Reconstruction of Quantum Field Theory as Extension of Wave Mechanics

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Abstract

The task to be carried out should be clear from the title. The reason for choosing wave mechanics as a starting point was the estimation that the present state of quantum field theory is not acceptable. There are two essential points in the construction presented here: First of all the role of interaction is adequately respected in it. Secondly a new attack is made to solve the old problem of describing elementary particles by stable wave packets, but this time with all means nowadays being available. Perturbation theory can be applied to solve the field equation and to derive the Feynman rules. But the prescription for the exchange terms cannot be deduced in quantum field theory, whatever version of it is chosen. The result of the present construction shall be called quantum wave theory. It reveals to be both, a field theory and a quantum theory.

1. What is the problem?

Quantum mechanics is a theory, which is well suited to describe the 'behaviour' of a quantum object under the influence of an external potential, as immediately can be seen by a glance at the Schrödinger equation or on the equation for the harmonic oscillator. However interaction between such objects is lacking. Hence Einstein was right with his claim that quantum mechanics is incomplete. But meanwhile quantum mechanics has been completed by quantum field theory. In that theory interaction is not only adequately respected, but also playing a central role, for instance, in the analysis of scattering processes.

So far all is quite right.

But now a problem arises. The usual version of quantum field theory, as it can be found in textbooks, is not acceptable. The reasons for such a far reaching thesis are given in the next section. In the present paper the bet is made on wave mechanics as a base for the extension to quantum field theory, although it is not much appreciated nowadays. In section 3 a short review of the history of wave mechanics is given and especially a report about Schrödinger's attempt to describe elementary particles by stable wave packets. As is well known this endeavour failed. Nevertheless a new attempt is made to describe elementary particles by stable wave packets, but this time under full consideration of the role of interaction resp. self interaction. In section 4 perturbation theory will be derived only by means of classical field theory. This is concerning the approximate solution of the field equation as well as the analysis of scattering processes. But no justification for the correct sign of exchange terms can be given.The result of the whole construction is presented in section 5. It shall be called quantum wave theory in order to demarcate it clearly from usual versions of quantum field theory. The question, whether quantum wave theory is a field theory or a quantum theory, shall be discussed in section 6. The summary in section 7 is striking the balance of the extension of wave mechanics to quantum wave theory. In the list of literature some older textbooks are quoted. They have the advantage of being thorough and explicit.

2. Why is the usual version of quantum field theory not acceptable?

A closer inspection of quantum field theory, as it is presented in the textbooks, would reveal that quantum field theory is blown up, sometimes even faulty, and in this version superfluous. This is the case especially for the following points

(a) perturbation theory

(b) the concept of particle in the Copenhagen interpretation

(c) quantum statistics for the ingoing particles of scattering processes

(d) quantum statistics for the outgoing particles of scattering processes.

In detail:

Perturbation theory

In quantum field theory a long distance must be covered, before one will arrive at perturbation theory. First of all the time ordered product must be derived in LSZ-reduction and redefined as T- product. Then the Theorem of Wick is to be applied. The detour over the interaction picture is only made in order to get explicitly the Hamilton operator of the interaction term. When after all this effort finally the result

$$< 0|T\{\phi(x_1)...\phi(x_n)\}|0 >= \frac{< 0|T\{\phi_I(x_1)...\phi_I(x_n)\exp[-i\int d^4x\mathcal{H}_I]\}|0 >}{< 0|T\{\exp[-i\int d^4x\mathcal{H}_I]\}|0 >} \tag{1}$$

appears, it is not yet the turn to begin. Then the exponential function must be expanded into a Taylor series and each term of it into a series according to the coupling constant. Not until in the coefficients

$$P_{r,n} = \phi_I(x_1)...\phi_I(x_r)\int dy_1...dy_n\mathcal{H}_I(y_1)...\mathcal{H}_I(y_n) \qquad \mathcal{H}_I(y_i) = \phi_I(y_i)^3 \tag{2}$$

for all pairs of factors ϕ_I contractions have been introduced, the perturbation series for the S-matrix can be deduced.

In section 4 it will be shown that a solution of the field equation can be arrived, and that scattering processes can be analyzed. But the prescription for the determination of the phase in connection with the occurrence of exchange terms can only be acknowledged as an empirical rule.

The concept of particle in the Copenhagen interpretation

According to the ideas of the Copenhagen interpretation we don't know what a particle is 'an sich', but only how it appears to us in an experiment: either as wave or as a corpusculum. The latter one is nothing else than the mass point of classical mechanics, while the waves are amplitudes belonging to the probability interpretation of Born. This curious idea must be estimated as an improper attempt to introduce the Transzendentalphilosophie of Kant into theoretical physics, or said a bit impolite: There is no place in theoretical physics for schizophrenic objects like the particles of the Copenhagen interpretation. On the other hand a concept of particle is lacking that describes those really existing particles, with which experimentally oriented physicists are working.

