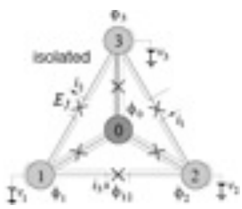
Tetrahedra and Physics

Frank Dodd (Tony) Smith, Jr. discussion with Klee Irwin

1 - Start with a regular Tetrahedron in flat 3-dim space



Tetrahedron Josephson Junction Quantum Computer Qubit

2 - Add 4 + 12 Tetrahedra sharing faces to get 17 Tetrahedra


The 4 fit face-to-face exactly in 3-dim,
but


the 12 do not fit exactly in 3-dim,
However, all 17 do fit exactly in curved 3-dim space which is naturally embedded in 4-dim space described by Quaternions.

3 - Add 4 half-Icosahedra (10 Tetrahedra each) to form a 40-Tetrahedron Outer Shell around the 17 Tetrahedra and so form a 57G

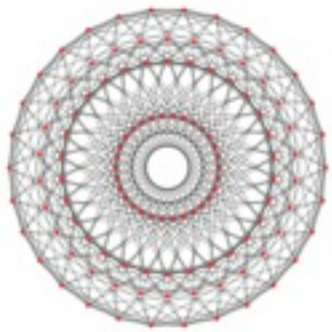
Like the 12 of 17, the Outer 40 do not exactly fit together in flat 3-dim space.

If you could force all 57 Tetrahedra to fit together exactly, you would be curving 3-dim space by a Dark Energy Conformal Transformation.

4 - The 57G can be combined with a Triangle of a Pearce D-Network

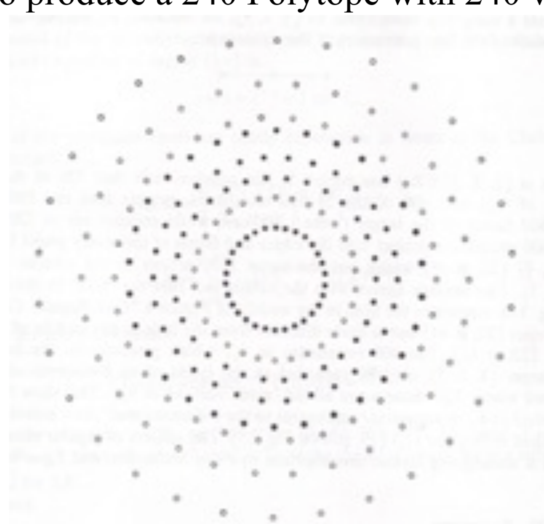
to form a 300-tetrahedron configuration 57G-Pearce

5 - Doubling the 300-cell 57G-Pearce produces a {3,3,5} 600-cell polytope

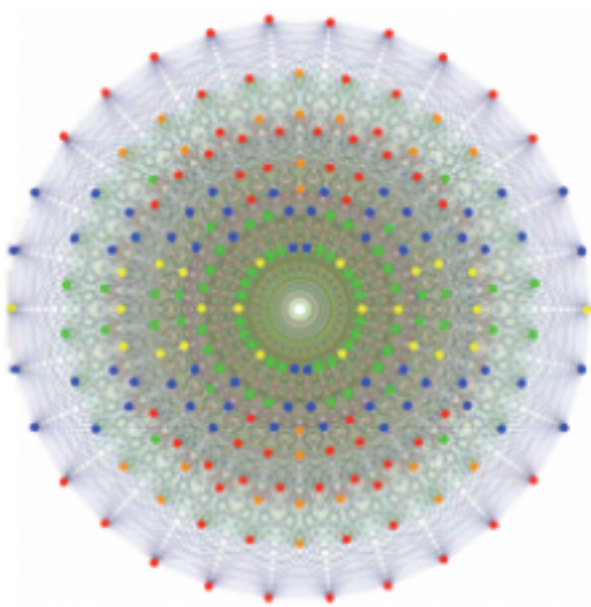


of 600 Tetrahedra and 120 vertices in 4-dim

6 - Adding a second {3,3,5} 600-cell displaced by a Golden Ratio screw twist uses four 57G-Pearce to produce a 240 Polytope with 240 vertices in 4-dim



7 - Extend 4-dim space to 4+4 = 8-dim space by considering the Golden Ratio algebraic part of 4-dim space as 4 independent dimensions, thus transforming the 4-dim 240 Polytope into the 240-vertex 8-dim Gosset Polytope



that represents the Root Vectors of the E8 Lie Algebra and the first shell of an 8-dim E8 Lattice

8 - The 240 Root Vectors of 248-dimensional E8 have structure inherited from the real Clifford Algebra Cl(16) = Cl(8) x Cl(8)

which structure allows construction of a E8 Physics Lagrangian from which realistic values of particle masses, force strengths, etc., can be calculated.

