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P vs NP: Solutions of NP Problems

Abstract

The simplest solution is usually the best solution---Albert Einstein

Best news. After over 30 years of debating, the debate is over. Yes, P is equal to NP.
For the first time, NP problems. including the classic traveling salesman problem have been solved in this paper. The general approach to solving the different types of NP problems are the same, except that sometimes, specific techniques may differ from each other according to the process involved in the problem. Another type of NP problems covered is the division of items of different sizes, masses, or values into equal parts. The techniques and formulas developed for dividing these items into equal parts are based on an extended Ashanti fairness wisdom as exemplified below. If two people A and B are to divide items of different sizes which are arranged from the largest size to the smallest size, the procedure would be as follows. In the first round, A chooses the largest size, followed by B choosing the next largest size. In the second round, B chooses first, followed by A. In the third round, A chooses first, followed by B and the process continues up to the last item. To abbreviate the sequence in the above choices, one obtains the sequence "AB, BA AB". Let A and B divide the sum of the whole numbers, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1 as equally as possible, by merely always choosing the largest number. Then A chooses 10, B chooses 9 and 8, followed by A choosing 7 and 6; followed by B choosing 5 and 4; followed by A choosing 3 an 2; and finally, B chooses 1. The sum of A's choices is 10 +7+ 6 + 3 + 2 = 28; and the sum of B's choices is 9 + 8 + 5 + 4 + 1 = 27, with error, plus or minus 0.5 . Observe the sequence "AB, BA, AB, BA, AB". Observe also that the sequence is **not** "AB, AB, AB, AB, AB as one might think. Te reason why the sequence is "AB, BA AB, BA, AB" is as follows. In the first round, when A chooses first, followed by B, A has the advantage of choosing the larger number and B has the disadvantage of choosing the smaller number. In the second round, if A were to choose first, A would have had two consecutive advantages, and therefore, in the second round, B will choose first to produce the sequence AB, BA. In the third round, A chooses first, because B chose first in the second round. After three rounds, the sequence would be AB, BA, AB. When his technique was applied to 100 items of different values or masses, by mere combinations, the total value or mass of A's items was equal to the total value or mass of B's items. Similar results were obtained for 1000 items. By hand, the techniques can be used to prepare final exam schedules for 100 or 1000 courses.

A new approach to solving the traveling salesman was used to determine the shortest route to visit nine cities and return to the starting city. The technique covered eliminates a shortcoming of the nearest neighbor approach as well as that of the grouping of the cities. The distances involved were arranged in increasing order and by inspection, ten distances were selected from a set of the shortest 14 distances, intead of the overall set of 45 distances involved. The selected distances were used to construct the shortest route.

Confirmed is the notion that an approach that solves one of these problems can also solve other NP problems. Since six problems from three different areas have been solved, all NP problems can be solved. If all NP problems can be solved, then all NP problems are P problems and therefore, P is equal to NP. The CMI Millennium Prize requirements have been satisfied.

Solutions of NP Problems

The following sample problems will be solved and analyzed. They are based on the suggested sample problems from the Wikipedia (Simple English) website. Many Thanks to Wikipedia.

Basis of the method used in solving the NP problems: **Ratios** p.3 To Page 3

Example 1 (Preliminaries)
By always choosing the largest number, A and B will divide the following set of numbers equally or nearly equally: 14,13,12,11,10,9,8,7, 6,5,4,3,2,1. To Page 4

Example 2a Consider the existence of dollar bills with denominations $100, $99, $98,...$2, down to $1. Suppose the bills are on a table with the $100 bill at the top, followed by the $99, $98, $97 bills, and so on with the $1 bill at the bottom of the stack. Now, by mere grabbing in turns always from the top of the stack, divide the total value of these dollar bills equally between A and B. To Page 6

Example 2b Consider the existence of dollar bills with denominations $100, $99, $98,...$2, down to $1. Suppose the bills are on a table with the $100 bill at the top, followed by the $99, $98, $97 bills, and so on with the $1 bill at the bottom of the stack. Now, by mere grabbing in turns always from the top of the stack, the total value of these dollar bills is to be divided equally between A and B.

Question: (a) If a computer costs $2,000, can A afford to buy this computer?
(b) If a computer costs $3,000, can A afford to buy this computer? To Page 8

Example 3 Let one randomly delete some of the bills in Example 2a, a previous example, and divide the remaining bills between A and B. After the deletion of some of the bills, there are 78 bills remaining. To Page 8

Example 4 A businessman wants to take 100 items of different masses to the market. These items are to be packed into boxes. Each box can only hold up to 560 units. The businessman would like to know if 10 boxes would be sufficient to carry all 100 items to the market. To Page 10

Example 5 A school offers 100 different courses, and each course requires one hour for the final exam. For each course, all students registered for that course must take the final exam at the same time. Since some students take more than one course, the final exam schedule must be such that students registered for two or more courses will be able to take the exams for all their registered courses. A teacher would like to know if it is possible to schedule all of the exams for the same day so that every student can take the exam for each course registered for. To Page 12

Example 6 A builder has 1000 concrete blocks of different masses arranged from 1000 units to one unit. The builder would like to divide the blocks into two piles A and B of equal masses. Prepare a list by masses of all the blocks in pile A, and all the blocks in pile B. To Page 14

Example 7 Solutions of Traveling Salesman Problems. To Page 19

To Overall Conclusion

Basis of the method used in solving the NP problems: Ratios

Method 2 below is the method used for the solutions of the **NP** problems.

Example 1: Divide \$12 between A and B in the ratio 1: 2

Method 1 (Usual arithmetic method)

Step 1: Fraction of the money A receives = $\frac{1}{1+2} = \frac{1}{3}$

Fraction of the money B receives = $\frac{2}{1+2} = \frac{2}{3}$

Step 2: Amount A receives = $\frac{1}{3} \times \frac{12}{1} = 4$

Amount B receives = $\frac{2}{3} \times \frac{12}{1} = 8$

Therefore, A receives \$4, and B receives \$8

(**Method 1** above is from the author's book entitled "Power of Ratios" by A. A. Frempong, and published by Yellowtextbooks.com.)

Method 2 (The process method)

The ratio 1:2 means whenever A receives \$1, B receives \$2.

Step 1: In the first round, A receives \$1, and B receives \$2.
After the first round, the amount of money remaining is \$12 - (\$1 + \$2) = \$9.

Step 2: In the second round, from this \$9, A receives \$1 and B receives \$2.
After the second round, the amount of money remaining = \$9 - (\$1+ \$2) = \$6

Step 3: In the third round, A receives \$1 and B receives \$2.
The amount remaining = \$6 - (\$1 + \$2) = \$3

Step 4: In the fourth and final round, A receives \$1 and B receives \$2.
The amount remaining = \$3 - (\$1 + \$2) = 0

Step 5: A's total = \$1 + \$1 + \$1 + \$1 = \$4
B's total = \$2 + \$2 + \$2 + \$2 = \$8

Example 2 Divide \$12 between A and B in the ratio 1: 1

Method 1 (Usual arithmetic method)

Step 1: Fraction of the money A receives = $\frac{1}{1+1} = \frac{1}{2}$

Fraction of the money B receives = $\frac{1}{1+1} = \frac{1}{2}$

Step 2: Amount A receives = $\frac{1}{2} \times \frac{12}{1} = 6$

Amount B receives = $\frac{1}{2} \times \frac{12}{1} = 6$

Therefore, A receives \$6, and B receives \$6.

Method 2 (The process method)

The ratio 1:1 means whenever A receives \$1, B receives \$1.

Step 1: In the first round, A receives \$1, and B receives \$1.
After the first round, the amount of money remaining is \$12 - (\$1 + \$1) = \$10

Step 2: In the second round, from this \$10, A receives \$1 and B receives \$1.
After the second round, the amount of money remaining = \$10 - (\$1+ \$1) = \$8

Step 3: In the third round, A receives \$1, and B receives \$1.
The amount remaining = \$8 - (\$1 + \$1) = \$6

Step 4: In the fourth round, A receives \$1 and B receives \$1
The amount remaining = \$6 - (\$1 + \$1) = 4

Step 5: In the fifth round, A receives \$1 and B receives \$1.
The amount remaining = \$4 - (\$1 + \$1) = 2

Step 6: In the sixth and final round, A receives \$1 and B receives \$1.
The amount remaining = \$2 - (\$1 + \$1) = 0

Step 7: A's total = \$1 + \$1 + \$1 + \$1 + \$1 + \$1 = \$6.
B's total = \$1 + \$1 + \$1 + \$1 + \$1 + \$1 = \$6.

Case 1: Only two devisors A and B

Example 1 (Preliminaries)
By always choosing the largest number, A and B will divide the following set of numbers equally or nearly equally.

14,13,12,11,10,9,8,7, 6,5,4,3,2,1.

Solution
For communication purposes, one will call the numbers to be divided the "dividends"; and one will call A and B the "divisors". Let the sum of A's choices be Q_A, and let the sum of B's choices be Q_B.

Step 1: Check to ensure that the numbers are arranged in decreasing order.
One will apply the wisdom method of the introduction.
That is, one applies "AB, BA, AB, BA, AB, BA, AB"

Method 1 Using braces
Step 2: A chooses the first element, 14
Step 3: B chooses the next two elements, 13 and 12.,
Step 4: A chooses the next two elements 11, and 10, and the alternating consecutive choices continue to the end.

$$\underbrace{14}_{A}, \underbrace{13, 12}_{B}, \underbrace{11, 10}_{A}, \underbrace{9, 8}_{B}, \underbrace{7, 6}_{A}, \underbrace{5, 4}_{B}, \underbrace{3, 2}_{A}, \underbrace{1}_{B} \qquad (1)$$

Step 5: Add the choices for A and add the choices for B.

$$Q_A = 14 + 11 + 10 + 7 + 6 + 3 + 2$$
$$= 53$$
$$Q_B = 13 + 12 + 9 + 8 + 5 + 4 + 1$$
$$= 52$$

The sum for A = 53; and the sum for B = 52.

Method 2 (Tabular form)
Step 1: List the dividends as shown below

14	13	12	11	10	9	8	7	6	5	4	3	2	1

Step 2: Write the divisors A, BB, AA, BB, AA, etc, above the numbers, This is the choosing step.

A	B	B	A	A	B	B	A	A	B	B	A	A	B
14	13	12	11	10	9	8	7	6	5	4	3	2	1

Note: $\overset{A}{\boxed{14}}$ means A chooses 14. $\overset{B}{\boxed{13}}$ means B chooses 13.

Step 3: Collect and add the corresponding (dividends) choices

Q_A	Q_B
14	13
11	12
10	9
7	8
6	5
3	4
2	1
Total: 53	52

Mathematical formulas for choosing the elements

Let $a_1 = 14,\ a_2 = 13,\ a_3 = 12,\ a_4 = 11,\ a_5 = 10,$
$a_6 = 9,\ a_7 = 8,\ a_8 = 7,\ a_9 = 6,\ a_{10} = 5,\ a_{11} = 4,\ a_{12} = 3,\ a_{13} = 2,\ a_{14} = 1$

By experimentation, one obtains the following formulas for A and B.

$$Q_A = a_1 + \sum_{n=2,4,6,}^{6} a_{2n} + a_{2n+1} \qquad (Q_A = a_1 + a_4 + a_5 + a_8 + a_9 + a_{12} + a_{13})$$

$$Q_B = \sum_{n=1,3,5.}^{5} a_{2n} + a_{2n+1} + a_{14} \qquad (Q_B = a_2 + a_3 + a_6 + a_7 + a_{10} + a_{11} + a_{14})$$

Apply the formulas to above (The above formulas are valid for only **two** divisors.)

$$\begin{aligned} Q_A &= a_1 + \sum_{n=2,4,6}^{6} a_{2n} + a_{2n+1} \\ &= 14 + 11 + 10 + 7 + 6 + 3 + 2 \\ &= 53 \end{aligned}$$

$$\begin{aligned} Q_B &= \sum_{n=1,3,5.}^{5} a_{2n} + a_{2n+1} + a_{14} \\ &= 13 + 12 + 9 + 8 + 5 + 4 + 1 \\ &= 52 \end{aligned}$$

Note that the above formulas using the sigma notation are valid for only two divisors, A and B. For three divisors A, B, and C, different formulas would have to be derived, based on the solutions of the problem.

Back to examples

Example 2a Consider the existence of dollar bills with denominations \$100, \$99, \$98,...\$2, down to \$1. Suppose the bills are on a table with the \$100 bill at the top, followed by the \$99, \$98, \$97 bills, and so on with the \$1 bill at the bottom of the stack. Now, by mere grabbing in turns always from the top of the stack, the total value of these dollar bills is to be divided equally between A and B.