The quantum statistics for the ingoing particles of scattering processes

For particles that are free in the sense that they satisfy the homogeneous part of a field equation, and moreover never have been in a common interaction, any correlation is a contradiction in itself. This especially is true for the ingoing particles of scattering processes, for they are generated by independent sources, and hence are independent themselves.

There is still another proof of the fact that that problems of quantum statistics are not valid for the ingoing particles. It is contained in the following quotation from [1] p. 149. "The relative minus sign between the direct and exchange terms is due to the Fermi statistics, which requires the amplitude to be antisymmetric under interchange of the two final electrons. It is also antisymmetric under interchange of the two initial electrons as required by the statistics." Assumed that this statement is true. Then one could in the electron-electron-scattering to second order interchange the two vertices of the exchange term, which would leave the photon propagator unchanged. By this procedure the exchange of the ingoing particles could be transmitted to the exchange of the outgoing particles and vice versa. Hence at most the outgoing particles are relevant for topics of quantum statistics.

The quantum statistics for the outgoing particles of scattering processes

If one is calculating the LSZ-reduction in detail, then at a certain place a temporal sequence of the field operators appears. It is valid for bosons as well as for fermions with the consequence, that a boson propagator and a sign of Bose statistics is attached to the fermions, too. In order to remove this fault first of all the symmetry or antisymmetry of the elements of the Fock space were applied, which were refuted already above. Secondly the temporal sequence of the field operators is changed into the T-product by introducing an additional sign per definition. But such a procedure cannot be admitted as a systematic deduction.

Another deduction of quantum statistics is relying on the violation of micro causality. This might be true. But it is irrelevant, as can be read off from the following quotation: "It is worthwhile observing that, if one quantizes, say, a Bose field with anticommutators, the violation of microscopic causality is sizable only at distances comparable to the Compton wavelength of the particle involved, generally $\sim 10^{-13}$cm." (see [2] p.172). This statement must be supplemented by the fact that a location below the Compton wavelength is needing such high energy that it would destroy the system. No measurement within this system is possible (cf. Thirring [7]). Hence the violation of micro causality may be true, but it cannot be realized by a measurement.

3. Wave mechanics

3.1 A historical remark

After Einstein in 1905 had inferred from experimental results and theoretical reasons that there must be a particle in correspondence to the electromagnetic field, which nowadays is called photon, de Broglie had the idea to invert this relation and to assign a wave to all material objects (cf. e. g. [9]). That was the beginning of wave mechanics. It was further elaborated by Schrödinger with the equation now bearing his name. Since then the harmonic oscillator is estimated as a classical example for the transition from classical mechanics to quantum mechanics.

The next step consisted in Schrödinger's [10] attempt to describe elementary particles by wave packets. For this purpose he developed a model, in which the wave packet is constructed by superposition of eigen functions of the harmonic oscillator. The decisive point of the construction is that such a wave packet is stable. But the hope that in a similar way the electrons in the orbit of an atom can be described, too, by packets of matter waves was in vain. For Heisenberg showed in the same paper [11], in which he published the uncertainty relations, that the model of Schrödinger has an equidistant energy spectrum and hence is the only example of a stable wave packet. The usual argument against such attempts nowadays is that the wave packets are dispersing, comparable with the dispersion of a heat pole on a heat leading material.

3.2 A new attempt to describe elementary particles by stable wave packets

The failure of Schrödinger's ansatz is probably caused by the insufficient means of the year 1926, for the applied equation of the harmonic oscillator is linear. Wave packets being built up of them disperse.

Now an attempt shall be undertaken, to repeat the original ansatz of Schrödinger, but this time by all means that nowadays are available.

For this purpose one has to leave the frame of quantum mechanics including the linear algebra of the Hilbert space as the corresponding mathematical tool, and to change to quantum field theory. For there the interaction between the particles is adequately respected. The theoretical and experimental analysis of scattering processes is just a main topic of this theory. Therefore it shall be tried now, to give a model for an electron on this base.

Provided, a wave packet is given and belonging to a free electron. Then this assumption is only an illusion. In reality the electron is indivisibly coupled to its own electromagnetic field. Hence interaction takes place, which may be considered, too, as the self interaction of the whole system. Within perturbation theory the electron is surrounding itself with a cloud of virtual photons, electrons and positrons, the expression 'virtual particle', of cause, to be read as 'scattering wave'. In a further step one can consider the originally given electron as one of the virtual particles. Under the assumption that the whole object proves to be stable even beyond perturbation theory, one has a model for a really existing electron. It is free in the sense that it does not interact with other particles. But it is not free in so far, as it does not yield the homogeneous part of a field equation.

3.3. Inferences

The draft of the last section is speculative insofar, as it, like in almost all other cases with interaction, cannot be assured by a concrete calculation. Nevertheless it seems to be a plausible alternative to the concept of particle in the Copenhagen interpretation of quantum mechanics. If the ansatz is true, it describes a stable wave packet being a model for a particle with finite extension. Its magnitude might be of the order of a Compton wave length. According to Thirring [7] this length is marking the region, in which further location is impossible. The attempt to confine an electron even more would afford such high energies that the particle would be destroyed. "...the Compton wave length of the electron ... is the smallest size within which the electron can be compressed." In a region free from external potentials such a particle can move uniform and rectilinear without dissipation. It therefore would be the 'really existing particle' the experimentally oriented physicists are working with.

