Clock Paradox and the Absolute Reference Frame

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Abstract

In this paper it is shown that the well-publicized claim about the Special Relativity Theory, which states that it is not possible, using only arguments contained within the theory, to detect an absolute motion. This is one of the fundamental conclusions of the theory and it is usually considered in connection with the so called Clock Paradox or the Twins' Paradox. There have been many papers published on this topic with the various proposed resolutions, however, the controversy related to these paradoxes continues to this day. This paper will clearly show that the resolution of the Clock Paradox is related to the clock synchronization procedure and that by introducing an absolute reference frame the paradox can be simply resolved. In addition it will be shown that it is possible to detect the absolute motion of particular reference frames relative to this absolute reference frame.

Introduction

To avoid possible misunderstandings and to refresh the definitions of the terms that will be used throughout this paper this section will start with the definition of the well-known Lorentz coordinate transformation [1]. This transformation relates the spatial and temporal coordinates between the laboratory coordinate system, which is considered not moving, and the moving coordinate system. The moving coordinate system variables will be designated for a moment as the primed variables. It is therefore well-known that the following relations hold:

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}, \tag{1}$$

$$t' = \frac{t - xv/c^2}{\sqrt{1 - v^2/c^2}}, \tag{2}$$

where v is the velocity measured in the laboratory coordinate system (t,x) and where c is the speed of light. For the purpose of studying the Clock Paradox (CP) and the Twin Paradox (TP), it is necessary to derive the relation between the stationary clocks in the moving coordinate system and the stationary clocks in the laboratory coordinate system. In order to simplify the derivations only the differentials of variables in question will be used. For the stationary clocks time interval in the moving coordinate system it is thus possible to write using Eq. 2 the following:

$$dt'_{x'=const} = \frac{dt - (dx)v/c^2}{\sqrt{1 - v^2/c^2}}. \tag{3}$$

From Eq.1 then follows that for x' = constant it holds that: $dx = vdt$. Substituting this relation into Eq.3 the following result is obtained:

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$$dt = \frac{dt'_{x'=const}}{\sqrt{1-v^2/c^2}}. \tag{4}$$

This is the famous time dilation effect formula of Special Relativity Theory (SRT). Here it is necessary to understand, however, and is strongly emphasized that the observed time differential *dt* is an apparent time dilation observed only in the laboratory coordinate system and that the actual time differential *dt'* in the moving coordinate system stays the same and is not changed. To simplify notation and to underscore that the time differential in the moving coordinate system is an invariant, sometimes called the proper time differential, this parameter will be designated as is customary by the variable $d\tau$. In most SRT publications it is thus typically found the following relation:

$$d\tau = dt\sqrt{1-v^2/c^2} \ . \tag{5}$$

Multiple coordinate systems moving in the laboratory reference frame.

There are several version of the CP that can be found in the literature [2]. The simplest description is obtained by considering the time dilation effect between the two independently moving coordinate systems in the laboratory. Assuming that their relative velocity is *u*, the time dilation of clocks placed in the first coordinate system and observed in the second coordinate system is:

$$dt_1 = d\tau / \sqrt{1-u^2/c^2} \ . \tag{6}$$

However, it is also typically claimed that according to SRT relativity principle for the time dilation of clocks placed in the second coordinate system and observed in the first coordinate system must follow the same formula:

$$dt_2 = d\tau / \sqrt{1-u^2/c^2} \ . \tag{7}$$

It is thus claimed that the clocks in the respective coordinate systems cannot be mutually delayed respective to each other and this results in a paradox. This contradiction is the essence of the CP and it has been resolved in many publications in various ways sometimes using a very contorted reasoning [3]. To find the logical fallacy in the above reasoning is very simple. When the observer is moved from the one coordinate system to the other the corresponding invariant $d\tau$ cannot remain the same. The invariant $d\tau$ was established only for the laboratory reference frame and when the observer's reference frame is moved to the first or to the second coordinate system the corresponding invariant of each coordinate reference frame must change.

