# Throwing objects off the Earth in the d20 system

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**Abstract**

We derive the minimum strength score require for a character to throw an object off the Earth in the d20 roleplaying system. By examining a "worst case" scenario involving air resistance, we show that the presence of the atmosphere induces a negligible correction to the minimum strength score, which is 114 for a Medium creature throwing another Medium creature off the Earth.

## 1 Introduction

Let us assume that the character trying to throw an object off the Earth makes a cabertoss-style throw with constant force equal to the maximum load they can lift.[1] If that force is denoted by $F$, and the height of the character plus the length of their arms is $h$, then the initial speed of an object thrown by them will be:

$$u_0 = \sqrt{\frac{2Fh}{m}} \tag{1.1}$$

For an object to escape the gravitational pull of the Earth (neglecting air resistance), it must achieve the escape velocity, given by:

$$u_{\text{esc}} = \sqrt{\frac{2GM}{R}} \tag{1.2}$$

$G$ is the gravitational constant, $M$ is the Earth's mass, and $R$ is the Earth's radius. By equating (1.1) and (1.2), we find that the minimum force the character needs to exert on the object to send it in orbit is:

$$F_{\text{min}} = \frac{GMm}{Rh} \tag{1.3}$$

To convert the force to its mass equivalent on the planet we divide by $g = GM/R^2$ to find $M_{\text{max}}$, the maximum mass a character can lift that will allow them to throw the object in space:

$$M_{\text{max}} = \frac{mR}{h} \tag{1.4}$$

[1] This is a reasonable approximation if the mass being thrown weighs significantly less than the maximum mass that the character can lift. Furthermore, as the character's maximum possible effort, it will give us the minimum strength required to perform the feat.

## 2 Air resistance corrections

Air resistance is quite difficult to incorporate in our treatment since it is a non-conservative force. However, with some simplifications we will be able to examine a "worst case" scenario, which we will use to show that air resistance can be neglected. The drag force in the high velocity domain is given by $\rho C_D A v^2/2$ where $A$ is the reference area while $C_D$ is the drag coefficient. Due to the thinning of the atmosphere and the deceleration of the object, we can see that the drag is constantly diminishing. Because the atmosphere is negligibly thin above the Kármán line ($a \approx 100$ km above sea level [1]), we will consider air resistance to be relevant only up to that point. We will also take the acceleration of gravity as well as the air density to be constant throughout the motion and deal with their variance later. The movement of the thrown object is described by the following equation:

$$m\frac{dv}{dt} = m\frac{dv}{dy}v = -mg - Bv^2 \tag{2.1}$$

Where $B = \rho C_D A/2$. Solving this equation gives us:

$$v^2 = v_0^2 + \frac{mg}{B}\left(e^{-2Ba/m} - 1\right) \tag{2.2}$$

We find that the total kinetic energy lost by passing through the atmosphere is $m^2 g\left(e^{-2Ba} - 1\right)/2B$. This includes a $mga$ decrease ($mga$ is gained in potential energy) and so we find that the work to overcome air drag is:

$$W = mga - \frac{m^2 g}{2B}\left(1 - e^{-2Ba/m}\right) \tag{2.3}$$

We have changed signs to keep $W$ positive. We can also observe that for $B \to 0$ the work required vanishes, while for $B \to \infty$ it tends to $mga$ (this is an unphysical result, as the mass would never reach $y = a$). Note that (2.3) does not depend on the initial velocity and is a strictly decreasing function of $B$. Thus, it becomes an upper bound estimate if we use the acceleration of gravity $g$ at height $y = a$ and the air density at sea level in calculating $B$. Applying conservation of energy to the scenario where the object reaches infinity with zero kinetic energy, we have:

$$\frac{1}{2}mv_0^2 - \frac{GMm}{R} - W = 0 \tag{2.4}$$

And we find:

$$v_0 = \sqrt{\frac{2GM}{R} + \frac{2W}{m}} \tag{2.5}$$

Using (1.1), (1.2) and $g = GM/R^2$, we find:

$$M_{\max} = \frac{mR}{h} + \frac{W}{gh} \tag{2.6}$$

We will now see that in the context of STR scores, the second term is negligible.

