

Dempster-Shafer Theory: combination of information using contextual knowledge

Mihai Cristian Florea

Thales Canada

Land & Joint Systems Division
1405, Boul. du Parc-Technologique
Quebec, G1P 4P5, CANADA
Mihai.Florea@ca.thalesgroup.com

Abstract – *The aim of this paper is to investigate how to improve the process of information combination, using the Dempster-Shafer Theory (DST). In presence of an overload of information and an unknown environment, the reliability of the sources of information or the sensors is usually unknown and thus cannot be used to refine the fusion process. In a previous paper [1], the authors have investigated different techniques to evaluate contextual knowledge from a set of mass functions (membership of a BPA to a set of BPAs, relative reliabilities of BPAs, credibility degrees, etc.). The purpose of this paper is to investigate how to use the contextual knowledge in order to improve the fusion process.*

Keywords: Dempster-Shafer Theory, Evidence Theory, Robust Combination Rule, Contextual Knowledge, Reliability Evaluation, Fusion Architecture.

1 Introduction

During crisis or emergency situations, the automatic information management systems are significantly overloaded with pieces of information of different natures (for example: signals intelligence - SIGINT, communications intelligence - COMINT, human intelligence - HUMINT, electronic intelligence - ELINT, imagery intelligence - IMINT, measurement and signature intelligence - MASINT, etc.), different structures (structured or unstructured data), different known reliabilities (reliable, partially reliable or even completely unreliable) or even unknown reliabilities. The pieces of information have to be rapidly handled, processed, interpreted, and combined, in order to rapidly create a situation awareness picture as accurate as possible.

In such a context, the information coming from different sensors can be imperfect and its imperfection is mainly due to the imperfection of the information itself and/or to the unreliability of the sensors/sources of information. Different aspects of the imperfection of the information (imprecision, uncertainty or a mix of both) can be modelled within the Dempster-Shafer

Éloi Bossé

Defence R&D Canada (DRDC) Valcartier
Decision Support Systems (DSS) Section
2459 Pie-XI Blvd North
Val-Bélair, G3J 1X5, CANADA
Eloi.Bosse@drdc-rddc.gc.ca

theory (DST) also known as Evidence Theory, which is a mathematical tool able to characterize and combine the imperfect information.

The goal of the combination of imperfect information is to find an accurate information, easily interpretable, which can resume the information set to be combined. The combination operation should be a computationally tractable process. Conjunctive, disjunctive or the normalized conjunctive (Dempster's) combination rules are some examples of blind combination rules, which consider the information set as equi-reliable and the contribution of each piece of information to the resulting combination is the same. This could be useful when combining several pieces of information, obtained at different instants from the same sensor which has an unchanged reliability¹. However, when combining several pieces of information from a set of sensors with different reliabilities (known or unknown), using a blind combination rule becomes inappropriate. To overcome this problem, Haenni [2] proposed to use discounted mass functions² before a blind combination, using *a priori* estimations of the reliabilities of the sensors. On the other hand, Dezert *et al.* [4] state that “*The discounting techniques must never be used as an artificial ad-hoc mechanism to update Dempster's result once problem has arisen. We strongly disagree with the idea that all problems with Dempster's rule can be solved beforehand by discounting techniques.*”

The *a priori* estimation of the reliability of the sensors/sources of information is a difficult process in a normal context and becomes more challenging in a crisis or emergency context. In [5], Florea *et al.* have showed that using incorrect *a priori* estimations for the reliabilities when discounting the mass functions, can

¹The situation of combining several pieces of information, obtained from the same sensor, at different instants, and the reliability of the sensor can change in time, can be assimilated to the combination of several pieces of information, obtained from different sensors with different reliabilities.

²The discounting operation was first introduced by Shafer [3] on belief functions.

lead to lower performances than a robust combination rule able to automatically account for the reliability of the pieces of information.

Usually, the estimation of the reliability of the sensors is realized using a priori knowledge about the sensors, or the environment. A relative reliability of the sensors can be considered to create more robust combination rules. The contextual knowledge (membership of a BPA to a set of BPAs, relative reliabilities of BPAs, credibility degrees, etc.), which can be obtained from a set of mass functions, can play an important role in the refinement of the combination process.