Interaction between two particles is taking place by penetration of the clouds of virtual particles. But the detail of this process is not observable on principle, for otherwise the impact of a measuring device would imply that a process between three participants would take place instead with the two partners of the process to be measured. As one can see, here, too, the important idea of quantum theory appears, stating that the influence of a measuring process on a quantum object must not be neglected. In a field theory as, for instance wave mechanics, it is particularly impressive, for in a world described by it all is consisting of waves, and waves cannot be grasped by waves with arbitrary precision.

Hence scattering processes are dependent on chance. Thus for the directions, into which the outgoing particles are moving, provided the ingoing particles are given, only a probability density can be given. Since the process is an interaction between waves, the Feynman integrals deliver probability amplitudes, the norms of which are the probabilities.

Diffraction at an obstacle is possible, if the extend of the obstacle is of the same order as the size of the particles. For this reason diffraction of electrons is only possible at the lattices of crystals. At this order of magnitude the crystals themselves are composed of particles, and that is to say of waves. Hence the diffraction of particles at the lattices of crystals may be considered as a case of interaction.

The uncertainty relations don't need any further discussion, because they already are valid in classical physics. The latitude of a wave packet is reciprocal proportional to the extension of the corresponding frequency range.

4. Perturbation theory

4.1 Solution of the field equation

Field theories usually are characterized by Lagrangian densities. The corresponding field equations can be derived from them as the Euler-Lagrangian equations. This process - as may be remarked by the way – can be reverted by the theorem of Darboux, reported in Bolza [12, p.37]. Any common or partial differential equation of at most second order may be considered as the Euler-Lagrangian equation of a suited Lagrangian function or density.

In the following a field equation may be given or even a system of coupled differential equations as for instance the Dirac-Maxwell system of equations. If only one single field equation is given, it shall have the form

$$D\varphi(x) = gP(\varphi(x)) \qquad x = x^\mu \qquad 0 \leq \mu \leq 3$$

with an at most quadratic function D of differential operators and a polynomial P of the functional values $\varphi(x)$. In this case a propagator function always can be found in the well known way by changing to the Fourier representation and then applying the theorem of Cauchy. All that, what must be done else, shall be demonstrated by the example of a model with the equation

$$(\partial_\mu \partial^\mu + m^2)\varphi(x) = g\varphi(x)^2 \tag{3}$$

This model is unrealistic, because it has no minimal energy. But it is suited to demonstrate the applied methods.

The propagator function Δ_F is satisfying the equation

$$(\partial_\mu \partial^\mu + m^2)\Delta_F(x) = \delta^{(4)}(x) \tag{4}$$

By help of it the field equation (3) can be formally integrated with the result

$$\varphi(x) = \varphi_0(x) + g \int dx \Delta_F(x-y)\varphi(y)^2 \tag{5}$$

Equation (5) can be represented diagrammatically by a small tree. The root is a point representing $\varphi(x)$ and the trunk above it a line going from x to y and representing $\Delta_F(x-y)$. The point y can serve as a common starting point for two branches, playing the same role for each of the two factors $\varphi(y)$ as the trunk does for $\varphi(x)$.

On the other hand the value $\varphi(x)$ of the function φ can be expanded into a Taylor series according to the coupling constant g with the result

$$\varphi(x) = \sum_{n=0}^{\infty} g^n \varphi_n(x) \tag{6}$$

Substituting $\varphi(x)$ in equation (5) by the right hand side of equation (6), doing the same for $\varphi(y)$, and comparing coefficients will give the recursion formula

$$\varphi_n(x) = \sum_{i=0}^{n-1} \int dy \Delta_F(x-y)\varphi_i(y)\varphi_{n-i-1}(y) \qquad n > 0 \tag{7}$$

The recursion formula can be iterated arbitrarily by a process that is ending at approximations of zeroth order. In graphical language this would result in a tree graph growing out of the small tree for equation (5) by growing and iterated branching. The recursion procedure will end with sums of products of free particle states φ_0^1, φ_0^2,...,φ_0^r and propagators.

Example: The approximations up to third order

order 0:

For the following calculation eight solutions φ_0^1, φ_0^2,...,φ_0^8 of the homogenous field equation are needed.

order 1:

(a) $\varphi_1'(y) = \int dy_1 \Delta_F(y-y_1)\varphi_0^1(y_1)\varphi_0^2(y_1)$

For the calculation also a variation of this contribution is needed. It is

(b) $\varphi_1''(y) = \int dy_2 \Delta_F(y-y_2)\varphi_0^3(y_2)\varphi_0^4(y_2)$

order 2:

(c) $\varphi_2(x) = \varphi_2'(x) + \varphi_2''(x)$

(d) $\varphi_2'(x) = \int dy \Delta_F(x-y)\varphi_1'(y)\varphi_0^5(y)$

(e) $\varphi_2''(x) = \int dy \Delta_F(x-y)\varphi_0^6(y)\varphi_1''(y)$

order 3:

