programmable controller.
2. The sum of the number of multiples $7m$, $11m$,… and the number of primes is the total number of terms of the sequence. It must match with the formula result: $\frac{x}{30}$ (page 2).
3. The sum of the number of multiples and the number of primes of form ($7m + 2$), ($11m + 2$),… is the total number of these terms. It must match with the number of multiples $7m$, $11m$,… of sequence **A** (page 4).
4. I used a calculator to obtain the following information:
5. $P_T(x)$ = Number of twin prime pairs greater than $\sqrt{x}$ and lesser than x. It must match with the $P_T(x)$ of sequence **A**.
   $P_T(x)$ = (Number of primes of sequence **B**) – (Number of primes of form ($7m + 2$), ($11m + 2$),… of sequence **B**)
6. k_{bx} = Number of multiples $7m$, $11m$,… divided by the total number of terms of sequence **B**.
7. k_{jx} = Number of multiples that there are in the terms ($7m + 2$), ($11m + 2$),… divided by the total number of these.

   Proposed formula for k_{jx}: $\boldsymbol{k(jx) = 1 - \frac{30\pi(bx)}{x - c(jx)\pi(ax)}}$ (page 10).
8. c_{jx} = Constant of proposed formula for k_{jx}. Solving: $\boldsymbol{c(jx) = \frac{x - \frac{30\pi(bx)}{1 - k(jx)}}{\pi(ax)}}$
9. k_{0x} = Minimum value of k_{jx} for which the conjecture is false: $\boldsymbol{k(0x) = 1 - \frac{30\pi(bx)}{x - 30\pi(ax)}}$ (pages 7 and 8).

Choosing the group ($30n + 19$) as an example, we will count the number of multiples that there are in each of the groups ($7m + 2$), ($11m + 2$), ($13m + 2$),… until the group ($307m + 2$). The obtained values are highlighted in **bold**.
Although for this analysis, any sequence of primes can be chosen, and to count each term only once, we will do it in ascending order (7, 11, 13, 17, 19, 23,…, 307).

1. Multiples that there are in the group ($7m + 2$): they are all included.
2. Multiples that there are in the group ($11m + 2$): not included those who are also ($7m + 2$).
3. Multiples that there are in the group ($13m + 2$): not included those who are also ($7m + 2$) or ($11m + 2$).

And so on until the group of prime 307.
The percentages indicated are relative to the total number of terms ($7m + 2$), ($11m + 2$), ($13m + 2$),…

$\underline{10^6}$ $(30n_1 + 11)$ and $(30n_1 + 13)$ 33.333 pairs

Highest prime to divide 997

Sequence **A** $(30n_1 + 11)$		Sequence **B** $(30n_1 + 13)$	
Total number of terms	33.333	Total number of terms	33.333
Multiples $7m$, $11m$,…	**23.545**	Multiples $7m$, $11m$,…	**23.529**
Primes greater than 10^3	**9.788**	Primes greater than 10^3	**9.804**
Number of terms $(7m – 2)$, $(11m – 2)$,…	23.529	Number of terms $(7m + 2)$, $(11m + 2)$,…	23.545
Multiples $(7m – 2)$, $(11m – 2)$,…	**16.464**	Multiples $(7m + 2)$, $(11m + 2)$,…	**16.464**
Primes $(7m – 2)$, $(11m – 2)$,…	**7.065**	Primes $(7m + 2)$, $(11m + 2)$,…	**7.081**

$P_T(x)$ = Number of twin prime pairs (finished in 1 and 3) between 10^3 and 10^6 $P_T(x)$ = 9.788 – 7.065 = 9.804 – 7.081 = 2.723
Not included the twin prime pairs (finished in 1 and 3) lesser than 10^3

k_{ax} = 0,706357063
k_{jx} = 0,699732245 k_{jx} / k_{ax} = 0,990621148
c_{jx} = 2,251409252
k_{0x} = 0,584008613 k_{0x} / k_{ax} = 0,826789514

k_{bx} = 0,705877058
k_{jx} = 0,699256742 k_{jx} / k_{bx} = 0,990621148
c_{jx} = 2,249996341
k_{0x} = 0,583611756 k_{0x} / k_{bx} = 0,826789522

$\underline{10^6}$ $(30n_2 + 17)$ and $(30n_2 + 19)$ 33.333 pairs

Highest prime to divide 997

Sequence **A** $(30n_2 + 17)$		Sequence **B** $(30n_2 + 19)$	
Total number of terms	33.333	Total number of terms	33.333
Multiples $7m$, $11m$,…	**23.546**	Multiples $7m$, $11m$,…	**23.563**
Primes greater than 10^3	**9.787**	Primes greater than 10^3	**9.770**
Number of terms $(7m – 2)$, $(11m – 2)$,…	23.563	Number of terms $(7m + 2)$, $(11m + 2)$,…	23.546
Multiples $(7m – 2)$, $(11m – 2)$,…	**16.501**	Multiples $(7m + 2)$, $(11m + 2)$,…	**16.501**
Primes $(7m – 2)$, $(11m – 2)$,…	**7.062**	Primes $(7m + 2)$, $(11m + 2)$,…	**7.045**

$P_T(x)$ = Number of twin prime pairs (finished in 7 and 9) between 10^3 and 10^6 $P_T(x)$ = 9.787 – 7.062 = 9.770 – 7.045 = 2.725
Not included the twin prime pairs (finished in 7 and 9) lesser than 10^3

k_{ax} = 0,706387063
k_{jx} = 0,700292832 k_{jx} / k_{ax} = 0,991372673
c_{jx} = 2,082267247
k_{0x} = 0,584651294 k_{0x} / k_{ax} = 0,827664214

k_{bx} = 0,706897069
k_{jx} = 0,700798437 k_{jx} / k_{bx} = 0,991372673
c_{jx} = 2,083663833
k_{0x} = 0,585073401 k_{0x} / k_{bx} = 0,827664206