Method 2a: Using the numerical values and braces

Apply, "AB, BA, AB, BA, AB, BA,..." (as in Method 1 of Example 1)

Step 1: A chooses the first \$100 bill. (Only a single item is removed).
Step 2: B chooses the next two bills, the \$99 and \$98 bills, (two items removed consecutively)
Step 3: A chooses the next two bills, the \$97 and \$96 bills, and the alternating removal continues to the end.

$\underbrace{100}_{A}, \underbrace{99\ 98}_{B}, \underbrace{97,\ 96}_{A}, \underbrace{95,\ 94}_{B}, \underbrace{93,\ 92}_{A}\ \underbrace{91,\ 90}_{B}, \underbrace{89,\ 88}_{A}, \underbrace{87\ 86}_{B}, \underbrace{85,\ 84}_{A}, \underbrace{83,\ 82}_{B}, \underbrace{81,\ 80}_{A}\ \underbrace{79,\ 78}_{B},$

$\underbrace{77,\ 76,}_{A}, \underbrace{75\ 74}_{B}, \underbrace{73,\ 72}_{A}, \underbrace{71,\ 70}_{B}, \underbrace{69,\ 68}_{A}\ \underbrace{67,\ 66}_{B}, \underbrace{65,\ 64}_{A}, \underbrace{63\ 62}_{B}, \underbrace{61,\ 60}_{A}, \underbrace{59,\ 58}_{B}, \underbrace{57,\ 56}_{A}\ \underbrace{55,\ 54}_{B},$

$\underbrace{53,\ 52}_{A}, ,\underbrace{51,\ 50}_{B}, \underbrace{49\ 48}_{A}, \underbrace{47,\ 46}_{B}, \underbrace{45,\ 44}_{A}, \underbrace{43,\ 42}_{B}\ \underbrace{41,\ 40}_{A}, \underbrace{39,\ 38}_{B}, \underbrace{37\ 36}_{A}, \underbrace{35,\ 34}_{B}, \underbrace{33,\ 32}_{A}, \underbrace{31,\ 30}_{B}$

$\underbrace{29,\ 28,}_{A}, \underbrace{27,\ 26}_{B}, \underbrace{25\ 24}_{A}, \underbrace{23,\ 22}_{B}, \underbrace{21,\ 20}_{A}, \underbrace{19,\ 18}_{B},\underbrace{17,\ 16}_{A}, \underbrace{15,\ 14}_{B},\underbrace{13,\ 12}_{A}, \underbrace{11,\ 10}_{B}, \underbrace{9,\ 8}_{A}, \underbrace{7,\ 6}_{B}, \underbrace{5,\ 4}_{A}$

$\underbrace{3,\ 2}_{B}, \underbrace{1}_{A},$

Step 4: Collect and add the choices (dividends) for A and B

$Q_A = 100 + 97 + 96 + 93 + 92 + 89 + 88 + 85 + 84 + 81 + 80 + 77 + 76 + 73 + 72 + 69 + 68 + 65 + 64 + 61 + 60 + 57 + 56 + 53 + 52 + 49 + 48 + 45 + 44 + 41 + 40 + 37 + 36 + 33 + 32 + 29 + 28 + 25 + 24 + 21 + 20 + 17 + 16 + 13 + 12 + 9 + 8 + 5 + 4 + 1 = \mathbf{2525}$.

$Q_B = 99 + 98 + 95 + 94 + 91 + 90 + 87 + 86 + 83 + 82 + 79 + 78 + 75 + 74 + 71 + 70 + 67 + 66 + 63 + 62 + 59 + 58 + 55 + 54\ 51 + 50 + 47 + 46 + 43 + 42 + 39 + 38 + 35 + 34 + 31 + 30 + 27 + 26 + 23 + 22 + 19 + 18 + 15 + 14 + 11 + 10 + 7 + 6 + 3 + 2 = \mathbf{2525}$.

Conclusion: A receives \$2525 and B receives \$2525, Note **t**he zero error for A and B.

Method 2b: Using tabular form

Step 1: Write the divisors A and B above the numbers (as done in Method 2 of Example 1)

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
100	99	98	97	96	95	94	93	92	91	90	89	88	87	86	85	84	83	82	81

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
80	79	78	77	76	75	74	73	72	71	70	69	68	67	66	65	64	63	62	61

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
60	59	58	57	56	55	54	53	52	51	50	49	48	47	46	45	44	43	42	41

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
40	39	38	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

Step 2: Collect and add the Choices (dividends)

$Q_A = 100 + 97 + 96 + 93 + 92 + 89 + 88 + 85 + 84 + 81 + 80 + 77 + 76 + 73 + 72 + 69 + 68 + 65 + 64 + 61 + 60 + 57 + 56 + 53 + 52 + 49 + 48 + 45 + 44 + 41 + 40 + 37 + 36 + 33 + 32 + 29 + 28 + 25 + 24 + 21 + 20 + 17 + 16 + 13 + 12 + 9 + 8 + 5 + 4 + 1 = \mathbf{2525.}$

$$Q_B = 99+98+95+94+91+90+87+86+83+82+79+78+75+74+71+70+67+66+$$
$$63+62+59+58+55+54+51+50+47+46+43+42+39+38+35+34+31+30+27+$$
$$26+23+22+19+18+15+14+11+10+7+6+3+2=\mathbf{2525}.$$

The above results are pleasantly astonishing. Of the 2^{100} possible ways to divide the above bills, the above technique and consequently the derived formulas divided the above mixture of bills into exactly two equal parts in value. Why has this technique been hiding for nearly 30 years? Note that the ratio $Q_A : Q_B$ is $1:1$.

Equations for above: $Q_A = a_1 + \sum_{n=2,4,6,...}^{48} a_{2n} + a_{2n+1} + a_{100}$ and $Q_B = \sum_{n=1,3,5,...}^{49} a_{2n} + a_{2n+1}$

Method 1b: Using term numbers and braces Apply, "AB, BA, AB, BA, AB,..."

$$\underbrace{a_1}_{A}, \underbrace{a_2\ a_3}_{B}, \underbrace{a_4, a_5}_{A}, \underbrace{a_6, a_7}_{B}, \underbrace{a_8, a_9}_{A}\ \underbrace{a_{10}, a_{11}}_{B}, \underbrace{a_{12}, a_{13}}_{A}, \underbrace{a_{14}\ a_{15}}_{B}, \underbrace{a_{16}, a_{17}}_{A}, \underbrace{a_{18}, a_{19}}_{B}, \underbrace{a_{20}, a_{21}}_{A}\ \underbrace{a_{22}, a_{23}}_{B},$$

$$\underbrace{a_{24}, a_{25}}_{A},, \underbrace{a_{26}, a_{27}}_{B}\ \underbrace{a_{28}, a_{29}}_{A}\ \underbrace{a_{30}, a_{31}}_{B}, \underbrace{a_{32}, a_{33}}_{A}\ \underbrace{a_{34}, a_{35}}_{B}, \underbrace{a_{36}, a_{37}}_{A}\ \underbrace{a_{38}\ a_{39}}_{B}\ \underbrace{a_{40}, a_{41}}_{A}, \underbrace{a_{42}, a_{43}}_{B}, \underbrace{a_{44}, a_{45}}_{A}$$

$$\underbrace{a_{46}, a_{47}}_{B}, \underbrace{a_{48}, a_{49}}_{A},, \underbrace{a_{50}, a_{51}}_{B}, \underbrace{a_{52}\ a_{53}}_{A}, \underbrace{a_{54}, a_{55}}_{B}\ \underbrace{a_{56}, a_{57}}_{A}\ \underbrace{a_{58}, a_{59}}_{B}\ \underbrace{a_{60}, a_{61}}_{A}, \underbrace{a_{62}, a_{63}}_{B}\ \underbrace{a_{64}\ a_{65}}_{A}, \underbrace{a_{66}, a_{67}}_{B}$$

$$\underbrace{a_{68}, a_{69}}_{A}\ \underbrace{a_{70}, a_{71}}_{B}\ \underbrace{a_{72}, a_{73}}_{A}, \underbrace{a_{74}, a_{75}}_{B}, \underbrace{a_{76}, a_{77}}_{A}\ \underbrace{a_{78}, a_{79}}_{B}\ \underbrace{a_{80}, a_{81}}_{A}\ \underbrace{a_{82}, a_{83}}_{B}, \underbrace{a_{84}, a_{85}}_{A}, \underbrace{a_{86}, a_{87}}_{B}, \underbrace{a_{88}, a_{89}}_{A},$$

$$\underbrace{a_{90}, a_{91}}_{B}, \underbrace{a_{92}, a_{93}}_{A}, \underbrace{a_{94}, a_{95}}_{B}, \underbrace{a_{96}, a_{97}}_{A}\ \underbrace{a_{98}, a_{99}}_{B}, \underbrace{a_{100}}_{A},$$

Using the term numbers and tabular form

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{60}

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{70}	a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{80}

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{90}	a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{100}

Collect the terms for A and add them ; and similarly collect the terms for B and add them.

$$Q_A = a_1 + a_4 + a_5 + a_8 + a_9 + a_{12} + a_{13} + a_{16} + a_{17} + a_{20} + a_{21} + a_{24} + a_{25} + a_{28} + a_{29} + a_{32} + a_{33}$$
$$+ a_{36} + a_{37} + a_{40} + a_{41} + a_{44} + a_{45} + a_{48} + a_{49} + a_{52} + a_{53} + a_{56} + a_{57} + a_{60} + a_{61} + a_{64} + a_{65}$$
$$+ a_{68} + a_{69} + a_{72} + a_{73} + a_{76} + a_{77} + a_{80} + a_{81} + a_{84} + a_{85} + a_{88} + a_{89} + a_{92} + a_{93} + a_{96} + a_{97} + a_{100}$$
$$Q_B = a_2 + a_3 + a_6 + a_7 + a_{10} + a_{11} + a_{14} + a_{15} + a_{18} + a_{19} + a_{22} + a_{23} + a_{26} + a_{27} + a_{30} + a_{31} + a_{34}$$
$$+ a_{35} + a_{38} + a_{39} + a_{42} + a_{43} + a_{46} + a_{47} + a_{50} + a_{51} + a_{54} + a_{55} + a_{58} + a_{59} + a_{62} + a_{63} + a_{66}$$
$$+ a_{67} + a_{70} + a_{71} + a_{74} + a_{75} + a_{78} + a_{79} + a_{82} + a_{83} + a_{86} + a_{87} + a_{90} + a_{91} + +a_{94} + a_{95} + a_{98} + a_{99}$$

Back to examples

Example 2b Consider the existence of dollar bills with denominations \$100, \$99, \$98,...\$2, down to \$1. Suppose the bills are on a table with the \$100 bill at the top, followed by the \$99, \$98, \$97 bills, and so on with the \$1 bill at the bottom of the stack. Now, by mere grabbing in turns always from the top of the stack, the total value of these dollar bills is to be divided equally between A and B.

Question: (a) If a computer costs \$2,000, can A afford to buy this computer?
(b) If a computer costs \$3,000, can A afford to buy this computer?

Answers: (a) From the solution of Example 2a, A received \$2,525, and therefore can afford to buy this computer.
Yes. A can afford to buy this \$2,000 computer.

(b) Since from the solution of Example 2a, A received \$2,525, and the computer costs \$3,000, A cannot afford to buy \$3,000 computer
No. A cannot afford to buy this \$3,000 computer.

Example 3

Let one randomly delete some of the bills in Example 2a, a previous example, and divide as equally as possible the remaining bills between A and B. After the deletion of some of the bills, there are 78 bills remaining.

Solution: Using the numerical values and braces

98.97,96,95,94,93,91,90,89,88,87,86.85,.81,80,79,78,77,76,
A B A B A B A B A B

75,74,73,72,69.68,67,66,65,64,62,61,60,58,56,55,54,53.51.50,
A B A B A B A B A B

49,47,46,45,43.,41,40,39,37,36,35.34,33.32,31,30,29,26,25.24,
A B A B A B A B A B

23,22,21,20,19,18,16,14,13,12,11,10,9,8,7,5,4,2,1
A B A B A B A B A B

$$Q_A = 98+95+94+90+89+86+85+79+78+75+74+69+68+65+64+60+58+54+53 +49+47+43+41+37+36+33+32+29+26+23+22+19+18+13+12+9+8+4+2 = \mathbf{1937}$$

$$Q_B = 97+96+93+91+88+87+81+80+77+76+73+72+67+66+62+61+56+55+51 +50+46+45+40+39+35+34+31+30+25+24+21+20+16+14+11+10+7+5+1 = \mathbf{1933}$$

Total of Q_A and $Q_B = 3870$. Division by 2 yields 1935. For Q_A, relative error = $\frac{2}{1935} = 0.0010$ or about 0.1% For Q_B, relative error = $\frac{-2}{1935} = -0.0010$	For equality, interchange the 47 bill in Q_A and the 45 bill in Q_B. Thus A gives \$2 to B, resulting in equality of **\$1,935** each. Other bills can be interchanged.