(f) $\varphi_3(x) = \varphi_3'(x) + \varphi_3''(x) + \varphi_3'''(x)$

(g) $\varphi_3'(x) = \int dy \Delta_F(x-y)\varphi_0^7(y)\varphi_2'(y)$

(h) $\varphi_3''(x) = \int dy \Delta_F(x-y)\varphi_1'(y)\varphi_1''(y)$

(i) $\varphi_3'''(x) = \int dy \Delta_F(x-y)\varphi_2''(y)\varphi_0^8(y)$

Finally the expressions must be plugged in recursively. For order 1 this already is done. For order 2 one will get

(d) $\varphi_2'(x) = \int dy dy_1 \Delta_F(x-y)\Delta_F(y-y_1)\varphi_0^1(y_1)\varphi_0^2(y_1)\varphi_0^5(y)$

(e) $\varphi_2''(yx) = \int dy dy_2 \Delta_F(x-y)\Delta_F(y-y_2)\varphi_0^3(y_2)\varphi_0^4(y_2)\varphi_0^6(y)$

For order 3 only the symmetrical part (h) shall be written down. The result is

(h) $\varphi_3'' = \int dy y_1 y_2 \Delta_F(x-y)\Delta_F(y-y_1)\Delta_F(y-y_2)\varphi_0^1(y_1)\varphi_0^2(y_1)\varphi_0^3(y_2)\varphi_0^4(y_2)$

The recursive procedure can be generalized to other field equations and to systems of such equations, as for instance the Dirac-Maxwell system

$$\begin{aligned} i\gamma^\mu\partial_\mu\psi(x) - m\psi(x) &= e\gamma^\mu A_\mu(x)\psi(x) \\ -i\partial_\mu\bar{\psi}(x)\gamma^\mu - m\bar{\psi}(x) &= e\bar{\psi}(x)\gamma^\mu A_\mu(x) \\ \partial_\mu\partial^\mu A^\nu(x) &= e\bar{\psi}(x)\gamma^\nu\psi(x) \end{aligned}$$

of equations for the three fields ψ, $\bar{\psi}$ und A.

4.2 Analysis of scattering processes

In a scattering process particles are generated and prepared in suited sources. They meet in a small region of the four dimensional space and interact there. Afterwards other particles leave the place of their emergence. They are registered and measured in detectors.

A total clearing up, of what exactly is happening during the phase of interaction, is neither theoretically nor experimentally possible. Therefore one generally confines oneself to justify the Feynman rules within the frame of perturbation theory. It is true that these rules are not correctly derived in the usual version of quantum field theory, but de facto they are true. For this reason they shall be derived in the following only by means of classical field theory.

It should be remarked already here that no reasons can be given for the correct choice of the relative phase in case of exchange terms. But that is no special deficiency of the present ansatz. The usual deduction of the so called quantum statistics has been revealed as faulty in section 2. Perhaps it cannot be justified at all, and must be taken as empirical knowledge.

The proof of the Feynman rules is given in two steps, in both cases by induction. First of all the tree graphs are derived and in a second step the diagrams with loops.

As means for the solution of this task only the recursion formula

$$\varphi_n(x) = \sum_{i=0}^{n-1} \int dy \Delta_F(x-y)\varphi_i(y)\varphi_{n-i-1}(y) \qquad n > 0 \tag{8}$$

of the previous section shall be used and the representation

$$\Delta_F = -i\int \frac{d^3k}{(2\pi)^3 2\omega_k}[\theta(x^0-y^0)e^{-ik(x-y)} + \theta(y^0-x^0)e^{ik(x-y)}] \tag{9}$$

of the propagator function, defined as a solution of equation (4).

In order to get the Feynman integrals some arrangements are to be made.

First of all the numerical factors and the combinatorial factors are left away. Moreover in this section the two concepts of 'Feynman diagram' and 'Feynman integral' are treated synonymously.

The multiple application of the recursion formula in the last section has created untruncated tree graphs with one special line and n vertices y_1 to y_n. These vertices shall be denoted as inner vertices. They are supplemented by r external vertices x_1 to x_r, which are far away from the center of interaction in four dimensional space. Every external vertex x_k, with the exception of x_m, is connected with an inner vertex y_i by a line charged with a propagator $\Delta_F(y_i - x_k)$ according to the equation

$$\varphi_0^i(y_i) = \int dx_k \Delta_F(y_i - x_k) u^k(x_k) \tag{10}$$

The function u^k is a solution of the homogeneous field equation and attached to the vertex x_k.

Physically this means that a scattering wave is coming from a vertex in the remote past and arriving at the vertex y_i, where it is participating in the interactions. Visa versa from a vertex y_j a scattering wave can be spread and will arrive at a vertex x_l in the far future. By this arrangement the ingoing and outgoing particles of a scattering process are described.

As a further measure the inner product of φ_n is taken with the solution u^m that is belonging to the external vertex x_m. Then a contribution

$$S_{n,m} = \int dx \varphi_n(x) u^m(x) \tag{11}$$

to the S-matrix will appear.

