Conjectures on Smarandache generalized Fermat numbers

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Abstract. In this paper I make few conjectures on few classes of generalized Fermat numbers, i.e. the numbers of the form F(k) = 2^(2^k) + n, where k is positive integer and n is an odd number, the numbers of the form F(k) = 4^(4^k) + 3 and the numbers of the form F(k) = m^(m^k) + n, where m + n = p, where p is prime, all subclasses of Smarandache generalized Fermat numbers, i.e. the numbers of the form F(k) = a^(b^k) + c, where a, b are integers greater than or equal to 2 and c is integer such that (a, c) = 1.

Conjecture 1:

Let be F(k) = 2^(2^k) + n, where k is positive integer and n is an odd number. Then, there exist an infinity of numbers n such that F(k) is prime for k = 0, k = 1 and k = 2.

Note:
For n = 1, the numbers F(k) are the Fermat numbers and it is known that the first five such numbers are primes (it is conjectured that there are not Fermat numbers that are primes for k ≥ 5). For n = 3 the three primes obtained are [5, 17, 19]. For n ≥ 3 obviously n can be only of the forms 30*m + 15 or 30*m + 27, otherwise one from the three numbers F(0), F(1) and F(3) would be divisible with 3 or 5.

Examples:
(Of such numbers n)

: For n = 15 are obtained the primes [17, 19, 31];
: For n = 27 are obtained the primes [29, 31, 43];
: For n = 57 are obtained the primes [59, 61, 73];
: For n = 135 are obtained the primes [137, 139, 151];
: For n = 147 are obtained the primes [149, 151, 163].

Note:
Obviously this conjecture implies the conjecture that there exist an infinity of primes of the forms 30*k + 1, 30*k + 13, 30*k + 17, 30*k + 19 respectively 30*k + 29, but I already conjectured in a previous paper (namely

"Twenty-four conjectures about the eight essential subsets of primes") that there exist an infinity of primes of the form 30*k + n, for any n equal to 1, 7, 11, 13, 17, 19, 23 or 29.

Conjecture 2:

There exist an infinity of quadruplets of primes of the form [30*k + 17, 30*k + 19, 30*k + 31, 30*k + 43]. Such primes are, as can be seen above, for instance, [17, 19, 31, 43] or [137, 139, 151, 163].

Comment:

I obviously could put the Conjecture 1 in a simpler form (i.e. Conjecture: there exist an infinity of odd numbers n such that the numbers n + 2, n + 4 and n + 16 are all three primes, but in this case it would appear like an arbitrary statement, which is not, but one from the many possible interesting related cojectures on Smarandache generalized Fermat numbers like the following ones:

Conjecture 3:

Let be F(k) = 4^(4^k) + 3, where k is positive integer. Then, there exist an infinity of numbers k such that F(k) is equal to 7*p, where p is prime.

Examples:
(Of such numbers F(k))

: F(1) = 259 = 7*37;
: F(2) = 4294967299 = 7*613566757;
: F(3) = 340282366920938463463374607431768211459 = 7*48611766702991209066196372490252601637

Conjecture 4:

Let be F(k) = m^(m^k) + n, where m is even and n is odd, such that m + n = p, where p is prime. Then, there exist at least a k, beside of course k = 0, for which F(k) has as a prime factor the number p.

Conjecture 5:

Let be F(k) = m^(m^k) + n, where m is odd and n is even, such that m + n = p, where p is prime. Then, there exist at least a k, beside of course k = 0, for which F(k) has as a prime factor the number p.

Reference:

Florentin Smarandache, ***Conjecture (General Fermat numbers)***, in Collected Papers, vol. II, Kishinev, 1997.