The Clock Paradox Resolution

The paradox faulty reasoning is best shown explicitly by writing down the details of the time dilation effects in reference to the laboratory coordinate system using the coordinate velocities of the respective systems. Referencing clocks to the laboratory coordinate system thus provides the necessary clock synchronization. For the first coordinate system clocks it is:

$$d\tau = dt_1\sqrt{1-v_1^2/c^2} \ , \tag{8}$$

and for the second coordinate system with the same clock differential $d\tau$ it is:

$$d\tau = dt_2\sqrt{1-v_2^2/c^2} \ . \tag{9}$$

For the mutual velocity u of the systems then holds, according to SRT velocity composition formula, the following well known relation:

$$u = \frac{v_1 - v_2}{1 - v_1 v_2 / c^2}. \tag{10}$$

It is now simple, for example, to select the coordinate system moving with the velocity v_2 as a reference system and use Eq.8 to express the observed time dilation of clocks that are at rest in the reference system moving with the velocity v_1.

$$dt_{12} = \frac{d\tau}{\sqrt{1 - v_2^2 / c^2}\sqrt{1 - u^2 / c^2}} = \frac{dt_2}{\sqrt{1 - u^2 / c^2}}. \tag{11}$$

The relation in Eq.11 is justified by the fact that when $u = 0$ the $dt_{12} = dt_2$. It is thus clear by comparing this result with Eq.6 that the new invariant $d\tau$ that corresponds to the selected reference coordinate system moving with the velocity v_2 is as follows:

$$d\tau_2 = dt_2. \tag{12}$$

This can be rewritten in a well-known customary form as:

$$dt_{12} = \frac{d\tau_2}{\sqrt{1 - u^2 / c^2}}. \tag{13}$$

This relation is further justified by the fact that the parameter $d\tau_2$ depends only on the velocity v_2, which is constant in that reference frame. Similarly as in Eq.13 it is now also simple to find the time dilation effect observed from the coordinate system moving with the velocity v_1:

$$dt_{21} = \frac{dt_1}{\sqrt{1 - u^2 / c^2}}. \tag{14}$$

These are interesting relations clearly indicating that Eq.6 and Eq.7 are not correct and that the CP actually does not exists because it generally holds that $dt_1 \neq dt_2$.

The Twin Paradox resolution

The above described procedure can now be extended also to resolve the Twin Paradox. This is accomplished by introducing the two more moving coordinate systems into the laboratory reference frame. These systems represent the returning paths for the observers. Repeating the previously described steps it is simple, after some algebra, to derive the following relations:

$$dt_{12} + dt_{32} = \frac{2dt_2}{\sqrt{1 - u^2 / c^2}}, \tag{15}$$

$$dt_{21} + dt_{41} = \frac{2dt_1}{\sqrt{1 - u^2 / c^2}}. \tag{16}$$

More details of derivation of formulas in Eq.15 and Eq.16 are given in the Appendix. From these relations one can then observe that each twin will return younger to the origin of their respective journeys, but with a different amount of aging. The difference depends on their original relative velocities as referenced to the laboratory coordinate system. It is thus clear that the laboratory coordinate system serves here as an absolute reference frame and allows providing the correct clocks synchronization. There is no acceleration effect necessary to be considered during the each twin turn around and the resolution of the paradoxes is obtained within the framework of the standard SRT. The only difference here is the removal of the standard conclusion that within SRT and with the inertial motion requirement one cannot determine the absolute motion relative to an absolute coordinate reference frame. This, generally accepted claim, is therefore false. This finding is interesting, because the respective differences in the twins' aging can now be used to determine the absolute velocities of respective moving coordinate systems.

Absolute reference frame

By introducing the following coefficient α:

$$\alpha = \frac{dt_{12} + dt_{32}}{dt_{21} + dt_{41}} = \frac{\sqrt{1 - v_1^2 / c^2}}{\sqrt{1 - v_2^2 / c^2}} . \tag{17}$$

Eq.17 can be solved with the help of Eq.10 and the following expression for the velocity v_1 can be obtained as is shown in the Appendix:

$$v_1 \approx \frac{c^2}{u}(1 - \alpha) + \frac{\alpha}{2} u \ . \tag{18}$$

Similar equation can also be obtained for the velocity v_2, or Eq.10 can be used to calculate the velocity v_2 after the velocity v_1 has been found when α has been measured. It is thus clear that the difference in aging of returning twins is an important parameter determining their velocities in reference to an absolute reference frame.