The result of these arrangements consists in non truncated tree graphs with no external line to be distinguished.

The question, whether the procedure delivers all tree graphs, will be cleared by induction. The vertex part as the only diagram of order one is a tree graph. Hence it can be taken as the anchoring of the induction. If, in the sense of induction hypothesis, the two factors $\varphi_i(y)$ and $\varphi_{n-i-1}(y)$ in the sum of equation (8) are already containing all tree graphs of lower order, then the sum itself is running over all tree graphs of order n. This is true, because every tree graph can be - eventually several times – represented by a contribution of equation (8).

In order to get contributions for diagrams with loops the tree graphs with at least two external lines are chosen as anchoring for the induction. This choice is possible, because every tree graph of at least second order has at least four external lines.

Provided that a graph with $r + 2$ external lines and l loops is given in the sense of the induction assumption. Then to every pair (y_i, y_j) of inner vertices, at which external lines are ending, a new graph shall be formed such that the two external lines at y_i and y_j are removed and substituted by an inner line between y_i and y_j charged with the propagator $\Delta_F(y_i - y_j)$.

For the time being this is merely a mathematical manipulation. What is the physical sense of it?

If, as an example, x_k is far away in the past and x_l is far away in the future, and if $y_i < y_j$, then the scattering wave, going out to x_l and participating at the interaction place y_j, is cancelled and substituted by a scattering wave coming from y_i. At y_i the interaction with the scattering wave coming from x_k is dropped and substituted by the interaction with the scattering wave going to y_j. Things are similar in case of $y_i > y_j$. Thus a new diagram emerges that has one loop more than the original graph. It is independent of the permutations of the different times. Hence the second proof by induction is completed.

By this procedure the same Feynman graphs occur as usual. For one can organize the building up of these graphs by exploiting the formulae (1) and (2) of section 2 according to the same building plan, as was used here.

Example : A graph of order 3 with a loop

It is sufficient to take the symmetrical graph (h) of the example of the last secton, because the other two graphs will give similar results. In order to compare the result with the corresponding expression coming out by the usual methods some arrangements are to be made. The graph (h) of example 1 is not truncated. But for the comparison it should be truncated. This is achieved by setting

$$\begin{aligned}\varphi_0^1(y_1) &= \int dx_1 \Delta_F(y_1 - x_1) u^1(x_1)\\ \varphi_0^3(y_1) &= \int dx_3 \Delta_F(y_1 - x_3) u^3(x_3)\\ \varphi_0^2(y_2) &= \int dx_2 \Delta_F(y_2 - x_2) u^2(x_2)\\ \varphi_0^4(y_2) &= \int dx_4 \Delta_F(y_2 - x_4) u^4(x_4)\end{aligned}$$

with the four solutions u^1, u^2, u^3, u^4 of the homogeneous field equation.

The inner product of $\varphi_3''(x)$ with u^5 is giving

$$B_3 = \int dx \varphi_3''(x) u^5(x) = \int dx dy \Delta_F(x-y) \varphi_1'(y) \varphi_1''(y) u^5(y)$$

Now x_3 may be situated in the past and far away from the scattering center, while x_2 may be situated in the future and far away from the scattering center, too. Then the interaction of the scattering wave u^3 coming from x_3 and arriving at y_1 is cancelled as well as the the interaction of the scattering wave u^2 going out from y_2 and arriving at x_2. Instead of the cancelled terms the propagator $\Delta_F(y_1 - y_2)$ is implemented. If $y_1 < y_2$, it describes a scattering wave spreading from y_1 to y_2. In case of $y_1 > y_2$ the wave is spreading from y_2 to y_1. Thus one has the element

$$\begin{aligned}B_3' = \int & dx dy dx_1 dx_4 dy_1 dy_2 \Delta_F(x-y) \Delta_F(y_1 - x_1) \Delta_F(y_2 - x_4)\\ & \times \Delta_F(y - y_1) \Delta_F(y - y_2) \Delta_F(y_1 - y_2) u^1(x_1) u^4(x_4) u^5(x)\end{aligned}$$

of the S-Matrix up to ugly combinatorial factors.The truncated version is

$$M_3 = \int dy y_1 y_2 \Delta_F(y - y_1) \Delta_F(y - y_2) \Delta_F(y_1 - y_2) u^1(x_1) u^4(x_4) u^5(x)$$

The corresponding expression according to the usual methods is the integral over the product of the contractions

$$(x, y), (y_1, x_1), (y_2, x_4), (y, y_1), (y, y_2), (y_1, y_2)$$

in the expression

$$P_{3,3} = \phi(x)\phi(x_1)\phi(x_4) \int dy dy_1 dy_2 \mathcal{H}(y)\mathcal{H}(y_1)\mathcal{H}(y_2) \qquad \mathcal{H}(y) = \phi(y)^3$$

The result is equal to M_3. The twelve factors ϕ, inherent in the expression $P_{3,3}$, are totally consumed by the six contractions.