Using term numbers

$$Q_A = a_1 + a_4 + a_5 + a_8 + a_9 + a_{12} + a_{13} + a_{16} + a_{17} + a_{20} + a_{21} + a_{24} + a_{25} + a_{28} + a_{29} + a_{32} + a_{33} + a_{36} + a_{37} + a_{40} + a_{41} + a_{44} + a_{45} + a_{48} + a_{49} + a_{52} + a_{53} + a_{56} + a_{57} + a_{60} + a_{61} + a_{64} + a_{65} + a_{68} + a_{69} + a_{72} + a_{73} + a_{76} + a_{77}$$

$$Q_B = a_2 + a_3 + a_6 + a_7 + a_{10} + a_{11} + a_{14} + a_{15} + a_{18} + a_{19} + a_{22} + a_{23} + a_{26} + a_{27} + a_{30} + a_{31} + a_{34} + a_{35} + a_{38} + a_{39} + a_{42} + a_{43} + a_{46} + a_{47} + a_{50} + a_{51} + a_{54} + a_{55} + a_{58} + a_{59} + a_{62} + a_{63} + a_{66} + a_{67} + a_{70} + a_{71} + a_{74} + a_{75} + a_{78}$$

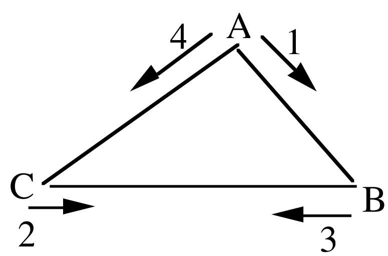
Observe above that the last term for Q_A is a_{77} and the last term for Q_B is a_{78} (there are 78 terms)

Case 2: Three or more divisors

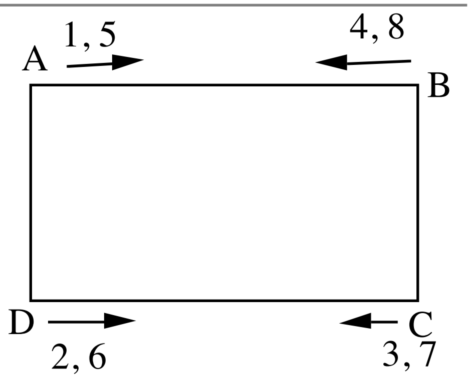
In the previous examples, for communication purposes, A and B were called the "divisors" and the numbers or terms to be divided were called "dividends". The concept of divisors A and B can be extended to three or more divisors such as A, B, C, or A, B, C, D, but in these cases, geometric figures will help keep track of the choices.

Geometric figures to keep track of the order and directions of the divisors
(For three or more divisors such as A, B, C; four divisors A, B, C, D)

The arrows are for directions



ABC / CBA / BCA / ACB



ABCD / DCBA / CDAB / BADC / ABCD

For ABC:

Step 1: Go Clockwise ABC (In the first round, A chooses first and C chooses last))
Step 2: Begin with C, reverse the direction in Step 1 and go CBA.
(Since C was at the largest disadvantage in the first round, by choosing last, C chooses first in the second round, followed by B)
Step 3: Begin with B, reverse previous direction (direction of C) and go clockwise BCA.
Step 4: Begin with A again, reverse previous direction (direction of B) and go counterclockwise ACB.

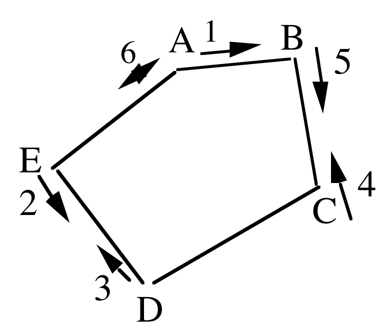
For ABCD

Step 1: Go Clockwise ABCD. (first round)
Step 2: Begin with D, reverse the direction in Step 1 (direction of A) and go DCBA.
Step 3: Begin with C, reverse previous direction (direction of D), and go clockwise CDAB.
Step 4: Begin with B, reverse previous direction (direction of C) and go counterclockwise BADC.
Step 5: Begin with A, reverse pevious direction, but by coincidence go clockwise ABCD. (5th round)

For five divisors A, B, C, D, E

ABCDE, *EDCBA*, *DEABC*, *CBAED* *BCDEA*

Step 1: Go Clockwise ABCDE
Step 2: Begin with E, reverse the direction in Step 1 and go EDCBA
Step 3: Begin with D, reverse previous direction and go clockwise DEABC
Step 4: Begin with C, reverse previous direction and go counterclockwise CBAED
Step 5: Begin with B, reverse previous direction and go clockwise BCDEA.
Step 6: Beginning again with A, reverse the direction (of B) and go counterclockwise AEDCB.



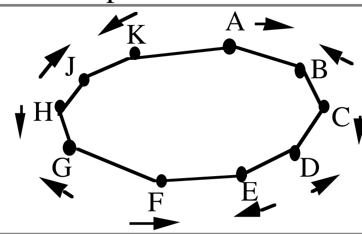
ABCDE, *EDCBA DEABC*
CBAED, *BCDEA*

Back to examples

Example 4: A businessman wants to take 100 items of different masses to the market. These items are to be packed into boxes. Each box can only hold up to 560 units. The businessman would like to know if 10 boxes would be sufficient to carry all 100 items to the market.

Step 1: Arrange the items in decreasing order of their masses. Let the mass of the first item (largest) be 100 units, and let the masses of the rest of the items be respectively, 99, 98, 97, and so on down to smallest item of mass 1 unit. Let the 10 boxes be labeled A, B, C, D, E, F, G, H, J, and K The ten boxes are to divide the 100 items. **Imitate** Example 2 but with 10 divisors.

Guide1; *ABCDEFGHJK*
Guide 2; *KJHGFEDCBA*
Guide 3 *JKABCDEFGH*
Guide 4; *HGFEDCBAKJ*
Guide 5; *GHJKABCDEF*
Guide 6; *FEDCBAKJHG*
Guide 7: *EFGHJKABCD*
Guide 8; *DCBAKJHGFE*
Guide 9; *CDEFGHJKAB*
Guide10: *BAKJHGFEDC*



A	B	C	D	E	F	G	H	J	K	K	J	H	G	F	E	D	C	B	A
100	99	98	97	96	95	94	93	92	91	90	89	88	87	86	85	84	83	82	81
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}

J	K	A	B	C	D	E	F	G	H	H	G	F	E	D	C	B	A	K	J
80	79	78	77	76	75	74	73	72	71	70	69	68	67	66	65	64	63	62	61
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}

G	H	J	K	A	B	C	D	E	F	F	E	D	C	B	A	K	J	H	G
60	59	58	57	56	55	54	53	52	51	50	49	48	47	46	45	44	43	42	41
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{60}

E	F	G	H	J	K	A	B	C	D	D	C	B	A	K	J	H	G	F	E
40	39	38	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{70}	a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{80}

C	D	E	F	G	H	J	K	A	B	B	A	K	J	H	G	F	E	D	C
20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{90}	a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{100}

Step 2: **Collect the choices for A, B, C, D, E, F, G, H, J, K**

Q_A	Q_B	Q_C	Q_D	Q_E	Q_F	Q_G	Q_H	Q_J	Q_K
100 a_1	99, a_2	98 a_3	97 a_4	96 a_5	95 a_6	94 a_7	93 a_8	92 a_9	91 a_{10}
81 a_{20}	82, a_{19}	83 a_{18}	84 a_{17}	85 a_{16}	86 a_{15}	87 a_{14}	88 a_{13}	89 a_{12}	90 a_{11}
78 a_{23}	77, a_{24}	76 a_{25}	75 a_{26}	74 a_{27}	73 a_{28}	72 a_{29}	71 a_{30}	80 a_{21}	79 a_{22}
63 a_{38}	64, a_{37}	65 a_{36}	66 a_{35}	67 a_{34}	68 a_{33}	69 a_{32}	70 a_{31}	61 a_{40}	62 a_{39}
56 a_{45}	55, a_{46}	54 a_{47}	53 a_{48}	52 a_{49}	51 a_{50}	60 a_{41}	59 a_{42}	58 a_{43}	57 a_{44}
45 a_{56}	46 a_{55}	47 a_{54}	48 a_{53}	49 a_{52}	50 a_{51}	41 a_{60}	42 a_{59}	43 a_{58}	44 a_{57}
34 a_{67}	33 a_{68}	32 a_{69}	31 a_{70}	40 a_{61}	39 a_{62}	38 a_{63}	37 a_{64}	36 a_{65}	35 a_{66}
27 a_{74}	28 a_{73}	29 a_{72}	30 a_{71}	21 a_{80}	22 a_{79}	23 a_{78}	24 a_{77}	25 a_{76}	26 a_{75}
12 a_{89}	11 a_{90}	20 a_{81}	19 a_{82}	18 a_{83}	17 a_{84}	16 a_{85}	15 a_{86}	14 a_{87}	13 a_{88}
9 a_{92}	10 a_{91}	1 a_{100}	2 a_{99}	3 a_{98}	4 a_{97}	5 a_{96}	6 a_{95}	7 a_{94}	8 a_{93}
Total: 505	**505**	**505**	**505**	**505**	**505**	**505**	**505**	**505**	**505**

Condition for sufficiency:
The 10 boxes would be sufficient to carry all the 100 items to the market if the mass of the contents of each box is equal to or less than 560 units. Since the mass of the contents in each box is 505 units, which is less than 560 units, each box satisfies this sufficiency condition. Therefore, the 10 boxes would be sufficient to carry the 100 items to the market.
Note above that the ratio

$Q_A : Q_B : Q_C : Q_D : Q_E : Q_F : Q_G : Q_H : Q_{J:} : Q_K = 1:1:1:1:1:1:1:1:1:1$

4b Using the term numbers

A	B	C	D	E	F	G	H	J	K	K	J	H	G	F	E	D	C	B	A
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}

J	K	A	B	C	D	E	F	G	H	H	G	F	E	D	C	B	A	K	J
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}

G	H	J	K	A	B	C	D	E	F	F	E	D	C	B	A	K	J	H	G
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{60}

E	F	G	H	J	K	A	B	C	D	D	C	B	A	K	J	H	G	F	E
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{70}	a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{80}

C	D	E	F	G	H	J	K	A	B	B	A	K	J	H	G	F	E	D	C
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{90}	a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{100}

Collect the terms for A, B, C, D, E, F, G, H, J, K

$Q_A = a_1 + a_{20} + a_{23} + a_{38} + a_{45} + a_{56} + a_{67} + a_{74} + a_{89} + a_{92}$

$Q_B = a_2 + a_{19} + a_{24} + a_{37} + a_{46} + a_{55} + a_{68} + a_{73} + a_{90} + a_{91}$

$Q_C = a_3 + a_{18} + a_{25} + a_{36} + a_{47} + a_{54} + a_{69} + a_{72} + a_{81} + a_{100}$

$Q_D = a_4 + a_{17} + a_{26} + a_{35} + a_{48} + a_{53} + a_{70} + a_{71} + a_{82} + a_{99}$

$Q_E = a_5 + a_{16} + a_{27} + a_{34} + a_{49} + a_{52} + a_{61} + a_{80} + a_{83} + a_{98}$

$Q_F = a_6 + a_{15} + a_{28} + a_{33} + a_{50} + a_{51} + a_{62} + a_{79} + a_{84} + a_{97}$

$Q_G = a_7 + a_{14} + a_{29} + a_{32} + a_{41} + a_{60} + a_{63} + a_{78} + a_{85} + a_{96}$

$Q_H = a_8 + a_{13} + a_{30} + a_{31} + a_{42} + a_{59} + a_{64} + a_{77} + a_{86} + a_{95}$

$Q_J = a_9 + a_{12} + a_{21} + a_{40} + a_{43} + a_{58} + a_{65} + a_{76} + a_{87} + a_{94}$

$Q_K = a_{10} + a_{11} + a_{22} + a_{39} + a_{44} + a_{57} + a_{66} + a_{75} + a_{88} + a_{93}$

Sub-Conclusion
The fairness wisdom method has performed perfectly.
Observe above in Step 2 that the totals for Q_A, Q_B, Q_C, Q_D, Q_E, Q_F, Q_G, Q_H, Q_J, Q_K are all the same. The technique applied picked combinations to produce these equal totals.
Note that in using the technique in this paper, the items involved must be arranged in decreasing order, preferably. Therefore, in programming, the first step should be to arrange the items in decreasing order, a task a computer performs very fast..