- can be seen as a practical way to estimate on the fly the relative reliability of the sensors.
- can be incorporated directly in the combination rule to increase its robustness. A first step in developing such a robust combination rule was realized in [5]. A weighted sum of the conjunctive and disjunctive combination rules was proposed, with weighting coefficients which are dependent of the conjunctive conflict between the mass functions to be combined. However, the robust combination rule (RCR) should not consider the conjunctive conflict as the only dissimilarity measure between mass functions.
- can be used to identify the unreliable sensors in order to refine the combination process accordingly.

In a previous paper [1], we have investigated and classified the different dissimilarity measures, according to different situations, in order to correctly understand the contextual knowledge obtained from a set of mass functions. Some techniques to relate the dissimilarity measures to relative reliability measures have also been investigated. In this paper we propose to investigate more thoroughly how to use the obtained contextual knowledge in order to improve the combination process.

This paper is divided as follows. In Section 2 we recall our classification of the dissimilarity measures, introduced in [5] and according to the different information sources (presented in Section 3). Section 4 presents an improved fusion mechanism, which considers the contextual knowledge in order to identify the defective sources of information. Section 6 is the conclusion.

2 Dissimilarity measures

The idea of measuring the dissimilarity between mass functions (BPA - basic probability assignments) in the DST is not new. A first measure of dissimilarity in the DST is the conjunctive conflict between BPAs and was first introduced by Shafer in [3]. However, Liu [6] and Martin *et al.* [7] show that the conjunctive conflict proposed by Shafer is not always an adequate metric to measure the dissimilarity between two BPAs. In

the last decades, different measures of conflict and distances have been proposed to better characterize the relations and the dissimilarities between BPAs [7–11]. Even more, some authors [7, 12–14] propose to characterize the intrinsic conflict of a BPA, before characterizing the conflict between several BPAs. In the rest of the paper we will generally refer to the entire set of metrics as to dissimilarity measures between BPAs. Shafer's conflict will be referred to as conjunctive dissimilarity, to make a clear difference with the global measures of conflict proposed by Liu [6] and Martin *et al.* [7]. The relation between the two pieces of information plays an important role in the definition of the dissimilarity measure between the two pieces of information. Thus, two different classes of the dissimilarity measures can be considered, based on the similar/dissimilar classification of sensors. A distance measure should be used to measure the dissimilarity between sensors providing information about the same attribute (*similar sensors*). Sensors providing information about different attributes are called *dissimilar sensors*, and the dissimilarity measure to be used should be a conjunctive dissimilarity measure. Luo and Kay [15] refer to these classes of sensors in terms of *redundant/complementary sensors*. A recent survey of the dissimilarity measures according to this classification was proposed by Florea and Bossé in [1].

3 Sensors, reliability and combination

A sensor capable to provide information about a specific characteristic/attribute of an object/a situation is called a **simple sensor**. A thermometer is an example of such a simple sensor. A sensor capable to provide information about distinct characteristics/attributes of the same object/situation, is called a **complex sensor** or a collection of simple sensors. A radar which can provide information about the range, altitude, direction, or speed of a moving target or a human which can provide information about the colours, dimensions, time, sounds, or even opinions, beliefs, etc. are examples of complex sensors.

While the simple sensors can be characterized as reliable or unreliable and the degrees of reliability of such sensors could be time-variant or time-constant, the complex sensors are more difficult to characterize from the reliability/unreliability point of view. If there is no *a priori* knowledge about the correlations between the simple sensors composing a complex sensor, the simple sensors should be considered completely independent.

3.1 Similar Sensors

We define a set of similar sensors as a set of simple sensors which are observing the same characteristic/attribute of the same static or dynamic situation/object. We do not need any *a priori* information

about the attribute studied by the sensors or any other *a priori* data bases, since we can rely on the corroboration of the sensors. Such fusion process can be seen as an **unsupervised fusion process**. The BPAs associated to pieces of information coming from similar sensors can and have to be compared through a distance measure and not through the conjunctive dissimilarity measure. One of the distance measures proposed for example in [7–11] can thus be used in such a situation.