Multiples $(7m + 2)$	**3.110**	13,208 %	Multiples $(43m + 2)$	**288**	1,223 %	Multiples $(89m + 2)$	**130**	0,552 %
Multiples $(11m + 2)$	**1.796**	7,628 %	Multiples $(47m + 2)$	**260**	1,104 %	Multiples $(97m + 2)$	**104**	0,442 %
Multiples $(13m + 2)$	**1.387**	5,891 %	Multiples $(53m + 2)$	**228**	0,968 %	Multiples $(101m + 2)$	**105**	0,446 %
Multiples $(17m + 2)$	**1.008**	4,281 %	Multiples $(59m + 2)$	**206**	0,875 %	Multiples $(103m + 2)$	**102**	0,433 %
Multiples $(19m + 2)$	**827**	3,512 %	Multiples $(61m + 2)$	**196**	0,832 %	Multiples $(107m + 2)$	**107**	0,454 %
Multiples $(23m + 2)$	**674**	2,862 %	Multiples $(67m + 2)$	**186**	0,79 %	Multiples $(109m + 2)$	**93**	0,395 %
Multiples $(29m + 2)$	**516**	2,191 %	Multiples $(71m + 2)$	**162**	0,688 %	Multiples $(113m + 2)$	**96**	0,408 %
Multiples $(31m + 2)$	**454**	1,928 %	Multiples $(73m + 2)$	**149**	0,633 %	Multiples $(127m + 2)$	**91**	0,386 %
Multiples $(37m + 2)$	**366**	1,554 %	Multiples $(79m + 2)$	**133**	0,565 %	Multiples $(131m + 2)$	**88**	0,374 %
Multiples $(41m + 2)$	**316**	1,342 %	Multiples $(83m + 2)$	**133**	0,565 %	Multiples $(137m + 2)$	**77**	0,327 %
Multiples $(139m + 2)$	**85**	0,361 %	Multiples $(193m + 2)$	**61**	0,259 %	Multiples $(251m + 2)$	**43**	0,183 %
Multiples $(149m + 2)$	**77**	0,327 %	Multiples $(197m + 2)$	**57**	0,242 %	Multiples $(257m + 2)$	**41**	0,174 %
Multiples $(151m + 2)$	**66**	0,28 %	Multiples $(199m + 2)$	**59**	0,251 %	Multiples $(263m + 2)$	**43**	0,183 %
Multiples $(157m + 2)$	**77**	0,327 %	Multiples $(211m + 2)$	**46**	0,195 %	Multiples $(269m + 2)$	**47**	0,199 %
Multiples $(163m + 2)$	**69**	0,293 %	Multiples $(223m + 2)$	**49**	0,208 %	Multiples $(271m + 2)$	**39**	0,166 %
Multiples $(167m + 2)$	**67**	0,284 %	Multiples $(227m + 2)$	**47**	0,199 %	Multiples $(277m + 2)$	**40**	0,17 %
Multiples $(173m + 2)$	**62**	0,263 %	Multiples $(229m + 2)$	**44**	0,187 %	Multiples $(281m + 2)$	**44**	0,187 %
Multiples $(179m + 2)$	**69**	0,293 %	Multiples $(233m + 2)$	**51**	0,217 %	Multiples $(283m + 2)$	**37**	0,157 %
Multiples $(181m + 2)$	**63**	0,267 %	Multiples $(239m + 2)$	**37**	0,157 %	Multiples $(293m + 2)$	**38**	0,161 %
Multiples $(191m + 2)$	**59**	0,251 %	Multiples $(241m + 2)$	**44**	0,187 %	Multiples $(307m + 2)$	**40**	0,17 %

Total number of multiples in the groups $(7m + 2)$ to $(307m + 2)$ 14.989 63,658 %

$\underline{10^6}$ $(30n_3 + 29)$ and $(30n_3 + 31)$ 33.333 pairs

Highest prime to divide 997

Sequence **A** $(30n_3 + 29)$		Sequence **B** $(30n_3 + 31)$	
Total number of terms	33.333	Total number of terms	33.333
Multiples $7m$, $11m$,…	**23.548**	Multiples $7m$, $11m$,…	**23.544**
Primes greater than 10^3	**9.785**	Primes greater than 10^3	**9.789**
Number of terms $(7m – 2)$, $(11m – 2)$,…	23.544	Number of terms $(7m + 2)$, $(11m + 2)$,…	23.548
Multiples $(7m – 2)$, $(11m – 2)$,…	**16.445**	Multiples $(7m + 2)$, $(11m + 2)$,…	**16.445**
Primes $(7m – 2)$, $(11m – 2)$,…	**7.099**	Primes $(7m + 2)$, $(11m + 2)$,…	**7.103**

$P_T(x)$ = Number of twin prime pairs (finished in 9 and 1) between 10^3 and 10^6 $P_T(x)$ = 9.785 – 7.099 = 9.789 – 7.103 = 2.686
Not included the twin prime pairs (finished in 9 and 1) lesser than 10^3

k_{ax} = 0,706447064		k_{bx} = 0,706327063	
k_{jx} = 0,698479442	k_{jx} / k_{ax} = 0,988721558	k_{jx} = 0,698360795	k_{jx} / k_{bx} = 0,988721558
c_{jx} = 2,700433393		c_{jx} = 2,700016271	
k_{0x} = 0,584401059	k_{0x} / k_{ax} = 0,827239701	k_{0x} = 0,58430179	k_{0x} / k_{bx} = 0,827239703

<u>10^7</u> $(30n_1 + 11)$ and $(30n_1 + 13)$ 333.333 pairs — Highest prime to divide 3.137

Sequence **A** $(30n_1 + 11)$		Sequence **B** $(30n_1 + 13)$	
Total number of terms	333.333	Total number of terms	333.333
Multiples $7m$, $11m$,…	**250.287**	Multiples $7m$, $11m$,…	**250.310**
Primes greater than $10^{3,5}$	**83.046**	Primes greater than $10^{3,5}$	**83.023**
Number of terms $(7m – 2)$, $(11m – 2)$,…	250.310	Number of terms $(7m + 2)$, $(11m + 2)$,…	250.287
Multiples $(7m – 2)$, $(11m – 2)$,…	**187.031**	Multiples $(7m + 2)$, $(11m + 2)$,…	**187.031**
Primes $(7m – 2)$, $(11m – 2)$,…	**63.279**	Primes $(7m + 2)$, $(11m + 2)$,…	**63.256**

$P_T(x)$ = Number of twin prime pairs (finished in 1 and 3) between $10^{3,5}$ and 10^7 $P_T(x)$ = 83.046 – 63.279 = 83.023 – 63.256 = 19.767
Not included the twin prime pairs (finished in 1 and 3) lesser than $10^{3,5}$

k_{ax} = 0,75086175		k_{bx} = 0,75093075	
k_{jx} = 0,747197475	k_{jx} / k_{ax} = 0,995119906	k_{jx} = 0,747266138	k_{jx} / k_{bx} = 0,995119906
c_{jx} = 1,74597435		c_{jx} = 1,746125574	
k_{0x} = 0,668227839	k_{0x} / k_{ax} = 0,889947902	k_{0x} = 0,668289246	k_{0x} / k_{bx} = 0,889947902

<u>10^7</u> $(30n_2 + 17)$ and $(30n_2 + 19)$ 333.333 pairs — Highest prime to divide 3.137

Sequence **A** $(30n_2 + 17)$		Sequence **B** $(30n_2 + 19)$	
Total number of terms	333.333	Total number of terms	333.333
Multiples $7m$, $11m$,…	**250.283**	Multiples $7m$, $11m$,…	**250.375**
Primes greater than $10^{3,5}$	**83.050**	Primes greater than $10^{3,5}$	**82.958**
Number of terms $(7m – 2)$, $(11m – 2)$,…	250.375	Number of terms $(7m + 2)$, $(11m + 2)$,…	250.283
Multiples $(7m – 2)$, $(11m – 2)$,…	**186.975**	Multiples $(7m + 2)$, $(11m + 2)$,…	**186.975**
Primes $(7m – 2)$, $(11m – 2)$,…	**63.400**	Primes $(7m + 2)$, $(11m + 2)$,…	**63.308**

$P_T(x)$ = Number of twin prime pairs (finished in 7 and 9) between $10^{3,5}$ and 10^7 $P_T(x)$ = 83.050 – 63.400 = 82.958 – 63.308 = 19.650
Not included the twin prime pairs (finished in 7 and 9) lesser than $10^{3,5}$

k_{ax} = 0,75084975		k_{bx} = 0,751125751	
k_{jx} = 0,74677983	k_{jx} / k_{ax} = 0,99457958	k_{jx} = 0,747054334	k_{jx} / k_{bx} = 0,99457958
c_{jx} = 1,937563656		c_{jx} = 1,938229798	
k_{0x} = 0,668297995	k_{0x} / k_{ax} = 0,890055569	k_{0x} = 0,66854365	k_{0x} / k_{bx} = 0,890055559