5. Intermediate result

The extension of wave mechanics to quantum field theory consists in four steps.

1. In a first step wave mechanics is tacitly raised onto the level of a relativistic theory.
2. The second step is essential. It consists in the inclusion of interaction.
3. An ansatz is given for the old problem to describe elementary particles by stable wave packets.
4. The solution of the field equation and the analysis of scattering processes are possible by means of classical field theory in the frame of classical field theory. But the prescription for the relative phase, if exchange terms occur, can only be stated as an empirically assured rule.

In order to distinguish the result of this construction from the usual version of quantum field theory it shall be called quantum wave theory.

6. Is quantum wave theory a field theory or a quantum theory?

A field shall be defined as an array of complex functions having the time and three spatial coordinates as their arguments. Since all concepts of the theory, including that of an elementary particle, are reduced to the concept of field, the answer is clear.

Quantum wave theory is a field theory.

But that is not all.

Quantum wave theory is also a field theory.

In order to justify this assertion first of all the concept of quantum must be cleared. There are two different kinds of definition.

On the one hand the eigenvalues of differential equations having discrete spectra of eigenvalues are considered to be quanta, so for instance the eigenvalues for the equation of the harmonic oscillator. But that shall not be adopted here for two reasons. First of all the Schrödinger equation, like similar equations, meanwhile is judged to be classical field equation. Secondly already in classical electrodynamics the expansion of the potentials into multipoles contains 'quantized' angular momenta. In another version stable elementary particles are considered to be quanta. That seems to be a reasonable concept and shall be adopted here.

Quantum wave theory is a quantum theory for two reasons. First of all the stable wave packets as the elementary particles of extended wave mechanics are clearly separated objects of the micro world and hence quanta. Moreover, for them an essential insight of quantum mechanics is valid: The interaction between quantum objects and a measuring device must not be neglected. Any measurement is disturbing the object to be measured and by this impact it can destroy the results of other measurements.

By the way: in a recent publication (Scientific American, August 2014, 29-33, p. 32) the existence of gravitational waves is celebrated as a proof that the general theory of gravitation is a quantum theory. According to this sentence already classical electrodynamics would be a quantum theory, because there are electromagnetic waves in it.

7. Summary

The task was to reconstruct the essential parts of quantum field theory and for this purpose to take wave mechanics as a starting point. The reason for this endeavour was the estimation that the present state of quantum field theory is not acceptable. One essential point of the extension consists in adequately respecting the role of interaction, another one in a new attack to the old problem to describe elementary particles by stable wave packets. But this time the attempt was made with all means nowadays being available. Since the propagator function already is given by the homogeneous part of the field equation, classical field theory reveals to be sufficient for solving the field equation with interaction term and to derive the Feynman rules, both in the frame of perturbation theory. But the rule for treating exchange terms cannot be deduced in either version of quantum field theory. The result of the present construction shall be called quantum wave theory. It reveals to be both, a field theory and a quantum theory.

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Appendix: Analysis of the LSZ-reduction

Survey and preparation of the calculation

The LSZ reduction is a procedure, by which expectation values for the transition between two multiple particle states are reduced to vacuum expectation values of time ordered products. In the original paper [8] this task is solved for a Klein-Gordon-field. But the problematic nature of this project can as well be demonstrated by special examples and for the Schrödinger equation in one spatial dimension. If this is done by a profound calculation, then it is easy to generalize the examples and to transfer them to other field equations.

Two remarks may be made in order to prepare the following calculation.

First of all the state $|p_{ein}>$ of an ingoing particle with the momentum p_{ein} is created out of the vacuum state $|0>$ by an operator $a_{ein}(p)$, which has the form

$$a_{ein}(p) = \int_{-\infty}^{+\infty} dx f^p(t,x) \varphi_{ein}(t,x)$$

with the operator φ_{ein} for the ingoing field and a solution f^p of the homogeneous Schrödinger equation

$$\frac{\partial}{\partial t} f(t,x) = \frac{i\hbar}{2m} \frac{\partial^2}{\partial x^2} f(t,x)$$

Secondly the asymptotic condition is granting the existence of the two limits

$$\lim_{t \to -\infty} < \phi|\varphi(t,x)|\psi > = < \phi|\varphi_{ein}(t,x)|\psi >$$

$$\lim_{t \to -+\infty} < \phi|\varphi(t,x)|\psi > = < \phi|\varphi_{aus}(t,x)|\psi >$$

for arbitrary states ϕ and ψ and the operator φ. Normalization factors are left away.