In the next example, Example 5, one will confirm the notion that a method that solves one of the NP problems can be used to solve other similar problems. One will use the results of the above example Example 4b to do the next problem.

Back to examples

Example 5: A school offers 100 different courses, and each course requires one hour for the final exam. For each course, all students registered for that course must take the final exam at the same time. Since some students take more than one course, the final exam schedule must be such that students registered for two or more courses will be able to take the exams for all their registered courses. A teacher would like to know if it is possible to schedule all of the exams for the same day so that every student can take the exam for each course registered for.

Step1: Final Exams 8AM – 6PM $\begin{cases} A = 8-9;\ B = 9-10;\ C = 10-11;\ D = 11-12; E = 12-1; \\ F = 1-2;\ G = 2-3;\ H = 3-4;\ \ J = 4-5;\ K = 5-6 \end{cases}$

Let the course numbers be a_1, a_2, a_3,... a_{100} **Using the result of Example 4b**

A	B	C	D	E	F	G	H	J	K	K	J	H	G	F	E	D	C	B	A
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}

J	K	A	B	C	D	E	F	G	H	H	G	F	E	D	C	B	A	K	J
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}

G	H	J	K	A	B	C	D	E	F	F	E	D	C	B	A	K	J	H	G
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{60}

E	F	G	H	J	K	A	B	C	D	D	C	B	A	K	J	H	G	F	E
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{70}	a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{80}

C	D	E	F	G	H	J	K	A	B	B	A	K	J	H	G	F	E	D	C
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{90}	a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{100}

Step 2: Collect the choices for A, B, C, D, E, F, G, H, J, K

Final Exam Schedule: 8 AM-6 PM

$Q_A = 8-9;\ Q_B = 9-10;\ Q_C = 10-11;\ Q_D = 11-12;\ Q_E = 12-1;$
$Q_F = 1-2;\ Q_G = 2-3;\ Q_H = 3-4;\ Q_J = 4-5;\ Q_K = 5-6$

8- 9	9-10	10-11	11-12	12-1	1-2	2-3	3-4	4-5	5-6
Q_A	Q_B	Q_C	Q_D	Q_E	Q_F	Q_G	Q_H	Q_J	Q_K
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
a_{20}	a_{19}	a_{18}	a_{17}	a_{16}	a_{15}	a_{14}	a_{13}	a_{12}	a_{11}
a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}	a_{21}	a_{22}
a_{38}	a_{37}	a_{36}	a_{35}	a_{34}	a_{33}	a_{32}	a_{31}	a_{40}	a_{39}
a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}	a_{41}	a_{42}	a_{43}	a_{44}
a_{56}	a_{55}	a_{54}	a_{53}	a_{52}	a_{51}	a_{60}	a_{59}	a_{58}	a_{57}
a_{67}	a_{68}	a_{69}	a_{70}	a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}
a_{74}	a_{73}	a_{72}	a_{71}	a_{80}	a_{79}	a_{78}	a_{77}	a_{76}	a_{75}
a_{89}	a_{90}	a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}
a_{92}	a_{91}	a_{100}	a_{99}	a_{98}	a_{97}	a_{96}	a_{95}	a_{94}	a_{93}

The final exam for every course has been scheduled. However, if a student takes for example, Course a_1 and course a_{20}, because the duration for the final exams for these two courses is 8-9 AM, the student cannot take the final exams for these two courses simultaneously. Therefore, it is **not** possible to prepare a schedule to allow every student to take the final exams for all registered courses on the same day. However, below is what is possible.

In order for every student to take the final exam for all courses registered for, ten days would be needed as shown below, where the course numbers are a_1, a_2, a_3,... a_{100} .

	8- 9	9-10	10-11	11-12	12-1	1-2	2-3	3-4	4-5	5-6
DAY	Q_A	Q_B	Q_C	Q_D	Q_E	Q_F	Q_G	Q_H	Q_J	Q_K
1	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
2	a_{20}	a_{19}	a_{18}	a_{17}	a_{16}	a_{15}	a_{14}	a_{13}	a_{12}	a_{11}
3	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}	a_{21}	a_{22}
4	a_{38}	a_{37}	a_{36}	a_{35}	a_{34}	a_{33}	a_{32}	a_{31}	a_{40}	a_{39}
5	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}	a_{41}	a_{42}	a_{43}	a_{44}
6	a_{56}	a_{55}	a_{54}	a_{53}	a_{52}	a_{51}	a_{60}	a_{59}	a_{58}	a_{57}
7	a_{67}	a_{68}	a_{69}	a_{70}	a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}
8	a_{74}	a_{73}	a_{72}	a_{71}	a_{80}	a_{79}	a_{78}	a_{77}	a_{76}	a_{75}
9	a_{89}	a_{90}	a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}
10	a_{92}	a_{91}	a_{100}	a_{99}	a_{98}	a_{97}	a_{96}	a_{95}	a_{94}	a_{93}

Observe how one used the results of the previous example (Example 4b) to solve the above problem, Example 5..
In the next example, one will cover an example involving 1000 items, which will be similar to Example 2a.

Back to examples

Example 6 A builder has 1000 concrete blocks of different masses arranged from 1000 units to one unit. The builder would like to divide the blocks into two piles A and B of equal masses. Prepare a list by masses of all the blocks in pile A, and all the blocks in pile B. Review Example 2a before proceeding

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
1000	999	998	997	996	995	994	993	992	991	990	989	988	987	986	985	984	983	982	981
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
980	979	978	977	976	975	974	973	972	971	970	969	968	967	966	965	964	963	962	961
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
960	959	958	957	956	955	954	953	952	951	950	949	948	947	946	945	944	943	942	941
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
940	939	938	937	936	935	934	933	932	931	930	929	928	927	926	925	924	923	922	921
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
920	919	918	917	916	915	914	913	912	911	910	909	908	907	906	905	904	903	902	901
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
900	899	898	897	896	895	894	893	892	891	890	889	888	887	886	885	884	883	882	881
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
880	879	878	877	876	875	874	873	872	871	870	869	868	867	866	865	864	863	862	861
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
860	859	858	857	856	855	854	853	852	851	850	849	848	847	846	845	844	843	842	841
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
840	839	838	837	836	835	834	833	832	831	830	829	828	827	826	825	824	823	822	821
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
820	819	818	817	816	815	814	813	812	811	810	809	808	807	806	805	804	803	802	801
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
800	799	798	797	796	795	794	793	792	791	790	789	788	787	786	785	784	783	782	781
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
780	779	778	777	776	775	774	773	772	771	770	769	768	767	766	765	764	763	762	761
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
760	759	758	757	756	755	754	753	752	751	750	749	748	747	746	745	744	743	742	741
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
740	739	738	737	736	735	734	733	732	731	730	729	728	727	726	725	724	723	722	721
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
720	719	718	717	716	715	714	713	712	711	710	709	708	707	706	705	704	703	702	701
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
700	699	698	697	696	695	694	693	692	691	690	689	688	687	686	685	684	683	682	681
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
680	679	678	677	676	675	674	673	672	671	670	669	668	667	666	665	664	663	662	661
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
660	659	658	657	656	655	654	653	652	651	650	649	648	647	646	645	644	643	642	641
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
640	639	638	637	636	635	634	633	632	631	630	629	628	627	626	625	624	623	622	621
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
620	619	618	617	616	615	614	613	612	611	610	609	608	607	606	605	604	603	602	601

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
600	599	598	597	596	595	594	593	592	591	590	589	588	587	586	585	584	583	582	581
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
580	579	578	577	576	575	574	573	572	571	570	569	568	567	566	565	564	563	562	561
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
560	559	558	557	556	555	554	553	552	551	550	549	548	547	546	545	544	543	542	541
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
540	539	538	537	536	535	534	533	532	531	530	529	528	527	526	525	524	523	522	521
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
520	519	518	517	516	515	514	513	512	511	510	509	508	507	506	505	504	503	502	501
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
500	499	498	497	496	495	494	493	492	491	490	489	488	487	486	485	484	483	482	481
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
480	479	478	477	476	475	474	473	472	471	470	469	468	467	466	465	464	463	462	461
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
460	459	458	457	456	455	454	453	452	451	450	449	448	447	446	445	444	443	442	441
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
440	439	438	437	436	435	434	433	432	431	430	429	428	427	426	425	424	423	422	421
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
420	419	418	417	416	415	414	413	412	411	410	409	408	407	406	405	404	403	402	401
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
400	399	398	397	396	395	394	393	392	391	390	389	388	387	386	385	384	383	382	381
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
380	379	378	377	376	375	374	373	372	371	370	369	368	367	366	365	364	363	362	361
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
360	359	358	357	356	355	354	353	352	351	350	349	348	347	346	345	344	343	342	341
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
340	339	338	337	336	335	334	333	332	331	330	329	328	327	326	325	324	323	322	321
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
320	319	318	317	316	315	314	313	312	311	310	309	308	307	306	305	304	303	302	301
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
300	299	298	297	296	295	294	293	292	291	290	289	288	287	286	285	284	283	282	281
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
280	279	278	277	276	275	274	273	272	271	270	269	268	267	266	265	264	263	262	261
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
260	259	258	257	256	255	254	253	252	251	250	249	248	247	246	245	244	243	242	241
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
240	239	238	237	236	235	234	233	232	231	230	229	228	227	226	225	224	223	222	221
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
220	219	218	217	216	215	214	213	212	211	210	209	208	207	206	205	204	203	202	201

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
200	199	198	197	196	195	194	193	192	191	190	189	188	187	186	185	184	183	182	181
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
180	179	178	177	176	175	174	173	172	171	170	169	168	167	166	165	164	163	162	161
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
160	159	158	157	156	155	154	153	152	151	150	149	148	147	146	145	144	143	142	141
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
140	139	138	137	136	135	134	133	132	131	130	129	128	127	126	125	124	123	122	121
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
120	119	118	117	116	115	114	113	112	111	110	109	108	107	106	105	104	103	102	101

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
100	99	98	97	96	95	94	93	92	91	90	89	88	87	86	85	84	83	82	81
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
80	79	78	77	76	75	74	73	72	71	70	69	68	67	66	65	64	63	62	61
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
60	59	58	57	56	55	54	53	52	51	50	49	48	47	46	45	44	43	42	41
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
40	39	38	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

Concrete masses for Pile A

Step 2: Collect and add the Choices (dividends) :

$$Q_{A1} = 1000+997+996+993+992+989+988+985+984+981+980+977+976+973 +972+969+968+965+964+961+960+957+956+953+952+949+948+945+ 944+941+940+937+936+933+932+929+928+925+924+921+920+917+ 916+913+912+909+908+905+904+901 = \mathbf{47{,}525}$$

$$Q_A = 900+897+896+893+892+889+888+885+884+881+880+877+876+873+872+ 869+868+865+864+861+860+857+856+853+852+849+848+845+844+841 +840+837+836+833+832+829+828+825+824+821+820+817+816+813+812 +809+808+805+804+801 = \mathbf{42{,}525}$$

$$Q_A = 800+797+796+793+792+789+788+785+784+781+780+777+776+773+772+ 769+768+765+764+761+760+757+756+753+752+749+748+745+744+741 +740+737+736+733+732+729+728+725+724+721+720+717+716+713+712 +709+708+705+704+701 = \mathbf{37{,}525}$$

$$Q_A = 700+697+696+693+692+689+688+685+684+681+680+677+676+673+672+ 669+668+665+664+661+660+657+656+653+652+649+648+645+644+641 +640+637+636+633+632+629+628+625+624+621+620+617+616+613+612 +609+608+605+604+601 = \mathbf{32{,}525}$$

$$Q_A = 600+597+596+593+592+589+588+585+584+581+580+577+576+573+572+ 569+568+565+564+561+560+557+556+553+552+549+548+545+544+541 +540+537+536+533+532+529+528+525+524+521+520+517+516+513+512 +509+508+505+504+501 = \mathbf{27{,}525}$$

$$Q_A = 500+497+496+493+492+489+488+485+484+481+480+477+476+473+472+ 469+468+465+464+461+460+457+456+453+452+449+448+445+444+441 +440+437+436+433+432+429+428+425+424+421+420+417+416+413+412 \; 409+408+405+404+401 = \mathbf{22{,}525}$$

$$Q_A = 400+397+396+393+392+389+388+385+384+381+380+377+376+373+372+ 369+368+365+364+361+360+357+356+353+352+349+348+345+344+341 +340+337+336+333+332+329+328+325+324+321+320+317+316+313+312 \; 309+308+305+304+301 = \mathbf{17{,}525}$$

$$Q_A = 300+297+296+293+292+289+288+285+284+281+280+277+276+273+272+ 269+268+265+264+261+260+257+256+253+252+249+248+245+244+241 +240+237+236+233+232+229+228+225+224+221+220+217+216+213+212 \; 209+208+205+204+201 = \mathbf{12{,}525}$$

$$Q_A = 200+197+196+193+192+189+188+185+184+181+180+177+176+173+172+ 169+168+165+164+161+160+157+156+153+152+149+148+145+144+141 +140+137+136+133+132+129+128+125+124+121+120+117+116+113+112 \; 109+108+105+104+101 = \mathbf{7{,}525}$$

$$Q_A = 100+97+96+93+92+89+88+85+84+81+80+77+76+73+72+ 69+68+65+64+61+60+57+56+53+52+49+48+45+44+41 +40+37+36+33+32+29+28+25+24+21+20+17+16+13+12 \; 9+8+5+4+1 = \mathbf{2{,}525}$$