Multiples $(7m + 2)$	**33.738**	13,48 %	Multiples $(43m + 2)$	**3.211**	1,283 %	Multiples $(89m + 2)$	**1.301**	0,52 %
Multiples $(11m + 2)$	**19.062**	7,616 %	Multiples $(47m + 2)$	**2.889**	1,154 %	Multiples $(97m + 2)$	**1.193**	0,477 %
Multiples $(13m + 2)$	**14.764**	5,899 %	Multiples $(53m + 2)$	**2.495**	0,997 %	Multiples $(101m + 2)$	**1.113**	0,445 %
Multiples $(17m + 2)$	**10.553**	4,216 %	Multiples $(59m + 2)$	**2.198**	0,878 %	Multiples $(103m + 2)$	**1.093**	0,437 %
Multiples $(19m + 2)$	**8.873**	3,545 %	Multiples $(61m + 2)$	**2.121**	0,847 %	Multiples $(107m + 2)$	**1.037**	0,414 %
Multiples $(23m + 2)$	**6.999**	2,796 %	Multiples $(67m + 2)$	**1.886**	0,753 %	Multiples $(109m + 2)$	**1.006**	0,402 %
Multiples $(29m + 2)$	**5.304**	2,119 %	Multiples $(71m + 2)$	**1.720**	0,687 %	Multiples $(113m + 2)$	**957**	0,382 %
Multiples $(31m + 2)$	**4.846**	1,936 %	Multiples $(73m + 2)$	**1.667**	0,666 %	Multiples $(127m + 2)$	**842**	0,336 %
Multiples $(37m + 2)$	**3.912**	1,563 %	Multiples $(79m + 2)$	**1.501**	0,6 %	Multiples $(131m + 2)$	**816**	0,326 %
Multiples $(41m + 2)$	**3.462**	1,383 %	Multiples $(83m + 2)$	**1.429**	0,571 %	Multiples $(137m + 2)$	**761**	0,304 %
Multiples $(139m + 2)$	**737**	0,294 %	Multiples $(193m + 2)$	**502**	0,2 %	Multiples $(251m + 2)$	**381**	0,152 %
Multiples $(149m + 2)$	**689**	0,275 %	Multiples $(197m + 2)$	**500**	0,2 %	Multiples $(257m + 2)$	**390**	0,156 %
Multiples $(151m + 2)$	**658**	0,263 %	Multiples $(199m + 2)$	**492**	0,197 %	Multiples $(263m + 2)$	**385**	0,154 %
Multiples $(157m + 2)$	**652**	0,261 %	Multiples $(211m + 2)$	**453**	0,181 %	Multiples $(269m + 2)$	**370**	0,148 %
Multiples $(163m + 2)$	**602**	0,241 %	Multiples $(223m + 2)$	**431**	0,172 %	Multiples $(271m + 2)$	**354**	0,141 %
Multiples $(167m + 2)$	**594**	0,237 %	Multiples $(227m + 2)$	**426**	0,17 %	Multiples $(277m + 2)$	**355**	0,142 %
Multiples $(173m + 2)$	**574**	0,229 %	Multiples $(229m + 2)$	**417**	0,167 %	Multiples $(281m + 2)$	**368**	0,147 %
Multiples $(179m + 2)$	**550**	0,22 %	Multiples $(233m + 2)$	**427**	0,171 %	Multiples $(283m + 2)$	**362**	0,145 %
Multiples $(181m + 2)$	**532**	0,213 %	Multiples $(239m + 2)$	**410**	0,164 %	Multiples $(293m + 2)$	**349**	0,139 %
Multiples $(191m + 2)$	**528**	0,211 %	Multiples $(241m + 2)$	**406**	0,162 %	Multiples $(307m + 2)$	**325**	0,13 %

Total number of multiples in the groups $(7m + 2)$ to $(307m + 2)$ 156.968 62,716 %

$\underline{10^7}$ $(30n_3 + 29)$ and $(30n_3 + 31)$ 333.333 pairs

Highest prime to divide 3.137

Sequence **A** $(30n_3 + 29)$

Total number of terms 333.333
Multiples $7m$, $11m$,… **250.369**
Primes greater than $10^{3,5}$ **82.964**
Number of terms $(7m – 2)$, $(11m – 2)$,… 250.383
Multiples $(7m – 2)$, $(11m – 2)$,… **186.899**
Primes $(7m – 2)$, $(11m – 2)$,… **63.484**

Sequence **B** $(30n_3 + 31)$

Total number of terms 333.333
Multiples $7m$, $11m$,… **250.383**
Primes greater than $10^{3,5}$ **82.950**
Number of terms $(7m + 2)$, $(11m + 2)$,… 250.369
Multiples $(7m + 2)$, $(11m + 2)$,… **186.899**
Primes $(7m + 2)$, $(11m + 2)$,… **63.470**

$P_T(x)$ = Number of twin prime pairs (finished in 9 and 1) between $10^{3,5}$ and 10^7
Not included the twin prime pairs (finished in 9 and 1) lesser than $10^{3,5}$

$P_T(x) = 82.964 – 63.484 = 82.950 – 63.470 = 19.480$

$k_{ax} = 0,751107751$
$k_{jx} = 0,746452434$ $k_{jx} / k_{ax} = 0,993802066$
$c_{jx} = 2,213587286$
$k_{0x} = 0,668652066$ $k_{0x} / k_{ax} = 0,890221231$

$k_{bx} = 0,751149751$
$k_{jx} = 0,746494174$ $k_{jx} / k_{bx} = 0,993802066$
$c_{jx} = 2,213701778$
$k_{0x} = 0,668689456$ $k_{0x} / k_{bx} = 0,890221231$

$\underline{10^8}$ $(30n_1 + 11)$ and $(30n_1 + 13)$ 3.333.333 pairs

Highest prime to divide 9.973

Sequence **A** $(30n_1 + 11)$

Total number of terms 3.333.333
Multiples $7m$, $11m$,… **2.613.173**
Primes greater than 10^4 **720.160**
Number of terms $(7m – 2)$, $(11m – 2)$,… 2.613.377
Multiples $(7m – 2)$, $(11m – 2)$,… **2.039.991**
Primes $(7m – 2)$, $(11m – 2)$,… **573.386**

Sequence **B** $(30n_1 + 13)$

Total number of terms 3.333.333
Multiples $7m$, $11m$,… **2.613.377**
Primes greater than 10^4 **719.956**
Number of terms $(7m + 2)$, $(11m + 2)$,… 2.613.173
Multiples $(7m + 2)$, $(11m + 2)$,… **2.039.991**
Primes $(7m + 2)$, $(11m + 2)$,… **573.182**

$P_T(x)$ = Number of twin prime pairs (finished in 1 and 3) between 10^4 and 10^8
Not included the twin prime pairs (finished in 1 and 3) lesser than 10^4

$P_T(x) = 720.160 – 573.386 = 719.956 – 573.182 = 146.774$

$k_{ax} = 0,783951978$
$k_{jx} = 0,780595757$ $k_{jx} / k_{ax} = 0,995718844$
$c_{jx} = 2,124723493$
$k_{0x} = 0,724433211$ $k_{0x} / k_{ax} = 0,924078554$

$k_{bx} = 0,784013178$
$k_{jx} = 0,780656695$ $k_{jx} / k_{bx} = 0,995718844$
$c_{jx} = 2,124877608$
$k_{0x} = 0,724489764$ $k_{0x} / k_{bx} = 0,924078554$

$\underline{10^8}$ $(30n_2 + 17)$ and $(30n_2 + 19)$ 3.333.333 pairs

Highest prime to divide 9.973

Sequence **A** $(30n_2 + 17)$

Total number of terms 3.333.333
Multiples $7m$, $11m$,… **2.613.261**
Primes greater than 10^4 **720.072**
Number of terms $(7m – 2)$, $(11m – 2)$,… 2.613.330
Multiples $(7m – 2)$, $(11m – 2)$,… **2.040.147**
Primes $(7m – 2)$, $(11m – 2)$,… **573.183**