The similarity of the sensors should also be reflected in the following aspects :

- The BPA obtained after the combination should be the closest (according to a specified distance measure) to the set of BPAs to be combined.
- A measure of relative reliability or membership degree of a BPA to the set of the BPAs should be based on the distance measure between each pair of BPAs.
- The initial BPAs which are not close (in terms of the distance measure) to the combined BPA, should be seen as “*unreliable*” and could be temporarily discarded from the combination process, in order to refine it.

3.2 Dissimilar Sensors

We define a set of dissimilar sensors as a set of simple or complex sensors which are observing the same static or dynamic situation but from several points of view (several characteristics/attributes). Thus, the corroboration of the sensors cannot be validated in absence of data bases and *a priori* knowledge. We can consider such a fusion process as a **supervised fusion process**. In fact, in this situations, the data bases and the *a priori* knowledge are needed to correctly discriminating the frame of discernment for the given fusion problem.

In this situation, a distance is inappropriate to be used to measure the dissimilarity between BPAs, since the dissimilar sensors are measuring different characteristics. Independently of the chosen metric proposed in [7–11], the distance between two pieces of information such as “the object is yellow” and “the object is round” is important. But this does not mean that the two pieces of information are not in agreement. In such a situation, when dissimilar information have to be fused, the agreement between the pieces of information should be measured through the conjunctive dissimilarity measure and not through a distance. The conjunction of information (*the object is yellow and round*) is then evaluated: *Is there any possible yellow and round object in our data base ?* If the data base contains round objects as well as yellow objects but there are no yellow and round objects, a conflict raises which is characterized by a conjunctive dissimilarity measure.

The dissimilarity of the sensors should thus be reflected in the combination process:

- The BPA obtained after the combination should be the closest (according to the conjunctive dissimilarity measure) to the set of BPAs to be combined.
- A measure of relative reliability or membership degree of a BPA to the set of the BPAs should not be based only on the conjunctive dissimilarity measure between each pair of BPAs. Given three BPAs m_1 , m_2 and m_3 , such that the conjunctive dissimilarity measures $k_2(m_1, m_2) = k_2(m_1, m_3) = k_2(m_2, m_3) = 0$, a total conflict between the three BPAs can sometimes arise ($k_3(m_1, m_2, m_3) = 1$). From this point of view, the conjunctive dissimilarity measure is completely different from a mean distance to be used for the similar sensors.
- The initial BPAs which are not close (in terms of the conjunctive dissimilarity measure) to the combined BPA, should be identified as “*unreliable*” and could be temporarily discarded from the combination process, in order to refine it.

3.3 Hybrid Sensors Fusion Model

Until now, the Fusion Community have concentrate its efforts to find the best combination rule which can perform in any given situation [5, 16, 17]. A general fusion model should rather depend on the problem we are facing, and thus should act to reflect the relation between the different sensors : similar or dissimilar.

We propose here a Hybrid Sensor Fusion (HSF) model which is represented by the architecture from Figure 1. First, information from similar sensors are fused together using a Similar Sensors Fusion (SSF) model and second, the resulting information is fused using a Dissimilar Sensors Fusion (DSF) model.

Thus, instead of trying to find a combination rule which adapt to most of the situations, it is important to correctly design the problem and use the appropriate fusion model for each situation. The HSF model depicted in Figure 1 allows :

- to integrate sensors which can provide only one piece of information.
- to integrate sensors which can provide more than one piece of information (over the time). When redundant pieces of information are provided, the resulting BPA should not be affected. The only change should be in the resulting reliability of the SSF process.
- to independently design a SSF model for a group of similar sensors without affecting the rest of the system. Such a subsystem should be easily plugged directly into the DSF model (see for example the SSF5). The combination rule to be used in a SSF model should be chosen according to the group of sensors and to the attribute they characterize. In