First example:

reduction of p'_{ein} in $< k|p'_{ein} >$ by a sequence of steps

First of all the state $|p'_{in} >$ is created out of the vacuum state $|0>$

$< k|p'_{ein} >=< k|a_{ein}(p')|0 >$

enlarging according to the scheme $a = b - (b - a)$

$< k|p'_{ein} >=< k|a_{aus}(p')|0 > -(< k|a_{aus}(p') - a_{ein}(p')|0 >)$

The first term is vanishing

$< k|p'_{ein} >= -(< k|a_{aus}(p') - a_{ein}(p')|0 >)$

smearing out with a test function

$< k|p'_{ein} >= - < k| \int_{-\infty}^{+\infty} dx'(\varphi'_{aus}(t', x') - \varphi_{ein}(t', x'))f^{p'}(t', x')|0 >$

application of the asymptotic condition

$< k|p'_{ein} >= - < k| \int_{-\infty}^{+\infty} dx' \lim_{t\to+\infty} \varphi'(t', x')f^{p'}(t, x)|0 >$

$+ < k| \int_{-\infty}^{+\infty} dx' \lim_{t\to-\infty} \varphi'(t', x')f^{p'}(t', x')|0 >$

combining the two terms

$< k|p'_{ein} >= - < k| \int_{-\infty}^{+\infty} dx'(\lim_{t'\to+\infty} - \lim_{t'\to-\infty})\varphi'(t', x')f^{p'}(t', x')|0 >$

transformation according to the scheme $f(b) - f(a) = \int_a^b dt' \frac{\partial}{\partial t'} f(t', x')dx'$

giving the intermediate result

$< k|p'_{ein} >= - < k| \int_{-\infty}^{+\infty} dx' \int_{-\infty}^{+\infty} dt' \frac{\partial}{\partial t'}(\varphi'(t', x')f^{p'}(t', x'))|0 >$

The derivative of the product is

$\frac{\partial}{\partial t'}(\varphi'(t', x')f^{p'}(t', x')) = (\frac{\partial}{\partial t'}\varphi'(t', x'))f^{p'}(t', x') + \varphi'(t', x')(\frac{\partial}{\partial t}f^{p'}(t', x'))$

Since $f^{p'}$ is a solution of the homogeneous Schrödinger equation, one has

$\frac{\partial}{\partial t'}f^{p'}(t', x') = \frac{i\hbar}{2m}\frac{\partial^2}{\partial x'^2}f^{p'}(t', x')$

and hence

$\frac{\partial}{\partial t'}(\varphi'(t', x')f^{p'}(t', x')) = (\frac{\partial}{\partial t'}\varphi'(t', x'))f^{p'}(t', x') + \frac{i\hbar}{2m}\varphi'(t', x')(\frac{\partial^2}{\partial x'^2}f^{p'}(t', x'))$

After plugging in according to this equation and integrating twice according to x with the boundary terms vanishing, one will get the result

$< k|p'e_{in} >= - \int_{-\infty}^{+\infty} dt' \int_{-\infty}^{+\infty} dx' \; f^{p'}(t', x')(\frac{\partial}{\partial t'} + \frac{i\hbar}{2m}\frac{\partial^2}{\partial x'^2}) < k|\varphi'(t', x')|0 >$

Second example:

transition of $< k|p'_{ein}p_{ein} >$ into a vacuum expectation value

with p_{ein} belonging to φ and p'_{ein} to φ'. By the same procedure as in the first example p'_{ein} can be eliminated. The beginning of the transformation is

$$< k|p'_{ein}p_{ein} >=< k|a_{ein}(p')|p_{ein} >$$

After a series of steps that are corresponding to those of the first example one will get

$$< k|p'_{ein}p_{ein} >= -\int_{-\infty}^{+\infty} dt' \int_{-\infty}^{+\infty} dx'\ f^{p'}(t',x')(\frac{\partial}{\partial t'} + \frac{i\hbar}{2m}\frac{\partial^2}{\partial x'^2}) < k|\varphi'(t',x')|p_{ein} >$$

The reduction of $< k|\varphi(t',x')|p_{ein} >$, too, can be carried out as in the first example. First of all the state $|p'_{in} >$ is created out of the vacuum state $|0 >$with the result

$$< k|\varphi(t',x')|p_{ein} > \quad =< k|\varphi(t',x')a_{ein}(p)|0 >$$

enlarging according to the scheme $a = b - (b - a)$

$$< k|\varphi(t',x')|p_{ein} >=< k|a_{aus}(p)\varphi(t',x')|0 > - < k|(a_{aus}(p)\varphi(t',x') - \varphi(t,x)a_{ein}(p))|0 >$$

After some steps one will arrive at

$$< k|\varphi(t',x')|p_{ein} >= - < k| \int_{-\infty}^{+\infty} dx \lim_{t\to+\infty} \varphi(t,x)\varphi'(t',x')f^{p}(t,x)|0 >$$
$$+ < k| \int_{-\infty}^{+\infty} dx' \lim_{t\to-\infty} \varphi'(t',x')\varphi(t,x)f^{p'}(t,x)|0 >$$

At this place the time ordered product is coming in. With the definition

$$Z(\varphi,\varphi') = \theta(t-t')\ \varphi(t,x)\varphi'(t',x') - \theta(t'-t)\ \varphi'(t',x')\varphi(t,x)$$

which intentionally is written $Z(x,y)$ instead of $T(x,y)$, one will get

after some immediate steps the result

$$< k|\varphi(t',x')|p_{ein} > \quad =< k| \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dt(\frac{\partial}{\partial t}(Z(\varphi(t,x)\varphi'(t',x'))f^{p}(t,x)|0 >$$

and then the final result

$$< 0|p_{ein}p'_{ein} >= \int_{-\infty}^{+\infty} dt \int_{-\infty}^{+\infty} dt' \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dx'$$
$$f^{p}(t,x)f^{p'}(t',x')(\frac{\partial}{\partial t} + \frac{i\hbar}{2m}\frac{\partial^2}{\partial x^2})(\frac{\partial}{\partial t'} + \frac{i\hbar}{2m}\frac{\partial^2}{\partial x'^2}) < k|Z(\varphi(t,x)\varphi'(t',x')|0 >$$

Thus the expectation value $< k|p'_{ein}p_{ein} >$ is reduced to the vacuum expectation value of a time ordered product.