Sequence **B** $(30n_2 + 19)$

Total number of terms 3.333.333
Multiples $7m$, $11m$,… **2.613.330**
Primes greater than 10^4 **720.003**
Number of terms $(7m + 2)$, $(11m + 2)$,… 2.613.261
Multiples $(7m + 2)$, $(11m + 2)$,… **2.040.147**
Primes $(7m + 2)$, $(11m + 2)$,… **573.114**

$P_T(x)$ = Number of twin prime pairs (finished in 7 and 9) between 10^4 and 10^8
Not included the twin prime pairs (finished in 7 and 9) lesser than 10^4

$P_T(x) = 720.072 – 573.183 = 720.003 – 573.114 = 146.889$

$k_{ax} = 0,783978378$
$k_{jx} = 0,78066949$ $k_{jx} / k_{ax} = 0,995779363$
$c_{jx} = 2,095325992$
$k_{0x} = 0,724461928$ $k_{0x} / k_{ax} = 0,924084067$

$k_{bx} = 0,783999078$
$k_{jx} = 0,780690103$ $k_{jx} / k_{bx} = 0,995779363$
$c_{jx} = 2,095377223$
$k_{0x} = 0,724481057$ $k_{0x} / k_{bx} = 0,924084067$

Multiples $(7m + 2)$	**356.180**	13,63 %	Multiples $(43m + 2)$	**33.369**	1,277 %	Multiples $(89m + 2)$	**13.765**	0,527%
Multiples $(11m + 2)$	**199.690**	7,641 %	Multiples $(47m + 2)$	**29.857**	1,143 %	Multiples $(97m + 2)$	**12.505**	0,478 %
Multiples $(13m + 2)$	**154.739**	5,921 %	Multiples $(53m + 2)$	**25.894**	0,991 %	Multiples $(101m + 2)$	**11.845**	0,453 %
Multiples $(17m + 2)$	**110.124**	4,214 %	Multiples $(59m + 2)$	**22.872**	0,875 %	Multiples $(103m + 2)$	**11.588**	0,443 %
Multiples $(19m + 2)$	**93.010**	3,559 %	Multiples $(61m + 2)$	**21.718**	0,831 %	Multiples $(107m + 2)$	**11.028**	0,422 %
Multiples $(23m + 2)$	**73.070**	2,796 %	Multiples $(67m + 2)$	**19.490**	0,746 %	Multiples $(109m + 2)$	**10.695**	0,409 %
Multiples $(29m + 2)$	**55.597**	2,127 %	Multiples $(71m + 2)$	**18.169**	0,695 %	Multiples $(113m + 2)$	**10.243**	0,392 %
Multiples $(31m + 2)$	**50.315**	1,925 %	Multiples $(73m + 2)$	**17.416**	0,666 %	Multiples $(127m + 2)$	**9.010**	0,345 %
Multiples $(37m + 2)$	**40.767**	1,56 %	Multiples $(79m + 2)$	**15.835**	0,606 %	Multiples $(131m + 2)$	**8.661**	0,331 %
Multiples $(41m + 2)$	**35.815**	1,371 %	Multiples $(83m + 2)$	**14.933**	0,571 %	Multiples $(137m + 2)$	**8.172**	0,313 %
Multiples $(139m + 2)$	**7.978**	0,305 %	Multiples $(163m + 2)$	**6.603**	0,253 %	Multiples $(181m + 2)$	**5.717**	0,219 %
Multiples $(149m + 2)$	**7.417**	0,284 %	Multiples $(167m + 2)$	**6.481**	0,248 %	Multiples $(191m + 2)$	**5.463**	0,209 %
Multiples $(151m + 2)$	**7.245**	0,277 %	Multiples $(173m + 2)$	**6.070**	0,232 %	Multiples $(193m + 2)$	**5.362**	0,205 %
Multiples $(157m + 2)$	**6.900**	0,264 %	Multiples $(179m + 2)$	**5.877**	0,225 %	Multiples $(197m + 2)$	**5.231**	0,2 %

Multiples ($199m + 2$)	**5.064**	0,194 %	Multiples ($239m + 2$)	**4.108**	0,157 %	Multiples ($271m + 2$)	**3.472**	0,133 %
Multiples ($211m + 2$)	**4.782**	0,183 %	Multiples ($241m + 2$)	**3.995**	0,153 %	Multiples ($277m + 2$)	**3.389**	0,13 %
Multiples ($223m + 2$)	**4.462**	0,171 %	Multiples ($251m + 2$)	**3.940**	0,151 %	Multiples ($281m + 2$)	**3.339**	0,128 %
Multiples ($227m + 2$)	**4.388**	0,168 %	Multiples ($257m + 2$)	**3.741**	0,143 %	Multiples ($283m + 2$)	**3.288**	0,126 %
Multiples ($229m + 2$)	**4.322**	0,165 %	Multiples ($263m + 2$)	**3.671**	0,14 %	Multiples ($293m + 2$)	**3.152**	0,121 %
Multiples ($233m + 2$)	**4.208**	0,161 %	Multiples ($269m + 2$)	**3.531**	0,135 %	Multiples ($307m + 2$)	**3.029**	0,116 %

Total number of multiples in the groups ($7m + 2$) to ($307m + 2$)   1.642.597   62,856 %

$\underline{10^8}$ ($30n_3 + 29$) and ($30n_3 + 31$)   3.333.333 pairs   Highest prime to divide 9.973

Sequence **A** ($30n_3 + 29$)

Total number of terms	3.333.333
Multiples $7m$, $11m$,…	**2.613.453**
Primes greater than 10^4	**719.880**
Number of terms ($7m - 2$), ($11m - 2$),…	2.613.501
Multiples ($7m - 2$), ($11m - 2$),…	**2.040.065**
Primes ($7m - 2$), ($11m - 2$),…	**573.436**

Sequence **B** ($30n_3 + 31$)

Total number of terms	3.333.333
Multiples $7m$, $11m$,…	**2.613.501**
Primes greater than 10^4	**719.832**
Number of terms ($7m + 2$), ($11m + 2$),…	2.613.453
Multiples ($7m + 2$), ($11m + 2$),…	**2.040.065**
Primes ($7m + 2$), ($11m + 2$),…	**573.388**

$P_T(x)$ = Number of twin prime pairs (finished in 9 and 1) between 10^4 and 10^8
Not included the twin prime pairs (finished in 9 and 1) lesser than 10^4

$P_T(x) = 719.880 - 573.436 = 719.832 - 573.388 = 146.444$

$k_{ax} = 0{,}784035978$
$k_{jx} = 0{,}780587036$ $k_{jx} / k_{ax} = 0{,}995601041$
$c_{jx} = 2{,}183711332$
$k_{0x} = 0{,}724553421$ $k_{0x} / k_{ax} = 0{,}924132873$

$k_{bx} = 0{,}784050378$
$k_{jx} = 0{,}780601373$ $k_{jx} / k_{bx} = 0{,}995601041$
$c_{jx} = 2{,}183748304$
$k_{0x} = 0{,}724566729$ $k_{0x} / k_{bx} = 0{,}924132873$

$\underline{10^9}$ ($30n_2 + 11$) and ($30n_2 + 13$)   33.333.333 pairs   Highest prime to divide 31.607   square root 31.622   50.847.534 primes lesser than 10^9

Sequence **A** ($30n_2 + 11$)

Total number of terms	33.333.333
Multiples $7m$, $11m$,…	**26.977.564**
Primes greater than $10^{4,5}$	**6.355.769**
Number of terms ($7m - 2$), ($11m - 2$),…	26.977.700
Multiples ($7m - 2$), ($11m - 2$),…	**21.762.981**
Primes ($7m - 2$), ($11m - 2$),…	**5.214.719**