Total for Q_A = 250,250 units

Concrete masses for Pile B

$Q_B = 999 + 998 + 995 + 994 + 991 + 990 + 987 + 986 + 983 + 982 + 979 + 978 + 975 + 974 + 971 + 970 + 967 + 966 + 963 + 962 + 959 + 958 + 955 + 954 + 951 + 950 + 947 + 946 + 943 + 942 + 939 + 938 + 935 + 934 + 931 + 930 + 927 + 926 + 923 + 922 + 919 + 918 + 915 + 914 + 911 + 910 + 907 + 906 + 903 + 902 = \mathbf{47{,}525}$

$Q_B = 899 + 898 + 895 + 894 + 891 + 890 + 887 + 886 + 883 + 882 + 879 + 878 + 875 + 874 + 871 + 870 + 867 + 866 + 863 + 862 + 859 + 858 + 855 + 854 + 851 + 850 + 847 + 846 + 843 + 842 + 839 + 838 + 835 + 834 + 831 + 830 + 827 + 826 + 823 + 822 + 819 + 818 + 815 + 814 + 811 + 810 + 807 + 806 + 803 + 802 = \mathbf{42{,}525}$

$Q_B = 799 + 798 + 795 + 794 + 791 + 790 + 787 + 786 + 783 + 782 + 779 + 778 + 775 + 774 + 771 + 770 + 767 + 766 + 763 + 762 + 759 + 758 + 755 + 754 + 751 + 750 + 747 + 746 + 743 + 742 + 739 + 738 + 735 + 734 + 731 + 730 + 727 + 726 + 723 + 722 + 719 + 718 + 715 + 714 + 711 + 710 + 707 + 706 + 703 + 702 \mathbf{= 37{,}525}$

$Q_B = 699 + 698 + 695 + 694 + 691 + 690 + 687 + 686 + 683 + 682 + 679 + 678 + 675 + 674 + 671 + 670 + 667 + 666 + 663 + 662 + 659 + 658 + 655 + 654 + 651 + 650 + 647 + 646 + 643 + 642 + 639 + 638 + 635 + 634 + 631 + 630 + 627 + 626 + 623 + 622 + 619 + 618 + 615 + 614 + 611 + 610 + 607 + 606 + 603 + 602 \mathbf{= 32{,}525}$

$Q_B = 599 + 598 + 595 + 594 + 591 + 590 + 587 + 586 + 583 + 582 + 579 + 578 + 575 + 574 + 571 + 570 + 567 + 566 + 563 + 562 + 559 + 558 + 555 + 554 + 551 + 550 + 547 + 546 + 543 + 542 + 539 + 538 + 535 + 534 + 531 + 530 + 527 + 526 + 523 + 522 + 519 + 518 + 515 + 514 + 511 + 510 + 507 + 506 + 503 + 502 = \mathbf{27{,}525}$

$Q_B = 499 + 498 + 495 + 494 + 491 + 490 + 487 + 486 + 483 + 482 + 479 + 478 + 475 + 474 + 471 + 470 + 467 + 466 + 463 + 462 + 459 + 458 + 455 + 454 + 451 + 450 + 447 + 446 + 443 + 442 + 439 + 438 + 435 + 434 + 431 + 430 + 427 + 426 + 423 + 422 + 419 + 418 + 415 + 414 + 411 + 410 + 407 + 406 + 403 + 402 = \mathbf{22{,}525}$

$Q_B = 399 + 398 + 395 + 394 + 391 + 390 + 387 + 386 + 383 + 382 + 379 + 378 + 375 + 374 + 371 + 370 + 367 + 366 + 363 + 362 + 359 + 358 + 355 + 354 + 351 + 350 + 347 + 346 + 343 + 342 + 339 + 338 + 335 + 334 + 331 + 330 + 327 + 326 + 323 + 322 + 319 + 318 + 315 + 314 + 311 + 310 + 307 + 306 + 303 + 302 \mathbf{= 17{,}525}$

$Q_B = 299 + 298 + 295 + 294 + 291 + 290 + 287 + 286 + 283 + 282 + 279 + 278 + 275 + 274 + 271 + 270 + 267 + 266 + 263 + 262 + 259 + 258 + 255 + 254 + 251 + 250 + 247 + 246 + 243 + 242 + 239 + 238 + 235 + 234 + 231 + 230 + 227 + 226 + 223 + 222 + 219 + 218 + 215 + 214 + 211 + 210 + 207 + 206 + 203 + 202 \mathbf{= 12{,}525}$

$Q_B = 199 + 198 + 195 + 194 + 191 + 190 + 187 + 186 + 183 + 182 + 179 + 178 + 175 + 174 + 171 + 170 + 167 + 166 + 163 + 162 + 159 + 158 + 155 + 154 + 151 + 150 + 147 + 146 + 143 + 142 + 139 + 138 + 135 + 134 + 131 + 130 + 127 + 126 + 123 + 122 + 119 + 118 + 115 + 114 + 111 + 110 + 107 + 106 + 103 + 102 = \mathbf{7{,}525}$

$Q_B = 99 + 98 + 95 + 94 + 91 + 90 + 87 + 86 + 83 + 82 + 79 + 78 + 75 + 74 + 71 + 70 + 67 + 66 + 63 + 62 + 59 + 58 + 55 + 54 + 51 + 50 + 47 + 46 + 43 + 42 + 39 + 38 + 35 + 34 + 31 + 30 + 27 + 26 + 23 + 22 + 19 + 18 + 15 + 14 + 11 + 10 + 7 + 6 + 3 + 2 \mathbf{= 2{,}525}$

Total for Q_B = 250,250 units

Back to examples

Example 7: Solutions of the Traveling Salesman Problem
Data Ordering and Route Construction Approach

The simplest solution is usually the best solution---Albert Einstein

Abstract

For one more time, yes, P is equal to NP. For the first time in history, the traveling salesman can determine by hand, with zero or negligible error, the shortest route from home base city to visit once, each of three cities, 10 cities, 20 cities, 100 cities, or 1000 cities, and return to the home base city. The formerly NP-hard problem is now NP-easy problem.
The general approach to solving the different types of NP problems are the same, except that sometimes, specific techniques may differ from each other according to the process involved in the problem. The first step is to arrange the data in the problem in increasing or decreasing order. In the salesman problem, the order will be increasing order, since one's interest is in the shortest distances. The main principle here is that the shortest route is the sum of the shortest distances such that the salesman visits each city once and returns to the starting city. The shortest route to visit nine cities and return to the starting city was found in this paper. It was also found out that even though the length of the shortest route is unique, the sequence of the cities involved is not unique.
Since an approach that solves one of these problems can also solve other NP problems. and the traveling salesman problem has been solved, all NP problems can be solved, provided one has an open mind and continues to think. If all NP problems can be solved, then all NP problems are P problems and therefore, P is equal to NP. The CMI Millennium Prize requirements have been satisfied.

Preliminaries

Given: The distances between each pair of cities.

Required : To find the shortest route to visit each of the cities once and return to the starting city. It is assumed that there is a direct route between each pair of cities.

Note

1. Number of distances required to travel to each city once and return equals the number of cities involved in the problem.

2 The symbol $C_{1,2}$ can mean the distance from City 1 to City 2.

The distance $C_{1,2}$ = the distance $C_{2,1}$.

Used as a sentence, $C_{1,2}$ can mean, from City 1, one visits City 2.

3. C_1 is the home base (starting city) of the traveling salesman.

4. $C_{1,2}(3)$ shows that the numerical value of $C_{1,2}$ is 3.

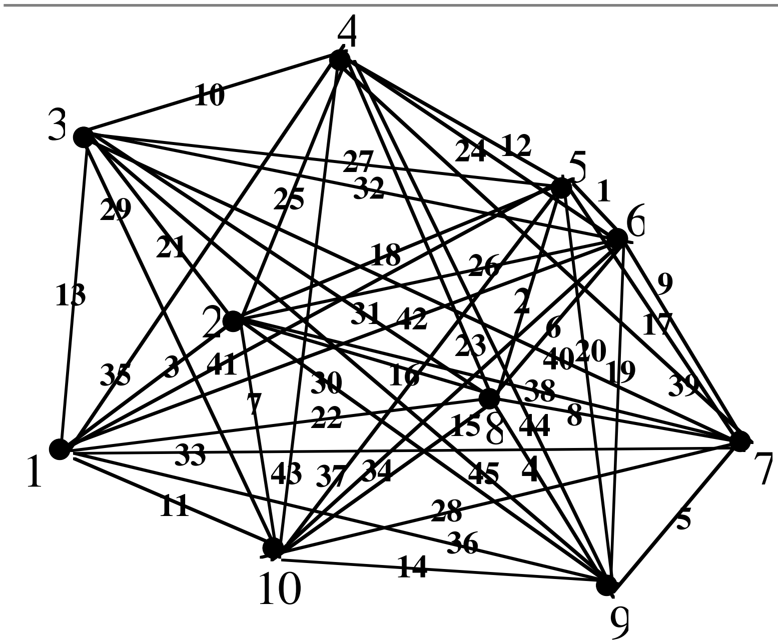
Determining the Shortest Route

Example From City 1, a traveling salesman would like to visit once each of **nine** other cities, namely, City 2, City 3, City 4, City 5, City 6, City 7, City 8, City 9, City 10; and return to City 1. Determine the shortest route**.**

As it was in the author's previous solutions of NP problems, the first step is to arrange the distances in this problem in increasing order. The main principle in this paper is that the shortest route is the minimum sum of the shortest distances such that the salesman visits each city once and returns to the starting city.

Since there are ten cities, ten distances are needed for the salesman to visit each of nine cities once and return to City 1.

For the departure from City 1, the first subscript of City 1 is 1, and for the return to City 1, the second subscript of the last city visited is 1.



Distances Between Each Pair of Cities

C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
$C_{1,2}$ **3**	$C_{2,3}$ **21**	$C_{3,4}$ **10**	$C_{4,5}$ **12**	$C_{5,6}$ **1**	$C_{6,7}$ **9**	$C_{7,8}$ **8**	$C_{8,9}$ **4**	$C_{9,10}$ **14**
$C_{1,3}$ **13**	$C_{2,4}$ **25**	$C_{3,5}$ **27**	$C_{4,6}$ **24**	$C_{5,7}$ **17**	$C_{6,8}$ **6**	$C_{7,9}$ **5**	$C_{8,10}$ **15**	
$C_{1,4}$ **35**	$C_{2,5}$ **18**	$C_{3,6}$ **32**	$C_{4,7}$ **39**	$C_{5,8}$ **2**	$C_{6,9}$ **19**	$C_{7,10}$ **28**		
$C_{1,5}$ **41**	$C_{2,6}$ **26**	$C_{3,7}$ **40**	$C_{4,8}$ **23**	$C_{5,9}$ **20**	$C_{6,10}$ **34**			
$C_{1,6}$ **42**	$C_{2,7}$ **38**	$C_{3,8}$ **31**	$C_{4,9}$ **44**	$C_{5,10}$ **37**				
$C_{1,7}$ **33**	$C_{2,8}$ **16**	$C_{3,9}$ **45**	$C_{4,10}$ **43**					
$C_{1,8}$ **22**	$C_{2,9}$ **30**	$C_{3,10}$ **29**						
$C_{1,9}$ **36**	$C_{2,10}$ **7**							
$C_{1,10}$ **11**								

Step A: Arrange the numerical values of the distances in increasing order

$C_{5,6}$ **1**	$C_{5,8}$ **2**	$C_{1,2}$ **3**	$C_{8,9}$ **4**	$C_{7,9}$ **5**	$C_{6,8}$ **6**	$C_{2,10}$ **7**	$C_{7,8}$ **8**	$C_{6,7}$ **9**
$C_{3,4}$ **10**	$C_{1,10}$ **11**	$C_{4,5}$ **12**	$C_{1,3}$ **13**	$C_{9,10}$ **14**	$C_{8,10}$ **15**	$C_{2,8}$ **16**	$C_{5,7}$ **17**	$C_{2,5}$ **18**
$C_{6,9}$ **19**	$C_{5,9}$ **20**	$C_{2,3}$ **21**	$C_{1,8}$ **22**	$C_{4,8}$ **23**	$C_{4,6}$ **24**	$C_{2,4}$ **25**	$C_{2,6}$ **26**	$C_{3,5}$ **27**
$C_{7,10}$ **28**	$C_{3,10}$ **29**	$C_{2,9}$ **30**	$C_{3,8}$ **31**	$C_{3,6}$ **32**	$C_{1,7}$ **33**	$C_{6,10}$ **34**	$C_{1,4}$ **35**	$C_{1,9}$ **36**
$C_{5,10}$ **37**	$C_{2,7}$ **38**	$C_{4,7}$ **39**	$C_{3,7}$ **40**	$C_{1,5}$ **41**	$C_{1,6}$ **42**	$C_{4,10}$ **43**	$C_{4,9}$ **44**	$C_{3,9}$ **45**

Step B: Interchange the first and second subscripts of each distance,
Note for example that the distance $C_{1,2}$ = the distance $C_{2,1}$.

