-

Relations of Energy Density and Its Entropic Density with Temperature concerning Schwarzschild Black Hole and Holographic Dark Energy

Yong Bao

Postbox 777, 100 Renmin South Road, Luoding 527200, Guangdong, China
E-mail: baoyong9803@163.com

We consider the relations of the energy density and its entropic density with the temperature of Schwarzschild black hole (SBH) and holographic dark energy (HDE). On the basis of literature [8], we obtain the energy density of the scalar field of SBH being proportional to the square of the Hawking-Unruh temperature near outside of the event horizon when the distance from the film to the horizon ε and the film thickness δ are constant, and that its entropic density is directly proportional to the Hawking-Unruh temperature when ε and n are constant. Basing on [16], we find the equation of gravitational energy density inside SBH; derive that the gravitational energy density is proportional to the square of the effective temperature far from the event horizon in SBH interior, whether the gravitational fields of SBH are Coulomb-like or wave-like; and their entropic density is directly proportional to the effective temperature. Basing on [20, 21, 22], we gain the HDE density being proportional to the square of the Gibbons-Hawking temperature, and that its entropic density is directly proportional to the Gibbons-Hawking temperature. These equations are similar and have relations with each other. We suggest that these relations are interesting and significant for SBH and HDE.

PACS numbers: 97.60.Lf, 04.70.Dy, 95.36. +x

I. Introduction

What is the relation of the energy density and its entropic density with the temperature of Schwarzschild black hole (SBH) and holographic dark energy (HDE)? It is an interesting and significant question. In the seventies J. D. Bekenstein, S. W. Hawking *et al* proposed a famous theory which the entropy of black hole is proportional to the area of horizon [1], later Zhao Z *et al* obtained the Hawking radiation [2] also [3], 't Hoot G. proposed a brick-wall model [4], soon after Liu W-B *et al* proposed a thin film model [5], in the 2000s, M. K. Parikh *et al* proposed the tunnel effect theory of black hole [6], lately Meng Q M, Deng D L *et al* [7, 8], Y. Wang *et al* [9], He T-M *et al* [10] and A. I. Fisenko *et al* [11] propose the multiform general Stefan-Boltzmann law [12]; A. Dutta *et al* [13] and Y-G Miao *et al* [14] proposed the different corrected Hawking temperature. These only relate to near the horizon of black hole. In SBH interior, C. A. Egan *et al* calculate the interior entropy [15]; T. Clifton *et al* proposed the gravitational energy density formula and the effective temperature expression respectively [16], but they are irrelative; A. C. Wall proposed the generalized entropy [17]. Moreover in order to explain the cosmic accelerated expansion [18], numerous theoretical models have been proposed [19], thereinto M. Li *et al* proposed the HDE density equation [20, 21, 22] which has no relation with the temperature.

This paper is organized as follows. In Sec. II, we get the energy density of the scalar field being proportional to the square of the Hawking-Unruh temperature [3, 23] near outside of the horizon of SBH, and that its entropic density is directly proportional to the Hawking-Unruh one. In Sec. III, we find that the gravitational energy density is proportional to the square of the effective temperature far from the horizon inside SBH, and its entropic density is directly proportional to the effective temperature. In Sec. IV, we obtain the HDE density being proportional to the square of the Gibbons-Hawking temperature [24, 25, 26, 27], and that its entropic density is directly proportional to the Gibbons-Hawking one. We conclude in Sec. V.

II. Energy Density of scalar field, Entropic Density and Hawking-Unruh Temperature near outside of Event

-

Horizon

In this section, we review [8]; obtain that the energy density of the scalar field is proportional to the square of the Hawking-Unruh temperature near outside of the event horizon of SBH, and the entropic density is directly proportional to the Hawking-Unruh one.

A. Energy density of scalar field and Hawking-Unruh temperature

First let us review [8] briefly. Tthe Schwarzschild coordinates is used, and the energy density ρ_{sf} of the scalar field of SBH is (we work with ħ = c = G = k = 1 units)

$$\rho_{sf} = 6\pi^2 M^2 T_{H-U}^4 / 45\varepsilon(\varepsilon+\delta) \tag{1}$$

Where $\delta = n\varepsilon$ is the film thickness, ε is the distance from the film to the horizon of SBH, and n = 1, 2, 3….