Sequence **B** ($30n_2 + 13$)

Total number of terms	33.333.333
Multiples $7m$, $11m$,…	**26.977.700**
Primes greater than $10^{4,5}$	**6.355.633**
Number of terms ($7m + 2$), ($11m + 2$),…	26.977.564
Multiples ($7m + 2$), ($11m + 2$),…	**21.762.981**
Primes ($7m + 2$), ($11m + 2$),…	**5.214.583**

$P_T(x)$ = Number of twin prime pairs (finished in 1 and 3) between $10^{4,5}$ and 10^9
Not included the twin prime pairs (finished in 1 and 3) lesser than $10^{4,5}$

$P_T(x) = 6.355.769 - 5.214.719 = 6.355.633 - 5.214.583 = 1.141.050$

$k_{ax} = 0{,}809326928$
$k_{jx} = 0{,}806702609$ $k_{jx} / k_{ax} = 0{,}996757406$
$c_{jx} = 2{,}136152$
$k_{0x} = 0{,}764406568$ $k_{0x} / k_{ax} = 0{,}944496644$

$k_{bx} = 0{,}809331008$
$k_{jx} = 0{,}806706676$ $k_{jx} / k_{bx} = 0{,}996757406$
$c_{jx} = 2{,}136161818$
$k_{0x} = 0{,}764410421$ $k_{0x} / k_{bx} = 0{,}944496644$

$\underline{10^9}$ ($30n_2 + 17$) and ($30n_2 + 19$)   33.333.333 pairs   Highest prime to divide 31.607   square root 31.622

Sequence **A** ($30n_2 + 17$)

Total number of terms	33.333.333
Multiples $7m$, $11m$,…	**26.977.923**
Primes greater than $10^{4,5}$	**6.355.410**
Number of terms ($7m - 2$), ($11m - 2$),…	26.978.760
Multiples ($7m - 2$), ($11m - 2$),…	**21.765.319**
Primes ($7m - 2$), ($11m - 2$),…	**5.213.441**

Sequence **B** ($30n_2 + 19$)

Total number of terms	33.333.333
Multiples $7m$, $11m$,…	**26.978.760**
Primes greater than $10^{4,5}$	**6.354.573**
Number of terms ($7m + 2$), ($11m + 2$),…	26.977.923
Multiples ($7m + 2$), ($11m + 2$),…	**21.765.319**
Primes ($7m + 2$), ($11m + 2$),…	**5.212.604**

$P_T(x)$ = Number of twin prime pairs (finished in 7 and 9) between $10^{4,5}$ and 10^9
Not included the twin prime pairs (finished in 7 and 9) lesser than $10^{4,5}$

$P_T(x) = 6.355.410 - 5.213.441 = 6.354.573 - 5.212.604 = 1.141.969$

$k_{ax} = 0{,}809337698$
$k_{jx} = 0{,}806757575$ $k_{jx} / k_{ax} = 0{,}996812056$
$c_{jx} = 2{,}101125023$
$k_{0x} = 0{,}764429131$ $k_{0x} / k_{ax} = 0{,}944511954$

$k_{bx} = 0{,}809362808$
$k_{jx} = 0{,}806782605$ $k_{jx} / k_{bx} = 0{,}996812056$
$c_{jx} = 2{,}101185605$
$k_{0x} = 0{,}764452848$ $k_{0x} / k_{bx} = 0{,}944511954$

Multiples ($7m$ + 2)	**3.702.682**	13,725 %	Multiples ($43m$ + 2)	**343.921**	1,275 %	Multiples ($89m$ + 2)	**141.398**	0,524 %
Multiples ($11m$ + 2)	**2.067.716**	7,664 %	Multiples ($47m$ + 2)	**307.617**	1,14 %	Multiples ($97m$ + 2)	**128.286**	0,475 %
Multiples ($13m$ + 2)	**1.600.794**	5,934 %	Multiples ($53m$ + 2)	**267.143**	0,99 %	Multiples ($101m$ + 2)	**121.875**	0,452 %
Multiples ($17m$ + 2)	**1.137.526**	4,216 %	Multiples ($59m$ + 2)	**235.591**	0,873 %	Multiples ($103m$ + 2)	**118.521**	0,439 %
Multiples ($19m$ + 2)	**960.190**	3,559 %	Multiples ($61m$ + 2)	**224.007**	0,83 %	Multiples ($107m$ + 2)	**113.007**	0,419 %
Multiples ($23m$ + 2)	**753.641**	2,793 %	Multiples ($67m$ + 2)	**200.462**	0,743 %	Multiples ($109m$ + 2)	**109.884**	0,407 %
Multiples ($29m$ + 2)	**573.335**	2,125 %	Multiples ($71m$ + 2)	**186.672**	0,692 %	Multiples ($113m$ + 2)	**105.072**	0,389 %
Multiples ($31m$ + 2)	**518.291**	1,921 %	Multiples ($73m$ + 2)	**179.001**	0,663 %	Multiples ($127m$ + 2)	**92.743**	0,344 %
Multiples ($37m$ + 2)	**421.045**	1,561 %	Multiples ($79m$ + 2)	**162.991**	0,604 %	Multiples ($131m$ + 2)	**89.318**	0,331 %
Multiples ($41m$ + 2)	**369.577**	1,37 %	Multiples ($83m$ + 2)	**153.412**	0,569 %	Multiples ($137m$ + 2)	**84.620**	0,314 %
Multiples ($139m$ + 2)	**82.723**	0,307 %	Multiples ($193m$ + 2)	**56.273**	0,208 %	Multiples ($251m$ + 2)	**41.261**	0,153 %
Multiples ($149m$ + 2)	**76.928**	0,285 %	Multiples ($197m$ + 2)	**54.948**	0,204 %	Multiples ($257m$ + 2)	**40.005**	0,148 %
Multiples ($151m$ + 2)	**75.245**	0,279 %	Multiples ($199m$ + 2)	**54.071**	0,201 %	Multiples ($263m$ + 2)	**39.013**	0,145 %
Multiples ($157m$ + 2)	**71.985**	0,267 %	Multiples ($211m$ + 2)	**50.626**	0,188 %	Multiples ($269m$ + 2)	**37.893**	0,14 %
Multiples ($163m$ + 2)	**68.907**	0,255 %	Multiples ($223m$ + 2)	**47.734**	0,177 %	Multiples ($271m$ + 2)	**37.431**	0,139 %
Multiples ($167m$ + 2)	**66.866**	0,248 %	Multiples ($227m$ + 2)	**46.668**	0,173 %	Multiples ($277m$ + 2)	**36.348**	0,135 %
Multiples ($173m$ + 2)	**64.006**	0,237 %	Multiples ($229m$ + 2)	**46.034**	0,171 %	Multiples ($281m$ + 2)	**35.794**	0,133 %
Multiples ($179m$ + 2)	**61.593**	0,228 %	Multiples ($233m$ + 2)	**44.986**	0,167 %	Multiples ($283m$ + 2)	**35.508**	0,132 %
Multiples ($181m$ + 2)	**60.634**	0,225 %	Multiples ($239m$ + 2)	**43.596**	0,162 %	Multiples ($293m$ + 2)	**34.053**	0,126 %
Multiples ($191m$ + 2)	**57.181**	0,212 %	Multiples ($241m$ + 2)	**43.000**	0,159 %	Multiples ($307m$ + 2)	**32.335**	0,12 %

Total number of multiples in the groups ($7m$ + 2) to ($307m$ + 2) 17.013.983 63,066 %