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Tetrahedra can be used as Josephson Junctions.

A very useful reference is the 2003 dissertation of Christopher Bell at St. John's College Cambridge entitled "Nanoscale Josephson devices", on the web at http://www.dspace.cam.ac.uk/bitstream/1810/34607/1/chris_bell_thesis.pdf

Feigelman, Ioffe, Geshkenbein, Dayal, and Blatter in cond-mat/0407663 say:
"... Superconducting tetrahedral quantum bits ...

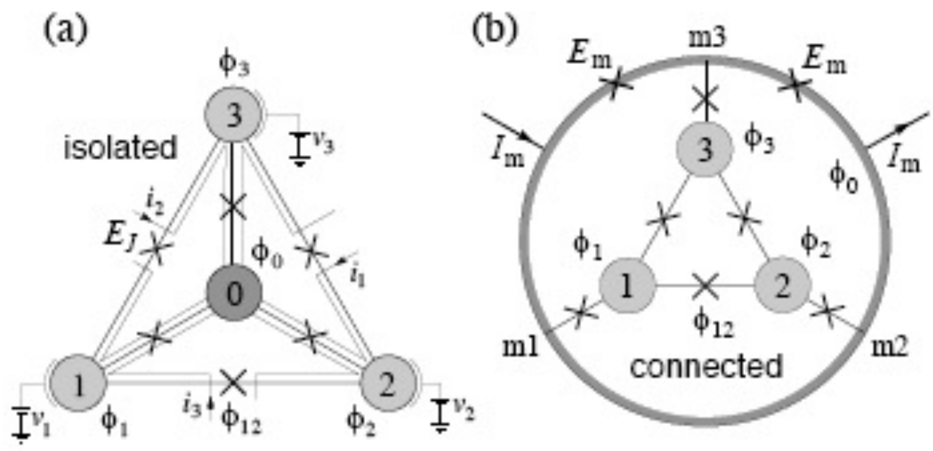


FIG. 1: (a) Tetrahedral superconducting qubit involving four islands and six junctions (with Josephson coupling E_J and charging energy E_C); all islands and junctions are assumed to be equal and arranged in a symmetric way. The islands are attributed phases ϕ_i, $i = 0, \ldots, 3$. The qubit is manipulated via bias voltages v_i and bias currents i_i. In order to measure the qubit's state it is convenient to invert the tetrahedron as shown in (b) — we refer to this version as the 'connected' tetrahedron with the inner dark-grey island in (a) transformed into the outer ring in (b). The measurement involves additional measurement junctions with couplings $E_m \gg E_J$ on the outer ring which are driven by external currents I_m (schematic, see Fig. 6 for details); the large coupling E_m effectively binds the ring segments into one island.

... The novel tetrahedral qubit design we propose below operates in the phase-dominated regime and exhibits two remarkable physical properties:

first, its non-Abelian symmetry group (the tetrahedral group Td) leads to the natural appearance of degenerate states and appropriate tuning of parameters provides us with a doubly degenerate groundstate. Our tetrahedral qubit then emulates a spin-1/2 system in a vanishing magnetic field, the ideal starting point for the construction of a qubit.
Manipulation of the tetrahedral qubit through external bias signals translates into application of magnetic fields on the spin;
the application of the bias to different elements of the tetrahedral qubit corresponds to rotated operations in spin space.
Furthermore, geometric quantum computation via Berry phases ... might be implemented through adiabatic change of external variables.
Going one step further, one may hope to make use of this type of systems in the future physical realization of non-Abelian anyons, thereby aiming at a new generation of topological devices ... which keep their protection even during operation ...