To a traditional researcher working with SRT it might be surprising that it is possible to find the absolute reference frame and the coordinate system velocities relative to this frame. The main reason for this fact is that the composition of velocities in SRT follows a nonlinear function.

Final remarks

In many publications [4], for example, even in the original Einstein's attempt to resolve the CP [5], it is claimed that this cannot be achieved within the SRT. In particular it is claimed that in order for the travelling twin to return it is necessary that he undergoes acceleration and this resolves the problem pointed out by Eq.6 and Eq.7. This claim is also false, since the author of this paper has shown that on the rotating platforms SRT is cancelled by the centrifugal force and the spacetime of such platforms is flat as in the laboratory reference frame [6]. It is thus possible to employ such a rotating platform at the end of the travelling twin journey and send him on the return trip without any additional time dilation effect.

Another claim that is typically used is that it is wrong to use only the time differentials in the derivations. This also does not make sense, since it is easy to integrate the differentials over some fixed period of time and arrive at the same result.

It thus seems that the only reasonable solution to the CP is the introduction of the absolute reference frame that provides an easily understandable clock synchronization procedure.

Conclusions

In this paper it was clearly shown that there is no CP and TP paradox within SRT. This was shown by using the standard SRT arguments and by introducing a third laboratory, stationary, coordinate reference frame that provided the suitable and simple method for the moving clocks synchronization. It was also shown that it is possible to detect the absolute motion relative to this laboratory coordinate reference system and that this coordinate system is actually an absolute reference frame.

References

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Appendix

Helpful identity formula:

$$\left(1+v_2u/c^2\right)\left(1-v_1u/c^2\right)=1-u^2/c^2\,. \tag{A1}$$

Addition of velocities formulas:

$$v_1=\frac{v_2+u}{1+v_2u/c^2}\,,\qquad v_2=\frac{v_1-u}{1-v_1u/c^2}\,. \tag{A2}$$

$$v_3=\frac{v_2-u}{1-v_2u/c^2}\,,\qquad v_4=\frac{v_1+u}{1+v_1u/c^2}\,. \tag{A3}$$

The Twins' return trip addition:

$$dt_{12}+dt_{32}=\frac{d\tau\left(1+v_2u/c^2\right)}{\sqrt{1-u^2/c^2}\sqrt{1-v_2^2/c^2}}+\frac{d\tau\left(1-v_2u/c^2\right)}{\sqrt{1-u^2/c^2}\sqrt{1-v_2^2/c^2}}=\frac{2dt_2}{\sqrt{1-u^2/c^2}}\,. \tag{A4}$$

$$dt_{21}+dt_{41}=\frac{d\tau\left(1-v_1u/c^2\right)}{\sqrt{1-u^2/c^2}\sqrt{1-v_1^2/c^2}}+\frac{d\tau\left(1+v_1u/c^2\right)}{\sqrt{1-u^2/c^2}\sqrt{1-v_1^2/c^2}}=\frac{2dt_1}{\sqrt{1-u^2/c^2}}\,. \tag{A5}$$

Velocity relative to the absolute reference frame:

$$\alpha=\frac{\sqrt{1-v_1^2/c^2}}{\sqrt{1-v_2^2/c^2}}=\frac{\sqrt{1-v_1^2/c^2}\left(1-v_1u/c^2\right)}{\sqrt{1-v_1^2/c^2}\sqrt{1-u^2/c^2}}=\frac{1-v_1u/c^2}{\sqrt{1-u^2/c^2}}\,. \tag{A6}$$

An approximation for the small velocity in comparison to c:

$$v_1=\left(1-\alpha\sqrt{1-u^2/c^2}\right)c^2/u\approx\left(1-\alpha\right)\,c^2/u+u\alpha/2\,. \tag{A7}$$