<u>10^9</u> ($30n_2 + 29$) and ($30n_2 + 31$) 33.333.333 pairs

Highest prime to divide 31.607 square root 31.622

Sequence **A** ($30n_2 + 29$)

Total number of terms	33.333.333
Multiples $7m$, $11m$,…	**26.977.414**
Primes greater than $10^{4,5}$	**6.355.919**
Number of terms ($7m - 2$), ($11m - 2$),…	26.978.563
Multiples ($7m - 2$), ($11m - 2$),…	**21.763.644**
Primes ($7m - 2$), ($11m - 2$),…	**5.214.919**

Sequence **B** ($30n_2 + 31$)

Total number of terms	33.333.333
Multiples $7m$, $11m$,…	**26.978.563**
Primes greater than $10^{4,5}$	**6.354.770**
Number of terms ($7m + 2$), ($11m + 2$),…	26.977.414
Multiples ($7m + 2$), ($11m + 2$),…	**21.763.644**
Primes ($7m + 2$), ($11m + 2$),…	**5.213.770**

$P_T(x)$ = Number of twin prime pairs (finished in 9 and 1) between $10^{4,5}$ and 10^9
Not included the twin prime pairs (finished in 9 and 1) lesser than $10^{4,5}$

$P_T(x)$ = 6.355.919 – 5.214.919 = 6.354.770 – 5.213.770 = 1.141.000

k_{ax} = 0,809322428
k_{jx} = 0,806701379 k_{jx} / k_{ax} = 0,996761428
c_{jx} = 2,133766434
k_{0x} = 0,764408544 k_{0x} / k_{ax} = 0,944504337

k_{bx} = 0,809356898
k_{jx} = 0,806735738 k_{jx} / k_{bx} = 0,996761428
c_{jx} = 2,133850341
k_{0x} = 0,764441101 k_{0x} / k_{bx} = 0,944504337

<u>268.435.456 = 2^{28}</u> ($30n_2 + 11$) and ($30n_2 + 13$) 8.947.849 pairs

Highest prime to divide 16.381 square root 16.384

Sequence **A** ($30n_2 + 11$)

Total number of terms	8.947.849
Multiples $7m$, $11m$,…	**7.119.033**
Primes greater than 2^{14}	**1.828.816**
Number of terms ($7m - 2$), ($11m - 2$),…	7.119.006
Multiples ($7m - 2$), ($11m - 2$),…	**5.642.375**
Primes ($7m - 2$), ($11m - 2$),…	**1.476.631**

Sequence **B** ($30n_2 + 13$)

Total number of terms	8.947.849
Multiples $7m$, $11m$,…	**7.119.006**
Primes greater than 2^{14}	**1.828.843**
Number of terms ($7m + 2$), ($11m + 2$),…	7.119.033
Multiples ($7m + 2$), ($11m + 2$),…	**5.642.375**
Primes ($7m + 2$), ($11m + 2$),…	**1.476.658**

$P_T(x)$ = Number of twin prime pairs (finished in 1 and 3) between 2^{14} and 2^{28}
Not included the twin prime pairs (finished in 1 and 3) lesser than 2^{14}

$P_T(x)$ = 1.828.816 – 1.476.631 = 1.828.843 – 1.476.658 = 352.185

k_{ax} = 0,795613895
k_{jx} = 0,792579048 k_{jx} / k_{ax} = 0,996185527
c_{jx} = 2,147564373
k_{0x} = 0,743107939 k_{0x} / k_{ax} = 0,934005732

k_{bx} = 0,795610878
k_{jx} = 0,792576042 k_{jx} / k_{bx} = 0,996185527
c_{jx} = 2,147556829
k_{0x} = 0,743105121 k_{0x} / k_{bx} = 0,934005732

<u>268.435.456 = 2^{28}</u> ($30n_2 + 17$) and ($30n_2 + 19$) 8.947.848 pairs

Highest prime to divide 16.381 square root 16.384

Sequence **A** ($30n_2 + 17$)

Total number of terms	8.947.848
Multiples $7m$, $11m$,…	**7.119.164**
Primes greater than 2^{14}	**1.828.684**
Number of terms ($7m - 2$), ($11m - 2$),…	7.119.581
Multiples ($7m - 2$), ($11m - 2$),…	**5.643.113**
Primes ($7m - 2$), ($11m - 2$),…	**1.476.468**

Sequence **B** ($30n_2 + 19$)

Total number of terms	8.947.848
Multiples $7m$, $11m$,…	**7.119.581**
Primes greater than 2^{14}	**1.828.267**
Number of terms ($7m + 2$), ($11m + 2$),…	7.119.164
Multiples ($7m + 2$), ($11m + 2$),…	**5.643.113**
Primes ($7m + 2$), ($11m + 2$),…	**1.476.051**

$P_T(x)$ = Number of twin prime pairs (finished in 7 and 9) between 2^{14} and 2^{28} $P_T(x)$ = 1.828.684 – 1.476.468 = 1.828.267 – 1.476.051 = 352.216
Not included the twin prime pairs (finished in 7 and 9) lesser than 2^{14}

k_{ax} = 0,795628624		k_{bx} = 0,795675228	
k_{jx} = 0,792618694	k_{jx} / k_{ax} = 0,996216915	k_{jx} = 0,792665121	k_{jx} / k_{bx} = 0,996216915
c_{jx} = 2,131026917		c_{jx} = 2,131142926	
k_{0x} = 0,743147263	k_{0x} / k_{ax} = 0,934037866	k_{0x} = 0,743190792	k_{0x} / k_{bx} = 0,934037866

<u>268.435.456 = 2^{28}</u> $(30n_2 + 29)$ and $(30n_2 + 31)$ 8.947.848 pairs   Highest prime to divide 16.381   square root 16.384

Sequence **A** $(30n_2 + 29)$		Sequence **B** $(30n_2 + 31)$	
Total number of terms	8.947.848	Total number of terms	8.947.848
Multiples $7m$, $11m$,…	**7.119.276**	Multiples $7m$, $11m$,…	**7.119.387**
Primes greater than 2^{14}	**1.828.572**	Primes greater than 2^{14}	**1.828.461**
Number of terms $(7m – 2)$, $(11m – 2)$,…	7.119.387	Number of terms $(7m + 2)$, $(11m + 2)$,…	7.119.276
Multiples $(7m – 2)$, $(11m – 2)$,…	**5.642.405**	Multiples $(7m + 2)$, $(11m + 2)$,…	**5.642.405**
Primes $(7m – 2)$, $(11m – 2)$,…	**1.476.982**	Primes $(7m + 2)$, $(11m + 2)$,…	**1.476.871**

$P_T(x)$ = Number of twin prime pairs (finished in 9 and 1) between 2^{14} and 2^{28} $P_T(x)$ = 1.828.572 – 1.476.982 = 1.828.461 – 1.476.871 = 351.590
Not included the twin prime pairs (finished in 9 and 1) lesser than 2^{14}

k_{ax} = 0,795641141		k_{bx} = 0,795653547	
k_{jx} = 0,792540846	k_{jx} / k_{ax} = 0,9961034	k_{jx} = 0,792553203	k_{jx} / k_{bx} = 0,9961034
c_{jx} = 2,193948468		c_{jx} = 2,193980089	
k_{0x} = 0,743155996	k_{0x} / k_{ax} = 0,934034147	k_{0x} = 0,743167582	k_{0x} / k_{bx} = 0,934034147

The programmable controller used is very slow to perform calculations with numbers greater than 10^9.

To know the approximate values of c_{jx} for higher numbers, we will use data (*) from Wikipedia concerning to number of primes and to number of twin prime pairs lesser than a given number (from 10^{10} to 10^{18}).