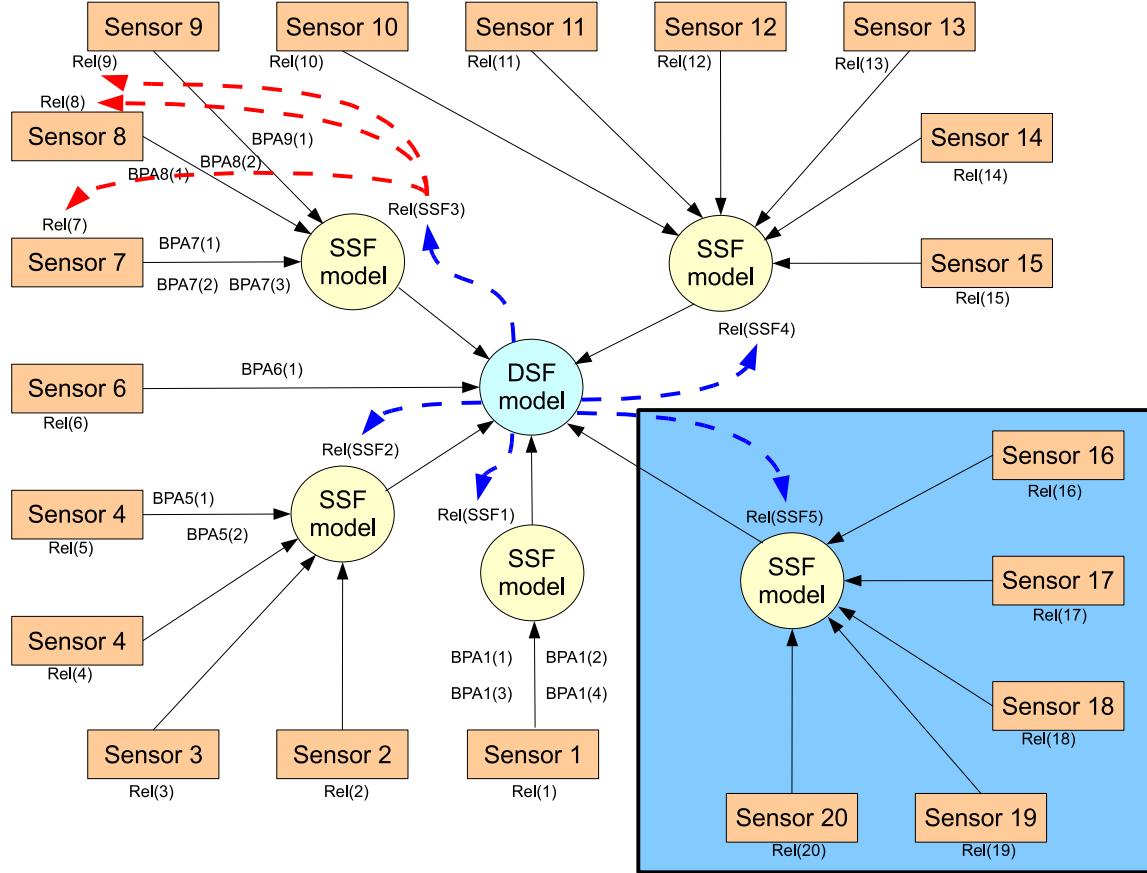


Figure 1: Hybrid Sensor Fusion (HSF) model

this way, the integration of new subsystems should be facilitated.

- to continuously refine the initial reliabilities of the sensors, according to the global combination process (the feedback is represented by the dashed lines).
- to combine the pieces of information at a SSF node in a sequential mode.
- to combine the pieces of information at the DSF node in a batch or a sequential mode. If the sequential mode is used, the time stamp associated to the piece of information provided by a SSF node could be (according to each application) :
 - the acquisition time of its last piece of information.
 - the average acquisition time of all of its pieces of information. If the average is equal for two or more SSF points, the acquisition time of their last pieces of information can then be considered.
- to combine the pieces of information at a SSF node

in a batch mode, if the similar sensors are providing only one piece of information.

- to design an order insensitive³ combination process by imposing any basic associative operators (conjunctive, disjunctive or Dempster's rules of combination) at any SSF and DSF nodes.
- to recover any basic associative combination rule, by setting identical combination rules in all the SSF and DSF models and chose not to use any feedback to refine the reliabilities.

This architecture has some drawbacks. The computation complexity of the combination process increases, especially due to the continue refinement. The order insensitive combination process, which are not associative, cannot be computed by a recursive algorithm, which leads also to an increase of the computation complexity.