$C_{5,6}$ or $C_{6,5}$ **1**	$C_{5,8}$ or $C_{8,5}$ **2**	$C_{1,2}$ or $C_{2,1}$ **3**	$C_{8,9}$ or $C_{9,8}$ **4**
$C_{7,9}$ or $C_{9,7}$ **5**	$C_{6,8}$ or $C_{8,6}$ **6**	$C_{2,10}$ or $C_{10,2}$ **7**	$C_{7,8}$ or $C_{8,7}$ **8**
$C_{6,7}$ o $C_{7,6}$ **9**	$C_{3,4}$ or $C_{4,3}$ **10**	$C_{1,10}$ or $C_{10,1}$ **11**	$C_{4,5}$ or $C_{5,4}$ **12**
$C_{1,3}$ or $C_{3,1}$ **13**	$C_{9,10}$ or $C_{10,9}$ **14**	$C_{8,10}$ or $C_{10,8}$ **15**	$C_{2,8}$ or $C_{8,2}$ **16**
$C_{5,7}$ or $C_{7,5}$ **17**	$C_{2,5}$ or $C_{5,2}$ **18**	$C_{6,9}$ or $C_{9,6}$ **19**	$C_{5,9}$ or $C_{9,5}$ **20**
$C_{2,3}$ or $C_{3,2}$ **21**	$C_{1,8}$ or $C_{8,1}$ **22**	$C_{4,8}$ or $C_{8,4}$ **23**	$C_{4,6}$ or $C_{6,4}$ **24**
$C_{2,4}$ or $C_{4,2}$ **25**	$C_{2,6}$ or $C_{6,2}$ **26**	$C_{3,5}$ or $C_{5,3}$ **27**	$C_{7,10}$ or $C_{10,7}$ **28**
$C_{3,10}$ or $C_{10,3}$ **29**	$C_{2,9}$ or $C_{9,2}$ **30**	$C_{3,8}$ or $C_{8,3}$ **31**	$C_{3,6}$ or $C_{6,3}$ **32**
$C_{1,7}$ or $C_{7,1}$ **33**	$C_{6,10}$ or $C_{10,6}$ **34**	$C_{1,4}$ or $C_{4,1}$ **35**	$C_{1,9}$ or $C_{9,1}$ **36**
$C_{5,10}$ or $C_{10,5}$ **37**	$C_{2,7}$ or $C_{7,2}$ **38**	$C_{4,7}$ or $C_{7,4}$ **39**	$C_{3,7}$ or $C_{7,3}$ **40**
$C_{1,5}$ or $C_{5,1}$ **41**	$C_{1,6}$ or $C_{6,1}$ **42**	$C_{4,10}$ or $C_{10,4}$ **43**	$C_{4,9}$ or $C_{9,4}$ **44**
$C_{3,9}$ or $C_{9,3}$ **45**			

Main Principle

The shortest route is the minimum sum of the shortest distances such that the salesman visits each city once, and returns to the starting city. Since there are ten cities, ten distances are needed to allow the salesman to visit once each of nine cities and return to the starting city. One will select ten distances, one at a time, to obtain ten well-connected distances to allow the salesman to visit each city once and return to City 1.

Since one is looking for short distances, for the moment, one will work with the ten numbers (distances) up to the value, 14 units in the above table. See the box with thicker lines in the table, below. If necessary, one will move up the table to add some higher numbers and continue.

A $C_{5,6}$ or $C_{6,5}$ **1**	**G** $C_{2,10}$ or $C_{10,2}$ **7**	**N** $C_{1,3}$ or $C_{3,1}$ **13**	**U** $C_{6,9}$ or $C_{9,6}$ **19**
B $C_{5,8}$ or $C_{8,5}$ **2**	**H** $C_{7,8}$ or $C_{8,7}$ **8**	**P** $C_{9,10}$ or $C_{10,9}$ **14**	**V** $C_{5,9}$ or $C_{9,5}$ **20**
C $C_{1,2}$ or $C_{2,1}$ **3**	**J** $C_{6,7}$ or $C_{7,6}$ **9**	**Q** $C_{8,10}$ or $C_{10,8}$ **15**	**W** $C_{2,3}$ or $C_{3,2}$ **21**
D $C_{8,9}$ or $C_{9,8}$ **4**	**K** $C_{3,4}$ or $C_{4,3}$ **10**	**R** $C_{2,8}$ or $C_{8,2}$ **16**	**X** $C_{1,8}$ or $C_{8,1}$ **22**
E $C_{7,9}$ or $C_{9,7}$ **5**	**L** $C_{1,10}$ or $C_{10,1}$ **11**	**S** $C_{5,7}$ or $C_{7,5}$ **17**	**Y** $C_{4,8}$ or $C_{8,4}$ **23**
F $C_{6,8}$ or $C_{8,6}$ **6**	**M** $C_{4,5}$ or $C_{5,4}$ **12**	**T** $C_{2,5}$ or $C_{5,2}$ **18**	**Z** $C_{4,6}$ or $C_{6,4}$ **24**

Solution

Step C: One will now try to construct a ten-distance route using the entries from A to K. If successful, one would surely have constructed the shortest route, since only the least ten numerical distances would have been used. That is, one would have found the sum of the least ten distances.

Note for example that $C_{1,2}(3)$ shows that the numerical value of $C_{1,2}$ is 3. Such notation makes one become aware of a distance size during a route construction.

Below is an attempt to construct a ten-distance route.

$C_{1,2}(3)C_{2,10}(7)--C_{3,4}(10)--C_{5,8}(2)C_{8,6}(6)C_{6,7}(9)C_{7,9}(5)--(A)$

$C_{1,2}(3)\ C_{2,10}(7)--C_{3,4}(10)--C_{5,6}(1)C_{6,7}(9)C_{7,9}(5)C_{9,8}(4)--(B)$

In trying to construct routes in (A), or (B), above, one is unable to complete a ten-distance route, since all the distances needed are not available within entries in boxes A-K. For example, after $C_{2,10}$, the first subscript of the next distance should be 10, (the second subscript of $C_{2,10}$); and there is no distance with this subscript within A-K. Similarly, after $C_{3,4}$, the firs subscript of the next distance should be 4; but there is no distance with this subscript within boxes A-K. However, if boxes L, M, N and P are added, the needed distances would be available. One will therefore construct a ten-distance route using boxes A-P. Within boxes A-K, there are only two possible first distances, namely, $C_{1,2}$ and $C_{1,10}$. One of these distances with subscript 1 will be the starting (departure) distance, and the other distance would be the return distance. After the above expansion to boxes A-P, another possible additional departure or return distance would be $C_{1,3}$. Since there would now be three distances with the subscript 1, one of these distances would be redundant, since one of them is the departure distance, and another is the return distance. The additional availability of distances would still allow for the construction of the shortest route, since the addition of distances is very minimum.

Step D:: The dashes above indicate missing distances. After including the entries in boxes L, M, N and P to obtain the entries in the boxes A -P as shown , above, by the box with thick lines. After this minimum addition, one successfully constructed the shortest route to visit nine cities and return to City 1. The shortest route from City 1 to visit nine cities and return to City 1 is given by

$$\boxed{C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)C_{9,10}(14)C_{10,2}(7)C_{2,1}(3) = 81}$$

The details of how the above route was obtained is shown below in Steps 1-11. One is interested in applying the entries in boxes A-P:

Begin from City 1 with $C_{1,2}$ or $C_{1,3}$ or $C_{1,10}$ and return to City 1 with $C_{2,1}$ or $C_{3,1}$ or $C_{10,1}$

Step 1: Begin with first city distance $C_{1,3}(13)$ (from box N, above)

Note: $C_{1,3}$ means distance from City 1 to City 3. (From City 1, salesman visits City 3.)

Step: 2: Since the second subscript of $C_{1,3}(13)$ is 3, the first subscript of the next distance will be 3. Inspect each of the above boxes to pick a distance whose first subscript is 3. Box K contains a distance with 3 as a first subscript. We choose the distance in box K, with the numerical value, 10. Connect the chosen distance with the distance in Step 1 to obtain the connected distance $C_{1,3}(13)C_{3,4}(10)$.

Step 3: Since the second subscript of the last distance is 4, the first subscript of the next distance should be 4. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 3), except that the first subscript of the next distance should be 4. Box M contains a distance with 4 as a first subscript. One chooses the distance $C_{4,5}(12)$ in box M, and attach to obtain the connected distances, $C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)$.

The excluded subscript numbers , except 1, represent the cities already visited.

Step 4: Since the second subscript of the last distance is 5, the first subscript of the next distance should be 5. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 3, 4), except that the first subscript of the next distance should be 5. One chooses the distance $C_{5,6}(1)$ in box A (with small numerical value, 1) to obtain the connected distances $C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)$.

Step 5: Since the second subscript of the last distance is 6, the first subscript of the next distance should be 6. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 3, 4, 5) except that the first subscript of the next distance should be 6, One chooses the distance $C_{6,7}(9)$ in box J to obtain the connected distances

$C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)$.

Step 6: Since the second subscript of the last distance is 7, the first subscript of the next distance should be 7. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 3, 4, 5, 6) except that the first subscript of the next distance should be 7. One chooses the distance $C_{7,8}(8)$ in box H to obtain the connected distances

$C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)$.

Step 7: Since the second subscript of the last distance is 8, the first subscript of the next distance should be 8. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 3, 4, 5, 6, 7) except that the first subscript of the next distance should be 8. One chooses the distance $C_{8,9}(4)$ in Box D to obtain the connected distances

$C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)$.

Step 8: Since the second subscript of the last distance 9, the first subscript of the next distance should be 9. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 3, 4, 5, 6, 7, 8), except that the first subscript of the next distance should be 9. One chooses the distance $C_{9,10}(14)$ in Box P, to obtain the connected distances $C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)C_{9,10}(14)$

Step 9: Since the second subscript of the last distance is 10, the first subscript of the next distance should be 10. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 3, 4, 5, 6, 7, 8, 9) except that the first subscript of the next distance should be 10. One chooses the distance $C_{10,2}(7)$ in box G to obtain the connected distances

$C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)C_{9,10}(14)C_{10,2}(7)$

Step 10: Since the second subscript of the last distance is 2, the first subscript of the next and last distance should be 2. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 3, 4, 5, 6, 7, 8, 9,10), except that the first subscript of the next distance should be 2 and the second subscript should be 1 (an exception) in order to return to City 1, the starting city. One chooses the distance $C_{2,1}(3)$ in box C to obtain the connected distances

$C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)C_{9,10}(14)C_{10,2}(7)C_{2,1}(3)$ (Ten distances)

Step 11: Add the distances in parentheses: 13 + 10 + 12 + 1 + 9 + 8 + 4 + 14 + 7 + 3 = 81 and obtain $\boxed{C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)C_{9,10}(14)C_{10,2}(7)C_{2,1}(3) = 81}$

The above in Step 11 is the shortest route of length 81 units.

EXTRA EXAMPLE (not the shortest route): Using $C_{1,2}$ as the first distance

Step 1 Begin with first city distance $C_{1,2}(3)$ (from box C, above)

Note: $C_{1,2}$ means distance from City 1 to City 2.