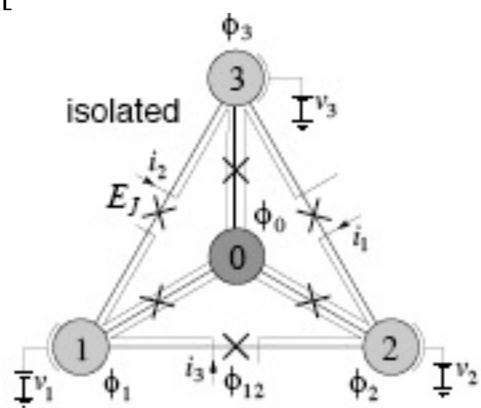
The second property we wish to exploit is geometric frustration:
In our tetrahedral qubit ... it appears in an extreme way by rendering the classical minimal states continuously degenerate along a line in parameter space. Semi-classical states then appear only through a fluctuation-induced potential, reminiscent of the Casimir effect ... and the concept of inducing
'order from disorder' ...
The quantum-tunneling between these semi-classical states defines the operational energy scale of the qubit, which turns out to be unusually large due to the weakness of the fluctuation-induced potential. Hence the geometric frustration present in our tetrahedral qubit provides a natural boost for the quantum fluctuations without the stringent requirements on the smallness of the junction capacitances, thus avoiding the disadvantages of both the charge- and the phase- device:
The larger junctions reduce the demands on the fabrication process and the susceptibility to charge noise and mesoscopic effects, while the large operational energy scale due to the soft fluctuation-induced potential reduces the effects of flux noise. Both types of electromagnetic noise, charge- and flux noise, appear only in second order ...

in order to benefit from a protected degenerate ground state doublet, the qubit design requires a certain minimal complexity; it seems to us that the tetrahedron exhibits the minimal symmetry requirements necessary for this type of protection and thus the minimal complexity necessary for its implementation. ...".

Construction of 57G in 3-dim space

Eric A. Lord, Alan L. Mackay, and S. Ranganathan in their book “New Geometries for New Materials” (Cambridge 2006) said:
“... The gamma-Brass cluster ... starts from a single tetrahedron

[



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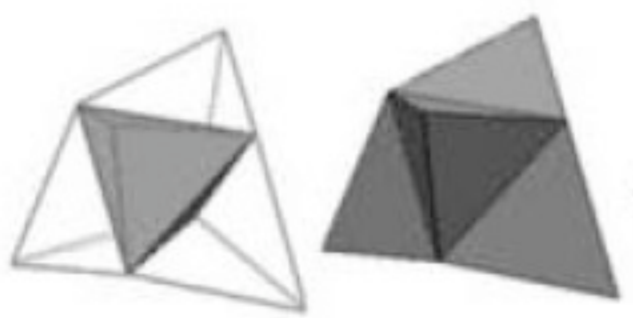
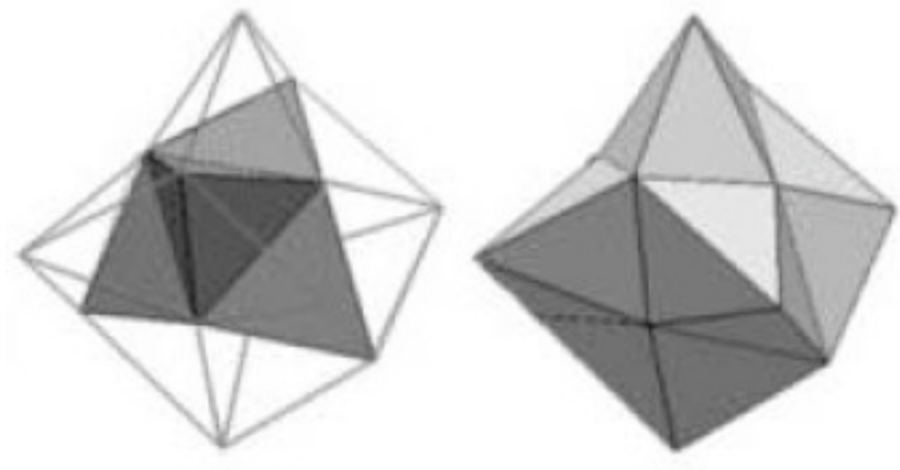
Place four spheres in contact.
Then place a sphere over each face of the tetrahedral cluster.
The centres and bonds then form a stella quadrangula



built from five regular tetrahedra ...[a total of 1+4 = 5 tetrahedra]...

Six more spheres [vertices] placed over the edges of the original tetrahedron form an octagonal shell. In terms of the network of centres and bonds we now have added 12 [= 2x6] more tetrahedra ...

There are now five tetrahedra around each edge of the original tetrahedron. ...



...[we now have 1+4+12 = 17 tetrahedra]...

[The 12 newly added tetrahedra]... are not quite regular ...[i.e.,
nonzero Fuller unzipping angles appear as described by Thomas Banchoff
in his book "Beyond the Third Dimension" (Scientific American Library 1990)
where he said:
"... in three-space .... we can fit five tetrahedra around an edge ...