³Here, the distinction between an associative and an order insensitive combination process becomes more clear. An associative combination process is a recursive order insensitive combination process, which allows to compute the final combined BPA, at time N , $m^N = m_1 \oplus m_2 \oplus \dots \oplus m_N$ from the last acquired BPA m_N and the final combined BPA from time $N - 1$, as follows: $m^N = m^{N-1} \oplus m_N = (m_1 \oplus m_2 \oplus \dots \oplus m_{N-1}) \oplus m_N$.

4 Reliability refinement

The effective evaluation of the reliability of the sensors is an difficult process. As shown in Florea *et al.* [5], the incorrect evaluation of the reliability degrees of the sensors can lead to lower performance results of the combination process. In literature, several situation have been observed:

- Each sensor is supposed to have a known reliability degree $Rel(i)$.
- If the reliability of the sensors is unknown, the sensors are considered equi-reliable, during the entire combination process. If the reliability degrees are required, a unitary reliability is then considered.
- The reliability of the sensors is not changing during the time.

These considerations are restrictive and prevent the fusion systems to be flexible. To overcome this problem, we propose to consider that each sensor is supposed to have an initial reliability degree $Rel_0(i)$, independent of the pieces of information it provides. We suppose that the initial reliability degrees can be refined during the combination process, from the contextual knowledge extracted from the relations between the BPAs to be combined. This continue refinement involves several distinct steps, which are closely related to the different feedback loops presented in Figure 1:

- FORWARD : We associate to the results obtained at each SSF node a reliability degree, since each SSF node can be considered as a new sensor for the DSF node. These reliability degrees should be strongly related to the dissimilarity measures (distance measures) associated to the combination results from each SSF node.
- BACKWARD : The first refinement of the reliability degrees of the sensors is performed at each SSF node. If the combination is realized in a batch mode, than the refinement is realized only once. If the combination is realized in a sequential mode, the refinement is realized at each update of the combination process.
- FORWARD : We associate to the results obtained at the DSF node a reliability degree. This reliability degree should be strongly related to the dissimilarity measure (conjunctive dissimilarity) associated to the combination result at the DSF node.
- BACKWARD : We refine the reliability degrees of the SSF nodes according to the relations between the conjunctive dissimilarities involved in the computation of the combination result at the DSF node.

- BACKWARD : We refine the reliability degrees of the sensors, according to the refined reliabilities of the SSF nodes.

In a static fusion process we can loop this algorithm several times until stabilization, while in a dynamic fusion process, the new reliabilities can be used for the next combination step.

5 Identifying reliable sensors

Let S_k , $1 \leq k \leq K$ be a set of sensors providing a set $\mathcal{M} = \{m_1, m_2, \dots, m_M\}$ of BPAs. The sensors should be either similar or dissimilar (corresponding to the SSF or the DSF models), but not a mix of both similar and dissimilar (corresponding to the HSF model). We denote by $S_k(\mathcal{M})$ the set of BPAs from \mathcal{M} which are generated by the sensor S_k . The number of mass functions does not necessarily correspond to the number of sensors, and some particular situations can be encountered:

- the entire set of BPAs is provided by the same sensor ($S_1(\mathcal{M}) = \mathcal{M}$), or
- each BPA is provided by a different sensor (each sensor is providing only one piece of information and $|S_k(\mathcal{M})| = 1, \forall 1 \leq k \leq K$).

We propose the algorithm depicted in Figure 2 in order to identify the “*reliable/unreliable*” sensors for static or dynamic modes of the SSF and DSF fusion models. The following steps are considered:

- Compute the membership degree $MD(m_i)$ of each BPA to the entire set \mathcal{M} of BPAs. Different techniques from [1] can be used for the SSF and DSF models respectively. If the $MD(m_i)$ exceeds a given threshold (depending on the technique employed), we say that m_i corroborates the set \mathcal{M} of BPAs. The set \mathcal{C} of BPAs which corroborates the set \mathcal{M} is then identified.
- For a given sensor S_k , and from the membership degrees associated to the pieces of information it generates, we propose several definitions for the **corroboration degree**:

- as the ratio between the number of corroborative BPAs inside $S_k(\mathcal{M})$ and the number of BPAs generated by S_k :

$$CRB_1(S_k) = \frac{|A_k|}{|S_k(\mathcal{M})|} \quad (1)$$

where $A_k = \{m_i | MD(m_i) \geq threshold, m_i \in S_k(\mathcal{M})\}$.