Substituting the Hawking-Unruh temperature $T_{H-U} = 1 / 8\pi M$ into (1), we obtain

$$\rho_{sf} = T_{H-U}^2 / 480\varepsilon^2(n+1) \tag{2}$$

Therefore the energy density of the scalar field is proportional to the square of Hawking-Unruh temperature near outside of the horizon when ε and n are constant. Recovering ħ, c, G and k, we obtain

$$\rho_{sf} = \mathrm{k}^2 T_{H-U}^2 / 480\varepsilon^2(n+1) \tag{3}$$

B. Entropic density and Hawking-Unruh temperature

The entropic density s_{sf} of the scalar field near outside of the horizon is [8]

$$s_{sf} = 8\pi^2 M^2 / 45\beta^3\varepsilon(\varepsilon+\delta) \tag{4}$$

where β is [8]

$$\beta = 8\pi M \tag{5}$$

Using (5) and $T_{H-U} = 1 / 8\pi M$ to (4), we obtain

$$s_{sf} = T_{H-U} / 360\varepsilon^2(\mathrm{n}+1) \tag{6}$$

Therefore the entropic density is directly proportional to the Hawking-Unruh temperature when ε and n are constant.

III. Gravitational Energy Density, Entropic Density and effective temperature far from inside Event Horizon

In this section, we review [16] briefly; find that the gravitational energy density is proportional to the square of the effective temperature far from the event horizon inside SBH, and its entropic density is directly proportional to the effective one.

A. Relations for Coulomb-like gravitational fields

In [16], the gravitational fields can be classified two types: Coulomb-like gravitational fields and wave-like ones. In general they are mixed. For the Coulomb-like gravitational fields

$$8\pi\rho_{grav} = 2\alpha\sqrt{2W/3} \text{ and } p_{grav} = 0 \tag{7}$$

where ρ_{grav} is the gravitational energy density, α is a constant, $W = T_{abcd}u^a u^b u^c u^d$, T_{abcd} is the Weyl tensor, u^a, u^b, u^c, u^d are the timelike unit vectors, and p_{grav} is the isotropic pressure. The Schwarzschild geometry can be written in Gullstrand–Painlevé coordinates as

$$\mathrm{d}s^2 = -[1-(2m / r)]\mathrm{d}t^2 - 2\sqrt{2m/r}\,\mathrm{d}r\mathrm{d}t + \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2 \tag{8}$$

where m is the constant mass parameter. The gravitational energy density and temperature is given at each point in the region $r < 2m$ by

$$\rho_{grav} = 2\alpha m / 8\pi r^3 \tag{9}$$

$$T_{grav} = m / 2\pi r^2\sqrt{|1-(2m/\mathrm{r})|} \tag{10}$$

where T_{grav} is the effective temperature. Taking (10) to (9), we find

$$\rho_{grav} = \alpha\pi[2-(r / m)]T_{grav}^2 \tag{11}$$

It is the equation concerning the gravitational energy density and the effective temperature in the region $\mathrm{r} < 2m$. Note that the isotropic pressure is zero. When $r \ll 2m$, we derive

$$\rho_{grav} = 2\alpha\pi T_{grav}^2 \tag{12}$$

So the gravitational energy density is proportional to the square of the effective temperature far from the horizon inside SBH. It includes two regions far from horizon inside SBH: Singularity and vacuum. In (11) when $r \to 0$, we gain (12) also, that is the gravitational energy density being proportional to the square of the effective temperature in the singularity and near the one.

B. Relations for the wave-like gravitational fields

In [16], for the wave-like gravitational fields

$$8\pi\rho_{grav} = \beta\sqrt{4W} \text{ and } p_{grav} = \rho_{grav} / 3 \quad (13)$$

For the SBH, the gravitational energy density is given at each point in region r < $2m$ by

$$\rho_{grav} = \sqrt{6}\beta m / r^3 \quad (14)$$

Using (10) to (14), we find

$$\rho_{grav} = \sqrt{6}\beta\pi[2-(r / m)]T_{grav}^2 \quad (15)$$

When $r << 2m$ and $r \rightarrow 0$, we obtain

$$\rho_{grav} = 2\sqrt{6}\beta\pi T_{grav}^2 \quad (16)$$

It is very similar to (12). Therefore the gravitational energy density is proportional to the square of the effective temperature far from the horizon inside SBH, whether the gravitational fields are Coulomb-like or wave-like.

C. Relations of entropic density

For the Coulomb-like gravitational fields, its entropic density s_{grav} is [16]

$$\delta s_{grav} = \delta (\rho_{grav} v) / T_{grav} \quad (17)$$

where $v = z^a \eta_{abcd} \mathrm{d}x^b \mathrm{d}x^c \mathrm{d}x^d$ and we can set an arbitrary constant to zero. Substituting (11) into (17) and integral, we get

$$s_{grav} = \alpha\pi[2-(r / m)]T_{grav} \quad (18)$$

This is the equation concerning the entropic density and the effective temperature in the region r < $2m$. When $r << 2m$ and $r \rightarrow 0$, we derive

$$s_{grav} = 2\alpha\pi T_{grav} \quad (19)$$

So the entropic density is directly proportional to the effective temperature far from the horizon inside SBH.