Step 2: Since the second subscript of $C_{1,2}(3)$ is 2, the first subscript of the next distance will be 2. Inspect each of the above boxes to pick a distance whose first subscript is 2. Box G contains, a distance with 2 as a first subscript. One chooses the distance in box G, with numerical value 7,

$\boxed{C_{1,2}(3)C_{2,10}(7)}$ Also Do: $C_{1,2}(3)C_{2,3}(21)$; $C_{1,2}(3)C_{2,5}(18)$ $C_{1,2}(3)C_{2,8}(16)$

However, since these connected distances contain values greater than 14, there is no need to continue their construction

Step 3: Since the second subscript of the last distance is 10, the first subscript of the next distance should be 10. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2), except that the first subscript of the next distance should be 10. Boxes G and L contain distances with excluded subscripts., One chooses the distance $C_{10,9}(14)$ in box P to obtain the connected distances, $\boxed{C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)}$

The excluded subscript numbers , except 1, represent the cities already visited.

Step 4: Since the second subscript of the last distance is 9, the first subscript of the next distance should be 9. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2, 10), except that the first subscript of the next distance should be 9. One chooses the distance $C_{9,8}(4)$ in box D (with numerical value, 4) to obtain the connected distances $\boxed{C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)}$ Also $C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,7}(5)$

$C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,5}(20)$ $C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,6}(19)$

Step 5 Since the second subscript of the last distance is 8, the first subscript of the next distance should be 8. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2, 10, 9), except that the first subscript of the next distance should be 8, One chooses the distance $C_{8,5}(2)$ in box B to obtain the connected distances

$\boxed{C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{8,5}(2)}$.

Also $C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{7,5}(17)$; $C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{7,8}(8)$;
$C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{7,9}(5)$

Step 6: Since the second subscript of the last distance is 5, the first subscript of the next distance should be 5. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2, 10, 9, 8) except that the first subscript of the next distance should be 5. One chooses the distance $C_{5,6}(1)$ in box A to obtain the connected distances

$\boxed{C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{8,5}(2)C_{5,6}(1)}$; Also
$C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{7,6}(9)C_{6,4}(24)$

Note that $C_{5,6}(1)$ has the least numerical value, 1, among the eligible distances.

Step 7: Since the second subscript of the last distance is 6, the first subscript of the next distance should be 6. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2, 10, 9, 8, 5) except that the first subscript of the next distance should be 6. One chooses the distance $C_{6,7}(9)$ in box J to obtain the connected distances

$\boxed{C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{8,5}(2)C_{5,6}(1)C_{6,7}(9)}$
. Also, $C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{7,6}(9)C_{6,5}(1)C_{5,4}(12)$

The excluded subscript numbers , except 1, represent the cities already visited.

Step 8: Since the second subscript of the last distance 7, the first subscript of the next distance should be 7. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2, 10, 9, 8, 5, 6), except that the first subscript of the next distance should be 7. One chooses the distance $C_{7,4}(39)$ from the original data table to obtain the connected distances $\boxed{C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{8,5}(2)C_{5,6}(1)C_{6,7}(9)C_{7,4}(39)}$

Note that $C_{7,4}(39)$ has a relatively large numerical value, 39, among the eligible distances. One went up to a larger range of numbers to accommodate $C_{7,4}$. Because a value greater 14 has been used, upon completion of the route construction, the route found would not be the shortest route. The excluded subscript numbers , except 1, represent the cities already visited.

Step 9: Since the second subscript of the last distance is 4, the first subscript of the next distance should be 4. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2, 10, 9, 8, 5, 6, 7), except that the first subscript of the next distance should be 4. One chooses the distance $C_{4,3}(10)$ in box K to obtain the connected distances

$\boxed{C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{8,5}(2)C_{5,6}(1)C_{6,7}(9)C_{7,4}(39)C_{4,3}(10)}$

Alsso: $C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{7,6}(9)C_{6,5}(1)C_{5,8}(2)C_{8,4}(23)C_{4,3}(10)$

However, since these connected distances contain values greater than 14, there is no need to continue their construction

Note that $C_{4,3}(10)$ has the least numerical value, 2, among the eligible distances.

Step 10: Since the second subscript of the last distance is 3, the first subscript of the next and last distance should be 3. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2, 10, 9, 8, 5, 6, 7, 4) except that the first subscript of the next distance should be 3 and the second subscript should be 1 (an exception) in order to return to City 1, the starting city. One chooses the distance $C_{3,1}(13)$ in box N to obtain the connected distances

$$\boxed{C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{8,5}(2)C_{5,6}(1)C_{6,7}(9)C_{7,4}(39)C_{4,3}(10)C_{3,1}(13)=102}$$

Also $C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{7,6}(9)C_{6,5}(1)C_{5,8}(2)C_{8,4}(23)C_{4,3}(10)C_{3,1}(13)$

Step 11: Add the distances in parentheses: 3 + 7 + 1 4 + 4 + 2 + 1 + 9 + 39 + 10 + 13 and obtain 102.

For comparison purposes, before proceeding to the discussion and conclusion of the material covered already, one will next summarize the shortcomings of some previous methods for solving the traveling salesman problem,

Shortcomings of the Nearest Neighbor Approach and Grouping of Cities Approach

Shortcoming of the Nearest Neighbor Approach

Consider four cities at A, B, C, D. Let the home base of the salesman be at A.

Case 1: Applying the nearest neighbor approach, one would depart from City A along AD of length 6 units (Note: $6 < 9 < 10$). To visit each of the three cities once and return to A, one would either travel the distances AD + DB + BC+ CA (6 + 4 +12+10 = 32 units) or the distances AD + DC + CB + BA (6 + 9 +12 + 7 = 34 units.

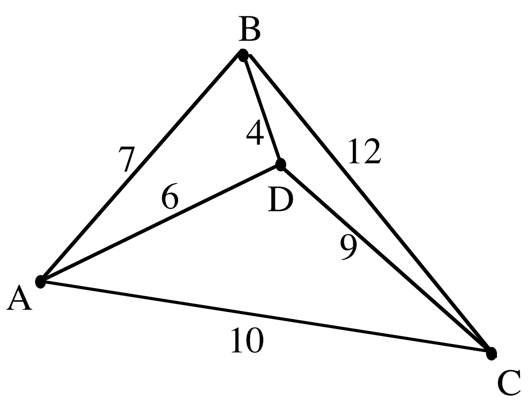
Case 2: If one departs along AB, one would either travel the distances
AB +BD + DC + CA (7 + 4 + 9 +10 = 30 units) or
AB + BC + CD + DA (7 + 12 + 9 + 6 = 34 units)

Case 3: If one departs along AC, one would travel either the distances
AC + CD + DB + BA (10 + 9 + 4 +7 = 30 units) or
AC + CB + BD + DA (10 + 12 + 4 + 6 = 32 units)

Observe above that the shortest route is **not** in Case 1, (of total distance 32 or 34 units) the nearest neighbor approach; but **is** in either **Case 2** or **Case 3 ,** of distance 30 units. Note that the totals in the first parts of Cases 2 and 3 are the same, the same individual distances, except for the order of the addition of the distances.

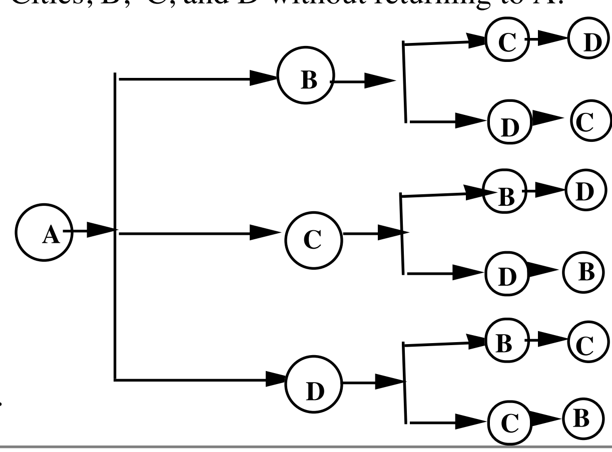
It is to be observed that departing to the nearest city at D, 6 units away, did not produce the shortest total distance. However, departing to either the city at B, or the city at C produced the shortest route of length 30 units, even though B or C is not the nearest neighbor.

The "culprit" is BC or CB of distance 12 units. If one departs to city at D, one is compelled to travel the longest distance of 12 units, since the options to visit the cities at B and D cannot avoid the 12 units distance. The error for Case 1 is about either 6% or 13%, respectively. As the number of cities increases, the errors will multiply.



$C_{A,B}$ = distance between City A and City B.

Possible routes for a salesman to visit each of the Cities, B, C, and D without returning to A.



$C_{A,B}$ **7**	$C_{A,B}$ **7**	$C_{A,C}$ **10**	$C_{A,C}$ **10**	$C_{A,D}$ **6**	$C_{A,D}$ **6**
$C_{B,C}$ **12**	$C_{B,D}$ **4**	$C_{C,B}$ **12**	$C_{C,D}$ **9**	$C_{D,B}$ **4**	$C_{D,C}$ **9**
$C_{C,D}$ **9**	$C_{D,C}$ **9**	$C_{B,D}$ **4**	$C_{D,B}$ **4**	$C_{B,C}$ **12**	$C_{C,B}$ **12**
28	**20**	**26**	**23**	**22**	**27**

Shortcoming of the Grouping of Cities Approach

Mini-Brute Force plus "Divide and Conquer" Approach

Example : Using a three-distance template to determine the shortest route

From City 1, a traveling salesman would like to visit once each of **nine** other cities, namely, City 2, City 3, City 4, City 5, City 6, City 7, City 8, City 9, City 10; and return to City 1.

Guidelines

Step 1: From City 1 (home base of salesman), consider the possible sub-routes for visiting three other cities (say, Cities 2, 3, and 4) without returning to City 1, and determine the shortest route from City 1 to visit once each of these cities.

Step 2: I From the last city visited by the shortest route, one will next determine the shortest sub-route for visiting three other cities, say, Cities 5, 6 and 7.

Step 3: From the last city visited according to the shortest route for visiting Cities 5, 6 and 7, one will determine the shortest route for visiting Cities 8, 9, and 10. The sums of the distances of the above shortest routes will added, and the distance from City 10 to City 1 will also be added the shortest routes sum. (Review example on previous page, and imitate)

Step 4: The results of Steps 1-3 can be combined into a single table as below (18 columns)

$C_{1,2}$ 3	$C_{1,2}$ 3	$C_{1,3}$ 13	$C_{1,3}$ 13	$C_{1,4}$ 35	$C_{1,4}$ 35	$C_{4,5}$ 12	$C_{4,5}$ 12	$C_{4,6}$ 24	$C_{4,6}$ 24
$C_{2,3}$ 21	$C_{2,4}$ 25	$C_{3,4}$ 10	$C_{3,2}$ 21	$C_{4,3}$ 10	$C_{4,2}$ 25	$C_{5,6}$ 1	$C_{5,7}$ 17	$C_{6,5}$ 1	$C_{6,7}$ 9
$C_{3,4}$ 10	$C_{4,3}$ 10	$C_{4,2}$ 25	$C_{2,4}$ 25	$C_{3,2}$ 21	$C_{2,3}$ 21	$C_{6,7}$ 9	$C_{7,6}$ 9	$C_{5,7}$ 17	$C_{7,5}$ 17
34	38	48	59	66	81	22	38	42	50

$C_{4,7}$ 39	$C_{4,7}$ 39	$C_{7,8}$ 8	$C_{7,8}$ 8	$C_{7,9}$ 5	$C_{7,9}$ 5	$C_{7,10}$ 28	$C_{7,10}$ 28
$C_{7,5}$ 17	$C_{7,6}$ 9	$C_{8,9}$ 4	$C_{8,10}$ 15	$C_{9,8}$ 4	$C_{9,10}$ 14	$C_{10,9}$ 14	$C_{10,8}$ 15
$C_{5,6}$ 1	$C_{6,5}$ 1	$C_{9,10}$ 14	$C_{10,9}$ 14	$C_{8,10}$ 15	$C_{10,8}$ 15	$C_{9,8}$ 4	$C_{8,9}$ 4
57	49	26	37	24	34	46	47

Step 5: Combine the boxed columns (shortest sub-routes) above, and add the distance $C_{10,1}$ ($C_{10,1}$ = 11, is the distance from the last city, City 10, to the home base city of the salesman.