- as the ratio between the overall membership degree and the number of BPAs generated by

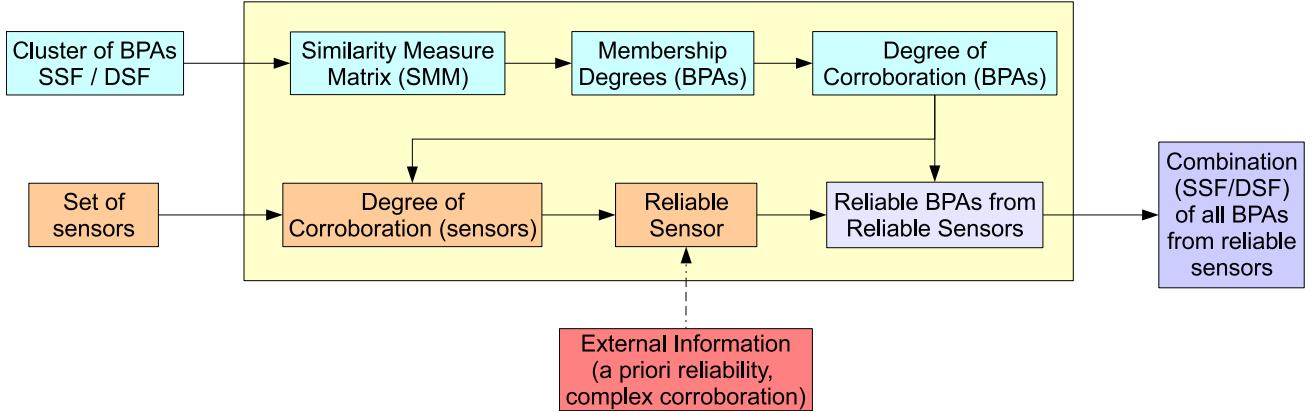


Figure 2: SSF/DSF fusion models with identification of reliable/unreliable sensors

S_k :

$$CRB_2(S_k) = \frac{\sum_{m_i \in S_k(\mathcal{M})} MD(m_i)}{|S_k(\mathcal{M})|} \quad (2)$$

- From the corroboration degree (CRB_1 or CRB_2 or even a combination of both), and from any external *a priori* knowledge (if available), we can update the reliability of the sensors (using a backward propagation described in Section 4).
- If the reliability degree of a sensor is inferior to a given threshold⁴, we state that S_k is a defective (unreliable) sensor and we discard from \mathcal{C} (or weight accordingly) all the BPAs provided by S_k . In a sequential fusion process, the evaluation of the reliability/unreliability of a sensor is a dynamic process which implies only a temporarily discard of the BPAs (until the sensor is providing a new BPA, and the system re-evaluates the reliability/unreliability of the sensor).

The most reliable result of the combination of the BPAs from the set \mathcal{M} is the BPA m^* resulting from the combination of the BPAs from the set \mathcal{C} . The combination technique is depending on the chosen model, SSF or DSF respectively.

A recent work by Blasch [18] investigates a fusion reliability metric, as a combination of accuracy, confidence and timeliness. This metric can be used in future works to demonstrate the performance of the proposed fusion system.

6 Conclusions

The main contribution of this paper is to consider the combination process not from the point of view of an unique combination rule to be used in all fusion situations. Instead, a hybrid and flexible fusion model is pro-

posed, as a mix of operators able to integrate both similar and dissimilar sensors. We have also investigated how to improve the combination process, by considering the contextual knowledge. The reliability degrees of the sensors are thus refined, using forward and backward propagations between the different components of the design. In a future paper we will compare the performances of this new design through Monte-Carlo simulations.

Acknowledgements

The authors would like to thank Thales Canada, Land & Joint Systems Division and DRDC Valcartier for the technical and financial support provided for this study.

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⁴We consider a reasonable threshold to be equal to 80%.

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