[image from Conway and Torquato PNAS 103 (2006) 10612-10617

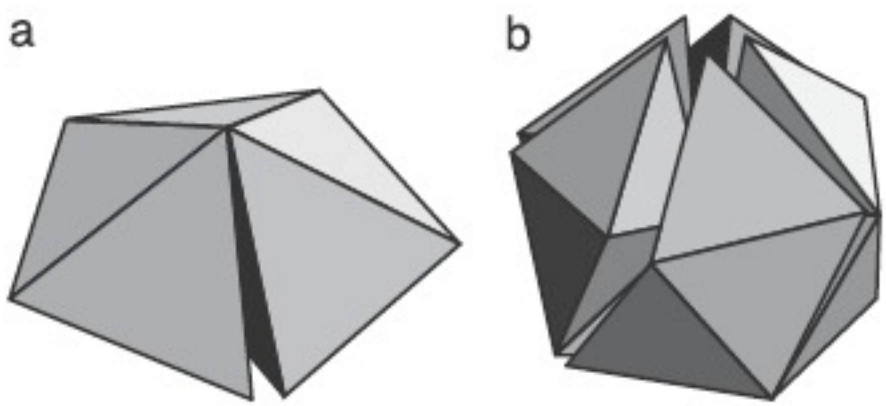


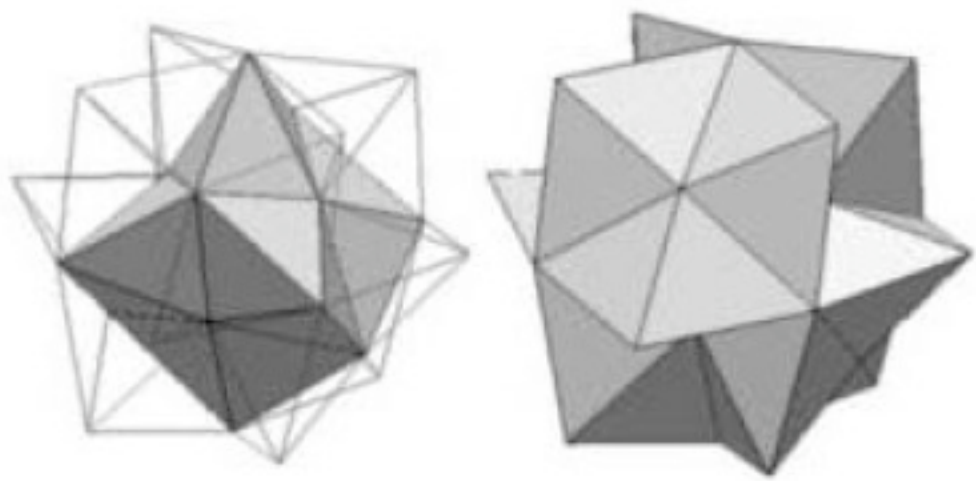
Fig. 1. Certain arrangements of tetrahedra. (*a*) Five regular tetrahedra about a shared edge. The angle of the gap is 7.36°. (*b*) Twenty regular tetrahedra about a shared vertex. The gaps amount to 1.54 steradians.
]

... with a ... small amount of room to spare,
which allows folding into 4-space ...[where the fit can be made exact]...".

Note that all the subsequently added tetrahedra of layers and structures further out from the center are also “not quite regular”, or, in other words, leave gaps among tetrahedra that are related to the Fuller unzipping angle.
The irregularity, or Fuller unzipping angle, can be visualized as the amount of curvature in a collection of tetrahedra by which it deviates from the flatness of 3-dim space described by the 3-dim Diamond Lattice.

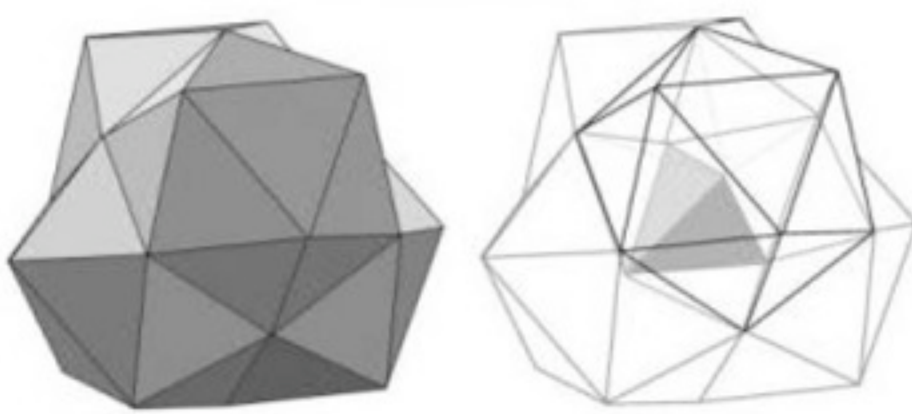
The irregularity goes away in curved 3-dim space,
which, if it is to be realized in a flat space, must be realized in 4-dim space
by adding a 4th dimension to 3-dim space.
However, for now, we will continue with construction of the 57G and
its Array in 3-dim space,
and leave additional dimensions to later sections of this paper.]