Third example:

Transition of $< p'_{aus}|p_{ein} >$ into a vacuum expectation value

In this case first of all the state $< p'_{aus}|$ is created according to

$< p'_{aus}|p_{ein} >=< 0|a_{aus}(p')|p_{ein} >$

This time the enlargement is following the scheme $a = b + (a - b)$ and thus

$< p'_{aus}|p_{ein} >=< 0|a_{ein}(p')|p_{ein} > + < 0|a_{aus}(p') - a_{ein}(p')|p_{ein} >$

The first term is vanishing and it remains

$< p'_{aus}|p_{ein} >=< 0|a_{aus}(p') - a_{ein}(p')|p_{ein} >$

As in the first example after some intermediate steps the operator between the two states will deliver a result with the operator $\varphi'(t', x')$. The evaluation of $< 0|\varphi'(t', x')|p_{ein} >$ will then bring out the time ordered product as in the second example.

Generalization and Transition to other fields

The three examples for the transition of an expectation value into a vacuum expectation value can easily be generalized to the case of two arbitrary multi particle states.

For a neutral scalar field modifications are to be done in connection with the Klein-Gordon equation. The smearing out with a test function f is done by the transition from $\varphi(t, x)$ to $\varphi^f(t, x)$, define by

$$\varphi^f(t,x) = \int_{-\infty}^{+\infty} dx \; (f(t,x)\frac{1}{c}\frac{\partial}{\partial t}\varphi(t,x) - \varphi(t,x)\frac{1}{c}\frac{\partial}{\partial t}f(t,x))$$

Because of the somewhat strange rule of derivation

$$\begin{aligned}
\frac{\partial}{\partial t}(f\frac{1}{c}\frac{\partial}{\partial t}\varphi - \varphi\frac{1}{c}\frac{\partial}{\partial t}f) &= f\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\varphi - \varphi\frac{1}{c^2}\frac{\partial^2}{\partial t^2}f \\
&= f\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\varphi - \varphi(\frac{\partial^2}{\partial x^2} - m^2)f \\
&= f\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\varphi - f(\frac{\partial^2}{\partial x^2} - m^2)\varphi + d \\
&= f(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} + m^2)\varphi + d \\
\frac{\partial}{\partial t}(f\frac{1}{c}\frac{\partial}{\partial t}\varphi - \varphi\frac{1}{c}\frac{\partial}{\partial t}f) &= f(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} + m^2)\varphi + d
\end{aligned}$$

with terms vanishing by the partial integration, in this case one has

$$(\lim_{t\to+\infty} - \lim_{t\to-\infty})\varphi^f(t,x) = \int_{-\infty}^{+\infty} dt\frac{1}{c}\frac{\partial}{\partial t}\varphi^f(t,x) = \int_{-\infty}^{+\infty} dt \int_{-\infty}^{+\infty} dx \; f(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} + m^2)\varphi$$

The result is corresponding to the Schrödinger equation, but with other differential operators.

For three spatial dimensions one has to select a test function satisfying the equation

$$(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2} - \frac{\partial^2}{\partial x_3^2} + m^2)f(t, x_1, x_2, x_3) = 0$$

and to extend the partial integration to all three spatial derivatives. In order to get expectation values for multi particle states one has to carry out the reduction for each particle separately. For the scalar field the result is

$$< q_1...q_m|p_1...p_n >= (-1)^{m-n}\int\prod_{i=1}^{m}d^4x_i\prod_{j=1}^{n}d^4y_j f_{p_i}(x_i)f^*_{q_j}(y_j)$$

$$\times(\Box_{x_i} + m^2)(\Box_{y_j} + m^2) < 0|T\{\varphi^*(x_1)...\varphi(y_m)^*\varphi(y_1)...\varphi(y_n)\}|0 >$$

for all $p_i \neq q_j$, where again normalization constants were left away.

If the homogeneous Dirac equation

$$(i\gamma^0\frac{1}{c}\frac{\partial}{\partial t} - i\gamma^i\partial_i - m)\psi = 0$$

is multiplied with $\frac{c}{i}\gamma^0$ and then written down for the coordinates a system

$$(\frac{\partial}{\partial t} + a^j\frac{\partial}{\partial x^j} + b)\psi_i = 0$$

of four linear equations will occur. For each of them one can proceed as in the former examples.