<u>10^{10}</u>   455.052.511* primes   27.412.679* twin prime pairs

Number of terms in each sequence **A** or **B**: 10^{10} / 30 = 333.333.333
Approximate number of primes in each sequence **A** or **B**: 455.052.511 / 8 = 56.881.563 (1)
Approximate number of twin prime pairs in the sequences **A**-**B** (1 combination of 3): 27.412.679 / 3 = 9.137.559
Approximate number of multiples $7m$, $11m$,… 333.333.333 – 56.881.563 = 276.451.770 (2)
Number of terms $(7m – 2)$, $(11m – 2)$,… is, approximately, equal to the number of multiples $7m$, $11m$,…
Approximate number of primes $(7m – 2)$, $(11m – 2)$,… 56.881.563 – 9.137.559 = 47.744.004 (3)
Approximate number of multiples $(7m – 2)$, $(11m – 2)$,… 276.451.770 – 47.744.004 = 228.707.766 (4)

Total number of terms in sequence **A**	333.333.333			
Multiples $7m$, $11m$,…	≈ 276.451.770	(2)	k_{ax} ≈ 0,82935531	
Primes greater than 10^5	≈ 56.881.563	(1)	k_{jx} ≈ 0,827297166	k_{jx} / k_{ax} ≈ 0,99751838
Number of terms $(7m – 2)$, $(11m – 2)$,…	≈ 276.451.770	(2)	c_{jx} ≈ 2,095100568	
Multiples $(7m – 2)$, $(11m – 2)$,…	≈ 228.707.766	(4)	k_{0x} ≈ 0,794244171	k_{0x} / k_{ax} ≈ 0,957664539
Primes $(7m – 2)$, $(11m – 2)$,…	≈ 47.744.004	(3)	k_{7x} ≈ 0,800914529	

<u>10^{11}</u>   4.118.054.813* primes   224.376.048* twin prime pairs

Number of terms in each sequence **A** or **B**: 10^{11} / 30 = 3.333.333.333
Approximate number of primes in each sequence **A** or **B**: 4.118.054.813 / 8 = 514.756.851 (1)
Approximate number of twin prime pairs in the sequences **A**-**B** (1 combination of 3): 224.376.048 / 3 = 74.792.016
Approximate number of multiples $7m$, $11m$,… 3.333.333.333 – 514.756.851 = 2.818.576.482 (2)
Number of terms $(7m – 2)$, $(11m – 2)$,… is, approximately, equal to the number of multiples $7m$, $11m$,…
Approximate number of primes $(7m – 2)$, $(11m – 2)$,… 514.756.851 – 74.792.016 = 439.964.835 (3)
Approximate number of multiples $(7m – 2)$, $(11m – 2)$,… 2.818.576.482 – 439.964.835 = 2.378.611.647 (4)

Total number of terms in sequence **A**	3.333.333.333			
Multiples $7m$, $11m$,…	≈ 2.818.576.482	(2)	k_{ax} ≈ 0,845572944	
Primes greater than $10^{5,5}$	≈ 514.756.851	(1)	k_{jx} ≈ 0,843905305	k_{jx} / k_{ax} ≈ 0,998027799
Number of terms $(7m – 2)$, $(11m – 2)$,…	≈ 2.818.576.482	(2)	c_{jx} ≈ 2,075447865	
Multiples $(7m – 2)$, $(11m – 2)$,…	≈ 2.378.611.647	(4)	k_{0x} ≈ 0,817369919	k_{0x} / k_{ax} ≈ 0,966646254
Primes $(7m – 2)$, $(11m – 2)$,…	≈ 439.964.835	(3)	k_{7x} ≈ 0,819835102	

10^{12} 37.607.912.018* primes 1.870.585.220* twin prime pairs

Number of terms in each sequence **A** or **B**: 10^{12} / 30 = 33.333.333.333
Approximate number of primes in each sequence **A** or **B**: 37.607.912.018 / 8 = 4.700.989.002 (1)
Approximate number of twin prime pairs in the sequences **A**-**B** (1 combination of 3): 1.870.585.220 / 3 = 623.528.406
Approximate number of multiples 7m, 11m,… 33.333.333.333 – 4.700.989.002 = 28.632.344.331 (2)
Number of terms (7m – 2), (11m – 2),… is, approximately, equal to the number of multiples 7m, 11m,…
Approximate number of primes (7m – 2), (11m – 2),… 4.700.989.002 – 623.528.406 = 4.077.460.596 (3)
Approximate number of multiples (7m – 2), (11m – 2),… 28.632.344.331 – 4.077.460.596 = 24.554.883.735 (4)

Total number of terms in sequence **A**	33.333.333.333		$k_{ax} \approx 0{,}85897033$	
Multiples 7m, 11m,…	≈ 28.632.344.331	(2)	$k_{jx} \approx 0{,}857592499$	$k_{jx} / k_{ax} \approx 0{,}99839595$
Primes greater than 10^6	≈ 4.700.989.002	(1)	$c_{jx} \approx 2{,}058134681$	
Number of terms (7m – 2), (11m – 2),…	≈ 28.632.344.331	(2)	$k_{0x} \approx 0{,}835815434$	$k_{0x} / k_{ax} \approx 0{,}973043427$
Multiples (7m – 2), (11m – 2),…	≈ 24.554.883.735	(4)	$k_{7x} \approx 0{,}835465384$	
Primes (7m – 2), (11m – 2),…	≈ 4.077.460.596	(3)	$k_{11x} \approx 0{,}844867362$	

10^{13} 346.065.536.839* primes 15.834.664.872* twin prime pairs

Number of terms in each sequence **A** or **B**: 10^{13} / 30 = 333.333.333.333
Approximate number of primes in each sequence **A** or **B**: 346.065.536.839 / 8 = 43.258.192.105 (1)
Approximate number of twin prime pairs in the sequences **A**-**B** (1 combination of 3): 15.834.664.872 / 3 = 5.278.221.624
Approximate number of multiples 7m, 11m,… 333.333.333.333 – 43.258.192.105 = 290.075.141.228 (2)
Number of terms (7m – 2), (11m – 2),… is, approximately, equal to the number of multiples 7m, 11m,…
Approximate number of primes (7m – 2), (11m – 2),… 43.258.192.105 – 5.278.221.624 = 37.979.970.481 (3)
Approximate number of multiples (7m – 2), (11m – 2),… 290.075.141.228 – 37.979.970.481 = 252.095.170.747 (4)

Total number of terms in sequence **A**	333.333.333.333			
Multiples 7m, 11m,…	≈ 290.075.141.228	(2)	$k_{ax} \approx 0{,}870225423$	
Primes greater than $10^{6,5}$	≈ 43.258.192.105	(1)	$k_{jx} \approx 0{,}869068509$	$k_{jx} / k_{ax} \approx 0{,}998670559$
Number of terms (7m – 2), (11m – 2),…	≈ 290.075.141.228	(2)	$c_{jx} \approx 2{,}042626025$	
Multiples (7m – 2), (11m – 2),…	≈ 252.095.170.747	(4)	$k_{0x} \approx 0{,}85087246$	$k_{0x} / k_{ax} \approx 0{,}977760977$
Primes (7m – 2), (11m – 2),…	≈ 37.979.970.481	(3)		