For the wave-like gravitational fields, substituting (15) into (17) and integral, we obtain

$$s_{grav} = \sqrt{6}\beta\pi[2-(r / m)]T_{grav} \quad (20)$$

When $r << 2m$ and $r \rightarrow 0$, we derive

$$s_{grav} = 2\sqrt{6}\beta\pi T_{grav} \quad (21)$$

Therefore the entropic density is directly proportional to the effective temperature far from the horizon inside SBH, whether the gravitational fields are Coulomb-like or wave-like.

IV. HDE Density, Entropic Density and Gibbons-Hawking Temperature

In this section, we review [20, 21, 22], obtain that the HDE density is proportional to the square of the Gibbons-Hawking temperature, and its entropic density is directly proportional to the Gibbons-Hawking one.

A. HDE density and Gibbons-Hawking temperature

In [20, 21, 22], the equation of HDE model can be rewritten as

$$\rho_{de} = 3c_L^2 \mathrm{M}_{\mathrm{pl}}^2 L^{-2} \quad (22)$$

where ρ_{de} is the HDE density, $c_L \geq 0$ is a dimensionless model parameter, $\mathrm{M}_{\mathrm{Pl}} \equiv 1 / \sqrt{8\pi \mathrm{G}} = 1 / \sqrt{8\pi}$ is the reduced Planck mass and L is the cosmic cutoff.

Substituting the Gibbons-Hawking temperature $T_{G-H} = 1 / 2\pi L$ into (22), we gain

$$\rho_{de} = 12\pi^2 c_L^2 \mathrm{M}_{\mathrm{pl}}^2 T_{G-H}^2 = 3\pi c_L^2 T_{G-H}^2 / 2 \quad (23)$$

where $\mathrm{M}_{\mathrm{P}} \equiv 1 / \sqrt{\mathrm{G}} = 1$ is the Planck mass. So the holographic dark energy density is proportionate to the square of the Gibbons-Hawking temperature.

B. HDE entropic density and Gibbons-Hawking temperature

In [22], the HDE entropy S_{de} is

$$S_{de} = \pi M_{pl}^2 L^2 \tag{24}$$

The HDE entropic density s_{de} is

$$s_{de} = \pi M_{pl}^2 L^2 / L^3 = \pi M_{pl}^2 / L = \pi T_{G-H} / 4 \tag{25}$$

Therefore the HDE entropic density is directly proportional to the Gibbons-Hawking temperature.

V. Conclusion

In this paper, we have obtained the energy density of the scalar field [8] of SBH being proportional to the square of the Hawking-Unruh temperature near outside of the horizon when the distance from the film to the horizon ε and the film thickness δ are constant, and that the entropic density is directly proportional to the Hawking-Unruh temperature when ε and n are constant; found the equation concerning the gravitational energy density and the effective temperature in the region $r < 2m$ [16] inside SBH; derived that the gravitational energy density is proportional to the square of the effective temperature far from the horizon in SBH interior, whether the gravitational fields of SBH are Coulomb-like or wave-like; obtained that their entropic density are directly proportional to the effective one. These equations are true in the singularity and near one of SBH also. Also we have got that the HDE density [20] is proportional to the square of the Gibbons-Hawking temperature, and its entropic density is directly proportional to the Gibbons-Hawking one.

Eq. (2), (12), (16), and (23) are similar. Eq. (2) belongs to the Hawking radiation; (12) and (16) belong to the quantum gravity. It is well-known that the Hawking radiation is produced by the quantum gravity, so Eq. (2) has the relation with (12) and (16). Eq. (12), (16) and (23) are similar also. Eq. (23) belongs to the dark energy which can produce the repulsion; therefore it has the relation with (12) and (16). Their entropic density is also. We suggest that these relations are interesting and significant for SBH and HDE.

References

[1] J. D. Bekenstein, Phys. Rev. D., 1993, **7** (2); 333; S. W. Hawking, Commun Math. Phys., 1975, 43, 199.

[2] Zhao Z, Zhang J H, Acta Phys. Sin., 1981, **31** (1): 508; Zhang J H, Meng Q M, Li C A, Acta Phys. Sin. 1996, **45** (9): 177.

[3] S. W. Hawking, Nature, 1974 (248): 30-31.

[4] 't Hoot G. Nucl. Phys. B, 1985 (256): 727-745.

[5] Liu W-B, Zhao Z, Chin. Phys. Lett, 2001, **18** (2): 310-312; Li X. Zhao Z, Chin. Phys. Lett, 2001, **18** (3): 463-465.