... 12 more spheres [vertices in addition to the 1+4+12 = 17] complete the rings of five tetrahedra around the edges of the four secondary tetrahedra ...[They add 2x12 = 24 more tetrahedra for a total of 1+4+12+24 = 41 tetrahedra]...



... Without increasing the number of vertices [which is now 26],

inserting 16 more tetrahedra reveals the structure to be
four interpenetrating icosahedra sharing a common tetrahedral building block ...



...[and gives a total of 41 + 16 = 57 tetrahedra]... and 26 vertices ... the model of the 26-atom gamma-brass cluster as four interpenetrating icosahedral clusters ...".

Note that each of the 4 interpenetrating icosahedra has:

10 tetrahedra to itself (each belongs to only 1)
6 tetrahedral shared with one other (each belongs to 2)
3 tetrahedra shared with two others (each belongs to 3)
1 tetrahedron shared with all three others (belongs to 4)

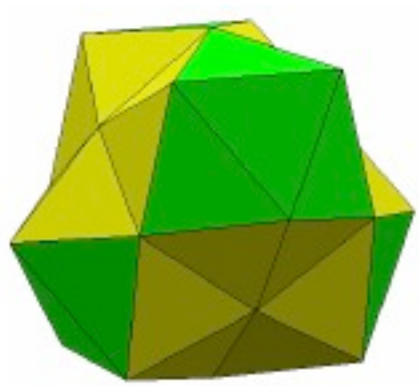
so

the total number of tetrahedra in a 57G
is 4x10 + 4x6/2 + 4x3/3 + 4x1/4 = 40+12+4+1 = 57.

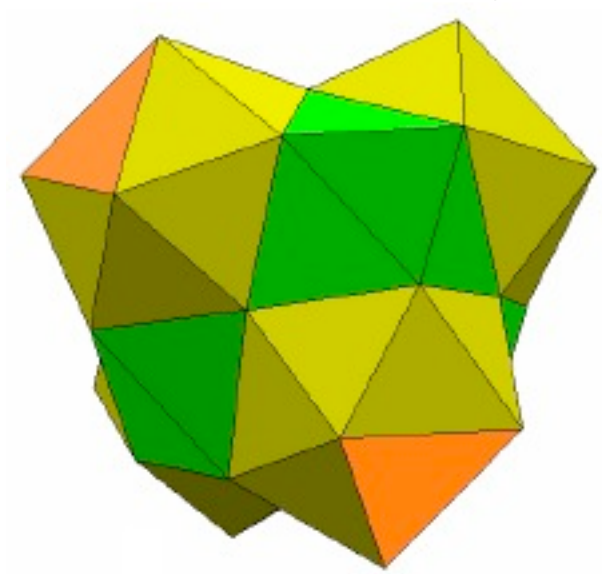
Extension of a 57G in 3-dim space

To extend a 57-tetrahedron 26-vertex 57G,
first construct some auxiliary structures,
the first of which is an 81-tetrahedron 38-vertex Pearce Cluster:

Begin with a 57-tetrahedron 26-vertex 57G



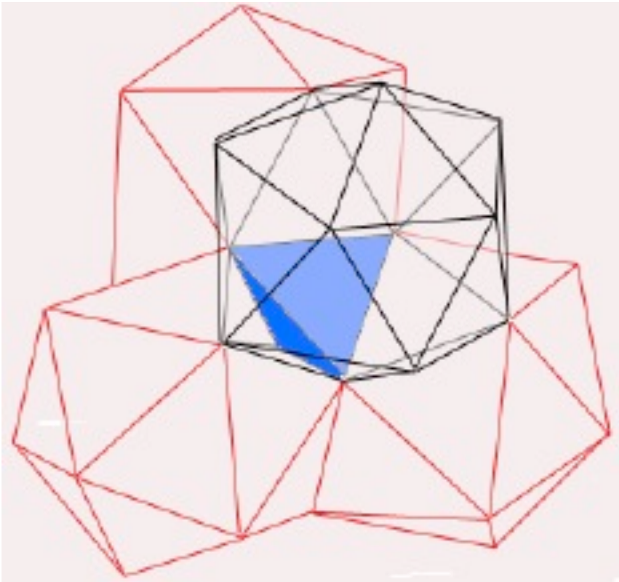
Then add 40 more tetrahedra and 12 more vertices to get



97 tetrahedra with 38 vertices.