10^{14} 3.204.941.750.802* primes 135.780.321.665* twin prime pairs

Number of terms in each sequence **A** or **B**: 10^{14} / 30 = 3.333.333.333.333
Approximate number of primes in each sequence **A** or **B**: 3.204.941.750.802 / 8 = 400.617.718.850 (1)
Approximate number of twin prime pairs in the sequences **A**-**B** (1 combination of 3): 135.780.321.665 / 3 = 45.260.107.221
Approximate number of multiples 7m, 11m,… 3.333.333.333.333 – 400.617.718.850 = 2.932.715.614.483 (2)
Number of terms (7m – 2), (11m – 2),… is, approximately, equal to the number of multiples 7m, 11m,…
Approximate number of primes (7m – 2), (11m – 2),… 400.617.718.850 – 45.260.107.221 = 355.357.611.629 (3)
Approximate number of multiples (7m – 2), (11m – 2),… 2.932.715.614.483 – 355.357.611.629 = 2.577.358.002.854 (4)

Total number of terms in sequence **A**	3.333.333.333.333			
Multiples 7m, 11m,…	≈ 2.932.715.614.483	(2)	$k_{ax} \approx 0{,}879814684$	
Primes greater than 10^7	≈ 400.617.718.850	(1)	$k_{jx} \approx 0{,}878829842$	$k_{jx} / k_{ax} \approx 0{,}998880626$
Number of terms (7m – 2), (11m – 2),…	≈ 2.932.715.614.483	(2)	$c_{jx} \approx 2{,}028807737$	
Multiples (7m – 2), (11m – 2),…	≈ 2.577.358.002.854	(4)	$k_{0x} \approx 0{,}863397011$	$k_{0x} / k_{ax} \approx 0{,}981339623$
Primes (7m – 2), (11m – 2),…	≈ 355.357.611.629	(3)		

10^{15} 29.844.570.422.669* primes 1.177.209.242.304* twin prime pairs

Number of terms in each sequence **A** or **B**: 10^{15} / 30 = 33.333.333.333.333
Approximate number of primes in each sequence **A** or **B**: 29.844.570.422.669 / 8 = 3.730.571.302.833 (1)
Approximate number of twin prime pairs in the sequences **A**-**B** (1 combination of 3): 1.177.209.242.304 / 3 = 392.403.080.768
Approximate number of multiples 7m, 11m,… 33.333.333.333.333 – 3.730.571.302.833 = 29.602.762.030.500 (2)
Number of terms (7m – 2), (11m – 2),… is, approximately, equal to the number of multiples 7m, 11m,…
Approximate number of primes (7m – 2), (11m – 2),… 3.730.571.302.833 – 392.403.080.768 = 3.338.168.221.065 (3)
Approximate number of multiples (7m – 2), (11m – 2),… 29.602.762.030.500 – 3.338.168.221.065 = 26.264.593.809.435 (4)

Total number of terms in sequence **A**	33.333.333.333.333			
Multiples 7m, 11m,…	≈ 29.602.762.030.500	(2)	$k_{ax} \approx 0{,}888082861$	
Primes greater than $10^{7,5}$	≈ 3.730.571.302.833	(1)	$k_{jx} \approx 0{,}887234568$	$k_{jx} / k_{ax} \approx 0{,}999044804$
Number of terms (7m – 2), (11m – 2),…	≈ 29.602.762.030.500	(2)	$c_{jx} \approx 2{,}016482789$	
Multiples (7m – 2), (11m – 2),…	≈ 26.264.593.809.435	(4)	$k_{0x} \approx 0{,}873978945$	$k_{0x} / k_{ax} \approx 0{,}984118693$
Primes (7m – 2), (11m – 2),…	≈ 3.338.168.221.065	(3)		

10^{16} 279.238.341.033.925* primes 10.304.195.697.298* twin prime pairs

Number of terms in each sequence **A** or **B**: 10^{16} / 30 = 333.333.333.333.333
Approximate number of primes in each sequence **A** or **B**: 279.238.341.033.925 / 8 = 34.904.792.629.240 (1)
Approximate number of twin prime pairs in the sequences **A-B** (1 combination of 3): 10.304.195.697.298 / 3 = 3.434.731.897.432
Approximate number of multiples $7m$, $11m$,… 333.333.333.333.333 – 34.904.792.629.240 = 298.428.540.704.093 (2)
Number of terms ($7m$ – 2), ($11m$ – 2),… is, approximately, equal to the number of multiples $7m$, $11m$,…
Approximate number of primes ($7m$ – 2), ($11m$ – 2),… 34.904.792.629.240 – 3.434.731.897.432 = 31.470.060.721.808 (3)
Approximate number of multiples ($7m$ – 2), ($11m$ – 2),… 298.428.540.704.093 – 31.470.060.721.808 = 266.958.479.982.285 (4)

			$k_{ax} \approx 0{,}895285622$	
Total number of terms in sequence **A**	333.333.333.333.333		$k_{jx} \approx 0{,}894547415$	$k_{jx} / k_{ax} \approx 0{,}999175451$
Multiples $7m$, $11m$,…	≈ 298.428.540.704.093	(2)	$c_{jx} \approx 2{,}005561339$	
Primes greater than 10^8	≈ 34.904.792.629.240	(1)	$k_{0x} \approx 0{,}883038021$	$k_{0x} / k_{ax} \approx 0{,}986319895$
Number of terms ($7m$ – 2), ($11m$ – 2),…	≈ 298.428.540.704.093	(2)	$k_{7x} \approx 0{,}877833225$	
Multiples ($7m$ – 2), ($11m$ – 2),…	≈ 266.958.479.982.285	(4)	$k_{11x} \approx 0{,}884814184$	
Primes ($7m$ – 2), ($11m$ – 2),…	≈ 31.470.060.721.808	(3)	$k_{13x} \approx 0{,}886559424$	

10^{18} 24.739.954.287.740.860* primes 808.675.888.577.436* twin prime pairs

Number of terms in each sequence **A** or **B**: 10^{18} / 30 = 33.333.333.333.333.333
Approximate number of primes in each sequence **A** or **B**: 24.739.954.287.740.860 / 8 = 3.092.494.285.967.607 (1)
Approximate number of twin prime pairs in the sequences **A-B** (1 combination of 3): 808.675.888.577.436 / 3 = 269.558.629.525.812
Approximate number of multiples $7m$, $11m$,… 33.333.333.333.333.333 – 3.092.494.285.967.607 = 30.240.839.047.365.726 (2)
Number of terms ($7m$ – 2), ($11m$ – 2),… is, approximately, equal to the number of multiples $7m$, $11m$,…
Approximate number of primes ($7m$ – 2), ($11m$ – 2),… 3.092.494.285.967.607 – 269.558.629.525.812 = 2.822.935.656.441.795 (3)
Approximate number of multiples ($7m$ – 2), ($11m$ – 2),… 30.240.839.047.365.726 – 2.822.935.656.441.795 = 27.417.903.390.923.931 (4)

			$k_{ax} \approx 0{,}907225171$	
Total number of terms in sequence **A**	33.333.333.333.333.333		$k_{jx} \approx 0{,}906651543$	$k_{jx} / k_{ax} \approx 0{,}999367712$
Multiples $7m$, $11m$,…	≈ 30.240.839.047.365.726	(2)	$c_{jx} \approx 1{,}987076711$	
Primes greater than 10^9	≈ 3.092.494.285.967.607	(1)	$k_{0x} \approx 0{,}897737814$	$k_{0x} / k_{ax} \approx 0{,}989542445$
Number of terms ($7m$ – 2), ($11m$ – 2),…	≈ 30.240.839.047.365.726	(2)	$k_{7x} \approx 0{,}8917627$	
Multiples ($7m$ – 2), ($11m$ – 2),…	≈ 27.417.903.390.923.931	(4)	$k_{11x} \approx 0{,}897947688$	
Primes ($7m$ – 2), ($11m$ – 2),…	≈ 2.822.935.656.441.795	(3)	$k_{13x} \approx 0{,}899493935$	

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