

# A note on the harmonic series and the logarithm

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## Abstract

A relationship between the harmonic series and the logarithm is presented.

Figure 1: Illustration of  $H(n)$  and  $\log(n)$  as part of  $H(n+n^2)$

$$H(n+n^2)$$

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} \dots \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} \dots \frac{1}{n+n^2}$$

$$\underbrace{\hspace{10em}}_{H(n)} \quad \underbrace{\hspace{10em}}_{\approx \log(n)}$$

Figure 2: Relationship between  $\log(n)$  and its approximation  $H(n+n^2)-H(n)$

$$\lim_{n \rightarrow \infty} n \cdot \left( \underbrace{H(n+n^2) - H(n)}_{\approx \log(n)} - \log(n) \right) = \frac{1}{2}$$