$C_{1,2}$ 3	$C_{4,5}$ 12	$C_{7,9}$ 5
$C_{2,3}$ 21	$C_{5,6}$ 1	$C_{9,8}$ 4
$C_{3,4}$ 10	$C_{6,7}$ 9	$C_{8,10}$ 15
34	22	24

Total = 34 + 22 + 24 + 11 = 91

Shortest route to visit each of the nine cities once and return =

$C_{1,2} + C_{2,3} + C_{3,4} + C_{4,5} + C_{5,6} + C_{6,7} + C_{7,9} + C_{9,8} + C_{8,10} + C_{10,1} = 91$ units.

Observe above that Cities, 2, 3, 4, 5, 6, 7, 8, 9, and 10 have been visited; and by $C_{8,10}$, the salesman is at City 10; and to return to City 1, one adds $C_{10,1}$.

Grouping of cities approach
$C_{1,2}(3)C_{2,3}(21)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,9}(5)C_{9,8}(4)C_{8,10}(15)C_{10,1}(11) =$ **91**

Comparison of Approaches for Finding Shortest Routes

Case 1: For the **Nearest Neighbor approach**, the error lies in being compelled to travel an avoidable longer distance as illustrated on page 28.

Case 2: For the **Grouping of Cities approach**, the error emanates from ignoring some of the shortest distances in determining the shortest route. The length of the shortest route by the grouping of cities approach was found to be 91 units, (for sample problem in this paper)

Case 3: For the **Data Ordering and Route Construction approach,** the length of the shortest route determined was 81 units. The error in Case 2 relative to Case 3 is about 13%, In observing the numerical values of the distances for the shortest routes in Cases 2 and 3 as well as the entries in the table used in the construction of the shortest route for Case 3, below, note that Case 3 used numerical values from the table in boxes A-P. (minimum boxes). Even though Case 2 was obtained by a different approach, one can observe that values 15 and 21 in Case 2 are from boxes beyond boxes A-P.

Case 3 **Shortest route**

Data ordering and route construction approach

$C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)C_{9,10}(14)C_{10,2}(7)C_{2,1}(3) = \mathbf{81}$ **<--- R1**

Numerical distances: 1, 3, 4, 7, 8, 9, 10, 12, 13, 14

Case 2

Grouping of cities approach

$C_{1,2}(3)C_{2,3}(21)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,9}(5)C_{9,8}(4)C_{8,10}(15)C_{10,1}(11) = \mathbf{91}$

Numerical distances: 1, 3, 4, 5, 9, 10, 11, 12, **15, 21**

A $C_{5,6}$ or $C_{6,5}$ **1**	**G** $C_{2,10}$ or $C_{10,2}$ **7**	**N** $C_{1,3}$ or $C_{3,1}$ **13**	**U** $C_{6,9}$ or $C_{9,6}$ **19**
B $C_{5,8}$ or $C_{8,5}$ **2**	**H** $C_{7,8}$ or $C_{8,7}$ **8**	**P** $C_{9,10}$ or $C_{10,9}$ **14**	**V** $C_{5,9}$ or $C_{9,5}$ **20**
C $C_{1,2}$ or $C_{2,1}$ **3**	**J** $C_{6,7}$ or $C_{7,6}$ **9**	**Q** $C_{8,10}$ or $C_{10,8}$ **15**	**W** $C_{2,3}$ or $C_{3,2}$ **21**
D $C_{8,9}$ or $C_{9,8}$ **4**	**K** $C_{3,4}$ or $C_{4,3}$ **10**	**R** $C_{2,8}$ or $C_{8,2}$ **16**	**X** $C_{1,8}$ or $C_{8,1}$ **22**
E $C_{7,9}$ or $C_{9,7}$ **5**	**L** $C_{1,10}$ or $C_{10,1}$ **11**	**S** $C_{5,7}$ or $C_{7,5}$ **17**	**Y** $C_{4,8}$ or $C_{8,4}$ **23**
F $C_{6,8}$ or $C_{8,6}$ **6**	**M** $C_{4,5}$ or $C_{5,4}$ **12**	**T** $C_{2,5}$ or $C_{5,2}$ **18**	**Z** $C_{4,6}$ or $C_{6,4}$ **24**

Note the following:

$C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)C_{9,10}(14)C_{10,2}(7)C_{2,1}(3) = 81$is equivalent to

$C_{1,3}(13) + C_{3,4}(10) + C_{4,5}(12) + C_{5,6}(1) + C_{6,7}(9) + C_{7,8}(8) + C_{8,9}(4) + C_{9,10}(14) + C_{10,2}(7) + C_{2,1}(3) = 81$

(From City 1 to City 3; from City 3 to City 4; from City 4 to City 5; from City 5 to City 6; from City 6 to City 7; from City 7 to City 8; from City 8 to City 9; from City 9 to City 10; .from City 10 to City 2; and finally, from City 2 to City 1.)

Discussion and Conclusion

The length of the shortest route was found to be 81 units; but the sequence of cities of the shortest route is not unique. One sequence of the cities of the shortest route is given by $C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)C_{9,10}(14)C_{10,2}(7)C_{2,1}(3)$, say R1 If the direction of travel of this route is reversed, one obtains the route given by $C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{8,7}(8)C_{7,6}(9)C_{6,5}(1)C_{5,4}(12)C_{4,3}(10)C_{3,1}(13)$. Another route of length 81 units is $C_{1,3}(13)C_{3,4}(10)$ $C_{4,5}(12)$ $C_{5,8}(2)C_{8,6}(6)C_{6,7}(9)C_{7,9}(5)C_{9,10}(14)C_{10,2}(7)C_{2,1}(3)$
Therefore, the sequence of cities of the shortest route is not unique, but the length of the route is unique.

Justification of the shortest route.

From City 1, ten distances are needed to visit nine cities and return to City 1.
If each of the distances, $C_{m,n}$, in the ten-distance route were from the least ten distances (i.e., box A-K) in the table, one could immediately conclude that such a ten-distance route is the shortest route. In observing the possible shortest route, $C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)C_{9,10}(14)C_{10,2}(7)C_{2,1}(3)$,R1, not all the distances are from the least ten distances in the table, and one cannot immediately conclude that R1 is the shortest route. However, the next three distances (except 11 which is not applicable here), 12, 13, and 14 are included in R1.
These additions are minimum additions, and therefore, the shortest route of length 81 units is given by $C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)C_{9,10}(14)C_{10,2}(7)C_{2,1}(3)$. Perhaps, one should say a shortest route, since the sequence of cities is not unique.
Observe below that any ten-distance route which contains a distance greater than 14 (largest distance in R1) is at least 6 units greater than that of R1.

$C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)C_{9,10}(14)C_{10,2}(7)C_{2,1}(3) =$ **81**	**R1**
Numerical distances: 1, 3, 4, 7, 8, 9, 10, 12, 13, 14	

$C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,7}(5)C_{7,6}(9)C_{6,5}(1)C_{5,8}(2)C_{8,4}(23)C_{4,3}(10)C_{3,1}(13) =$ **87**	**R2**
Numerical distances: 1, 2, 3, 5, 7, 9, 10, 13, 14, **23**	

1

$C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,9}(5)C_{9,8}(4)C_{8,2}(16)C_{2,10}(7)C_{10,1}(11) =$ **88**	**R4**
Numerical distances 1, 4, 5, 7, 9, 10, 11, 12, 13, **16**	

A	$C_{5,6}$ or $C_{6,5}$	**1**	**G**	$C_{2,10}$ or $C_{10,2}$	**7**	**N**	$C_{1,3}$ or $C_{3,1}$	**13**	**U**	$C_{6,9}$ or $C_{9,6}$	**19**
B	$C_{5,8}$ or $C_{8,5}$	**2**	**H**	$C_{7,8}$ or $C_{8,7}$	**8**	**P**	$C_{9,10}$ or $C_{10,9}$	**14**	**V**	$C_{5,9}$ or $C_{9,5}$	**20**
C	$C_{1,2}$ or $C_{2,1}$	**3**	**J**	$C_{6,7}$ or $C_{7,6}$	**9**	**Q**	$C_{8,10}$ or $C_{10,8}$	**15**	**W**	$C_{2,3}$ or $C_{3,2}$	**21**
D	$C_{8,9}$ or $C_{9,8}$	**4**	**K**	$C_{3,4}$ or $C_{4,3}$	**10**	**R**	$C_{2,8}$ or $C_{8,2}$	**16**	**X**	$C_{1,8}$ or $C_{8,1}$	**22**
E	$C_{7,9}$ or $C_{9,7}$	**5**	**L**	$C_{1,10}$ or $C_{10,1}$	**11**	**S**	$C_{5,7}$ or $C_{7,5}$	**17**	**Y**	$C_{4,8}$ or $C_{8,4}$	**23**
F	$C_{6,8}$ or $C_{8,6}$	**6**	**M**	$C_{4,5}$ or $C_{5,4}$	**12**	**T**	$C_{2,5}$ or $C_{5,2}$	**18**	**Z**	$C_{4,6}$ or $C_{6,4}$	**24**

The future in the approach to solving the traveling salesman problem lies in the approach (data ordering and route construction)) whereby one concentrates on the smallest distances, and by judicious selection, construct the shortest route. Such an approach reduces the redundant use of brute-force. For the nine cities visit, using brute-force, one would have to consider about 362,880 possibilities. Each possibility would be a column of nine distances. One of these 362,880 columns would be the shortest route to visit the nine cities without returning to City1.

Bye-bye: nearest neighbor approach. You compelled the salesman to travel a longer distance.
Bye-bye: grouping of cities approach. You ignored some of the shortest distances.
Welcome: Data Ordering and Route Construction. Continue to refine and you would always be welcome
The error in the shortest route of length 81 units determined is zero or negligible.

Overall Conclusion

Three different types of NP problems were solved, The first type involved the division of items of different sizes, lengths, masses, volumes, or values into equal parts by combinations only. The second type covered possible final exam schedules for schools. The third type was the traveling salesman problem. The general approach for solving the different types of NP problems were the same, except that sometimes, specific techniques may differ from each other according to the process involved in the problem. The first step in these problems is to arrange the given data in either increasing or decreasing order. For the solutions of the first type of NP problems, an extended Ashanti fairness wisdom technique was applied to a set of 100 items of different values or masses. Two people A and B were able to divide items equally by merely choosing in turns from a set of ordered items. The total value or mass of A's items was found to be equal total value or mass of B's items, and these results are combinations of the items of different values or masses. It is very pleasing that such a simple technique can produce desired combinations. High school and middle school graduates could be taught the technique involved. From the solutions, formulas or simple equations were produced to help programmers apply the techniques. Note that in using the technique in this paper, the items involved must be arranged in decreasing order, preferably. Therefore, in programming, the first step should be to arrange the items in decreasing order. The technique was also applied to 1000 items; and the results were perfect, just like the results for the 100 items. Therefore, the technique covered does not care whether there are 2^{100} or 2^{1000} possibilities. There are social consequences of the method and principles used to divide the set of items into equal totals. The results can be applied by government agencies in the distribution of goods and services. Management personnel should be aware of the principles involved in the above technique. From the elementary school, through high school, and perhaps college, students should be taught the principles in the above wisdom technique, since throughout life, one is going to encounter situations in which two or more people are asked to choose in turns, from items of different values or sizes, and in this case, the sequence by which the choices are made matters; one may be either a participant or one may be in charge of the distribution process.

By hand, the techniques can be used to prepare final exam schedules for 100 or 1000 courses. School secretaries and office assistants can learn and apply the techniques covered.

A new approach to solving the traveling salesman was used to determine the shortest route to visit nine cities and return to the starting city. The distances involved were arranged in increasing order and by inspection, ten distances were selected from a set of the shortest 14 distances, instead of the overall set of 45 distances; and used to construct the shortest route. Finally, if an approach can solve one NP problem, that approach can also solve other NP problems. Since three different types NP problems (seven problems) were solved using the same approach outlined above, all NP problems can be solved. The formerly NP problems are now P problems, and therefore, it is concluded that P is equal to NP.

Perhaps, one may make the following statements.

1. NP plus human ability equals P. **2**. NP plus human inability is **not** equal to P.
3. NP minus human inability equals P.

References:

For paper edition of the above paper, see Appendix 6 of the book entitled "Power of Ratios" by A. A. Frempong, published by Yellowtextbooks.com. After solving the NP problems and reviewing the solutions, the author realized that a ratio process had been applied in solving the NP solutions, and in the beginning, did not consider including the solutions of NP problems in the "Power of Ratios" book which contains also the author's previous solutions of the Navier-Stokes equations plus solutions of the magnetohydrodynamic equations (viXra.org). The " Power of Ratios" book covers definition of ratio and applications of ratios in mathematics, science, pharmacology, engineering, economics and business fields.

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