## 3 STR corrections

In the d20 system, strength (STR) is measured by an integer number. Most crucially, strength limits the weight a character can lift above their head, and informs us of the maximum force they can exert throughout the cabertoss motion, i.e. $M_{\text{max}}$. As can be seen in [2], the maximum load is multiplied by 4 for every increase of 10 in STR. This means that if the maximum load at STR 10 (the human average) is $M_{10} \approx 45.4\,\text{kg}$, the maximum load for arbitrary STR is given by:

$$M_{\text{max}} = sM_{10} \times 4^{[\text{STR}]/10} \tag{3.1}$$

$s$ is the size coefficient, equal to 1 for Medium creatures and ranging from 1/8 for Fine to 16 for Colossal. Conversely, the minimum strength required to lift the load is:

$$[\text{STR}] = 10 \log_4 \left( \frac{M_{\text{max}}}{sM_{10}} \right) \tag{3.2}$$

Substituting (2.6) in (3.2), we find:

$$[\text{STR}] = 10 \log_4 \left( \frac{m}{sM_{10}} \frac{R}{h} + \frac{W}{sM_{10}gh} \right) \tag{3.3}$$

Assuming that $W/mgR \ll 1$, the correction in strength can be approximated by a first-order power series:

$$[\text{STR}] = 10 \log_4 \left( \frac{m}{sM_{10}} \frac{R}{h} \right) + \frac{1}{\ln 4} \frac{W}{mgR} \tag{3.4}$$

We ask that the second term is smaller than 1 so that the air resistance correction to STR can be neglected. Let us again consider a worst-case scenario: assume that the thrown object is a very non-aerodynamic cuboid ($C_D = 2.1$ [3]) with $A = 400\,\text{m}^2$ and $m = 1.25 \times 10^5\,\text{kg}$ (typical values for a roughly symmetrical Colossal creature [4]) and that the thrower is a Fine creature with height of about 0.1 m. If we use the air density at sea level ($\rho = 1.2\,\text{kg/m}^3$), we find $B = 504\,\text{kg/m}$. Using typical values for the radius of the Earth and (2.3), we find $W = 1.25 \times 10^{11}$ J and $W/mgR = 0.016$, meaning that our use of the Taylor approximation has been justified, and that the second term in (3.4) is 0.0115, a negligible correction to the STR score in the context of the d20 system where attribute scores can only take integer values. This analysis holds for a wide range of thrown object sizes, as $W/mgR$ is dominated by the $a/R$ term (which is much smaller than 1) even for very light objects since $B \propto m^{2/3}$ (by square-cube law scaling arguments). Finally:

$$[\text{STR}] = 10 \log_4 \left( \frac{m}{sM_{10}} \frac{R}{h} \right) \tag{3.5}$$

## 4 Conclusion

We have shown that air resistance can be neglected when calculating the minimum STR score required for throwing an object off the Earth. Using (3.5) (and rounding up) as well as typical (mean) values for various size categories from [4] (approximating $h$ as $1.5\times$ [height]), the minimum STR required for a character to throw another object off a planet with the same radius as Earth is given by the following table.

**Table 1**. Minimum STR required to throw an object off the Earth.

| **Thrown** | F | D | T | S | M | L | H | G | C |
|---|---|---|---|---|---|---|---|---|---|
| **Thrower** | | | | | | | | | |
| Fine | 91 | 102 | 117 | 131 | 147 | 157 | 172 | 186 | 191 |
| Diminutive | 83 | 94 | 109 | 123 | 139 | 149 | 164 | 179 | 183 |
| Tiny | 73 | 84 | 99 | 113 | 129 | 139 | 154 | 169 | 173 |
| Small | 65 | 76 | 91 | 106 | 121 | 132 | 146 | 161 | 165 |
| Medium | 58 | 69 | 84 | 98 | 114 | 124 | 139 | 154 | 158 |
| Large | 48 | 59 | 74 | 88 | 104 | 114 | 129 | 144 | 148 |
| Huge | 38 | 49 | 64 | 78 | 94 | 104 | 119 | 134 | 138 |
| Gargantuan | 28 | 39 | 54 | 68 | 84 | 94 | 109 | 124 | 128 |
| Colossal | 21 | 32 | 47 | 60 | 77 | 97 | 102 | 116 | 121 |

This table has a few interesting features. Note that it is Gargantuan creatures that have the hardest time throwing a creature of their own size off the Earth. Colossal creatures can throw a Fine creature off the Earth with just 21 STR, though this is a feature of the d20 system, in which STR is an intensive property (a Colossal creature can exert a much larger force than a Medium creature with the same strength). Of course, Table 1 is only to be used as a rule of thumb. In borderline cases or if the setting in question features a planet with a radius drastically different from Earth's, (3.5) should be used with $m$ replaced by $(g_{\mathrm{planet}}/g_{\mathrm{Earth}})m$ since the given value of $M_{10}$ holds only for a planet with acceleration of gravity $g_{\mathrm{Earth}}$.

# References


[1] `http://www.fai.org/icare-records/100km-altitude-boundary-for-astronautics`

[2] `http://www.d20srd.org/srd/carryingCapacity.htm`

[3] `http://www.engineeringtoolbox.com/drag-coefficient-d_627.html`

[4] `http://www.d20srd.org/srd/combat/movementPositionAndDistance.htm`