[6] M. K. Parikh, F, Wilczek, Phys. Rev. Lett, 2000, 85 (4): 5042-5047.

[7] Meng Q M, Jiang J J, Liu J L, *et al*, Physical Journal, 2009, **58** (1): 78-82; Meng Q M, Acta Phys. Sin. 2005, 54 (1): 471-474; Meng Q M, Jiang J J, Chinese Physics G, 2008, 38 (2): 171-177; Meng Q M, Wang S, Jiang J J and Deng D L, Chinese Physics B, 2008, 17 (12): 2811-2816; Meng Q M, Jiang J J, Wang S, Acta Phys. Sin. 2009, 58 (15): 7486-7489; Jiang J-J, Meng Q-M and Wang S, Chin. Phys. B, 18 (2009), 457; Deng D L, Journal of Guizhou University (Natural Sciences), 2010, 27 (2): 119-121; Meng Q M, Jiang J J, Li C A, Acta Phys. Sin. 2010, 59 (2): 778-782.

[8] DENG De-Li, Journal of Guizhou University (Natural Sciences), 27, **4** (2010), 1000-5269(2010)04-0022-03.

[9] Y. Wang et al., arXiv: 1311.6707 [astro-ph.SR].

[10] He T-M, Yang J-B and Zhang J-Y, Chin. Phys. Lett. **31** (2014), 080401.

[11] A. I. Fisenko, V. Lemberg, arXiv: 1403.6868.

[12] J. Stefan, Wien. Ber., **79** (1879), 391; L. E. Boltzmann, Wied. Ann. Phys., **22** (1884), 291.

[13] A. Dutta, S. Gangopadhyay, arXiv: 1408.3928 [gr-qc].

[14] Y-G Miao, Y-Y Zhao, and S-J Zhang, arXiv: 1410.4115 [hep-th].

[15] C. A. Egan and C. H. Lineweaver, The Astrophysical Journal, 2010, 710: 1825-1834.

[16] T. Clifton, G. F. R. Ellis and R. Tavakol, Class. Quantum Grav.**30** (2013) 125009 (15pp).

[17] A. C. Wall, Class. Quantum Grav. **30** (2013), 165003 (35pp).

[18] A. G. Riess *et al.*, Astron. J, **116** (1998) 1009, Astrophys. J, 607 (2004) 665; S. Perlmutter *et al.*, Astrophys. J, **517** (1999) 565; R. A. Knop *et al.*, Astrophys. J, **598** (2003) 102; C. L. Bennett *et al.*, Astrophys. J. Suppl, **148** (2003) 1, arXiv: 0302207 [astro-ph]; D. N. Spergel *et al.*, Astrophys.J.Suppl, **148** (2003) 175, arXiv: 0302209 [astro-ph]; H. V. P. Peiris *et al.*, arXiv: 0302225 [astro-ph]; M. Tegmark *et al.*, Phys. Rev. D, **69** (2004) 103501; M. Tegmark *et al.*, Astrophys. J, **606** (2004) 702; S. W. Allen *et al.*, Mon. Not. Roy.

Astron. Soc. J, **353** (2004) 457.

[19] H. Wei, R-g. Cai, Science & Technology Review, **23**, 12 (2005) 28-32; M. Li, X-D. Li, Sh. Wang and Y. Wang, Commun. Theor. Phys. **56** (2011) 525-604; arXiv: 1103.5870 [astro-ph]; M. Li *et al.*, arXiv: 1209.0922 [astro- ph]; X. Lei *et al.*, Commun. Theor. Phys. 2013, **59** (02) 249-252; M. Sharif, A. Jawad, Commun. Theor. Phys. 2013, **60** (02), 183-188.

[20] M. Li, Phys. Lett. B, **603** (2004) 1; arXiv: 0403127 [hep-th].

[21] K.Ke, M. Li, Phys. Lett. B **606** (2005) 173-176.

[22] Q-G. Huang and M. Li, arXiv: 0404229 [astro-ph].

[23] W.G. Unruh, Phys.Rev.D**10**, (1974) 3194.

[24] G.W. Gibbons and S.W. Hawking, Phys. Rev. D **15**, 2738 (1977).

[25] J.W. Lee, J. Lee, and H.C. Kim, J. Cosmol. Astropart. Phys. **0708** (2007) 005.

[26]LI Miao, LI Xiao-Dong, WANG Shuang, and WANG Yi, Commun. Theor. Phys. **56** (2011) 525–604.

[27] S. Mohanty and A. Nautiyal, arXiv: 1404.2222 [hep-ph].