Then remove the 4x4 = 16 tetrahedra of the type of those colored green
(note that this does not remove any vertices)

to get the 81-tetrahedron 38-vertex Pearce Cluster



which has the configuration of four icosahedra in face contact with a central tetrahedron and with each other.

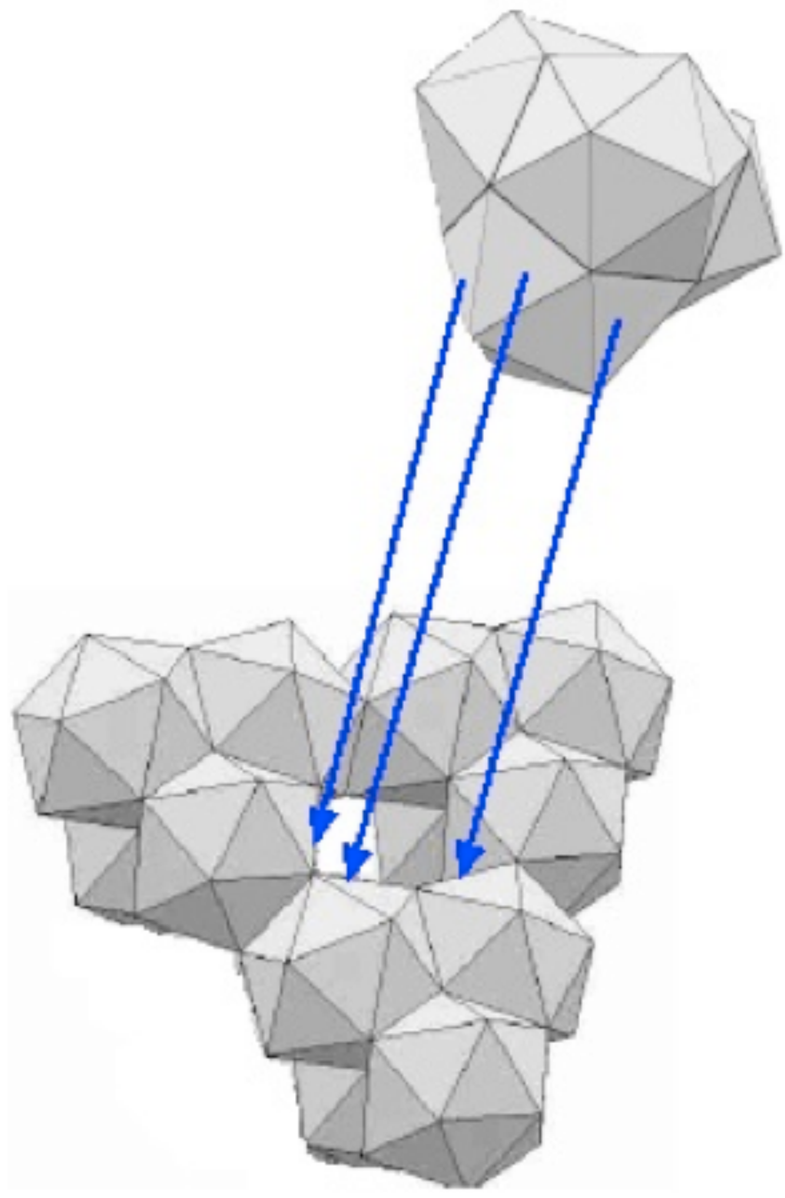
Then note that 4 Pearce Clusters can be put in face contact with each other to form the basis of what Lord and Ranganathan in Eur. Phys. J. D 15 (2001) 335-343 describe as “... a D [Diamond] network open packing in which a regular tetrahedron is centered at each node ...

... and is linked to neighboring nodes by oblate icosahedra. ...”.

Then, consider a Pearce Triangle formed by 3 Pearce Clusters

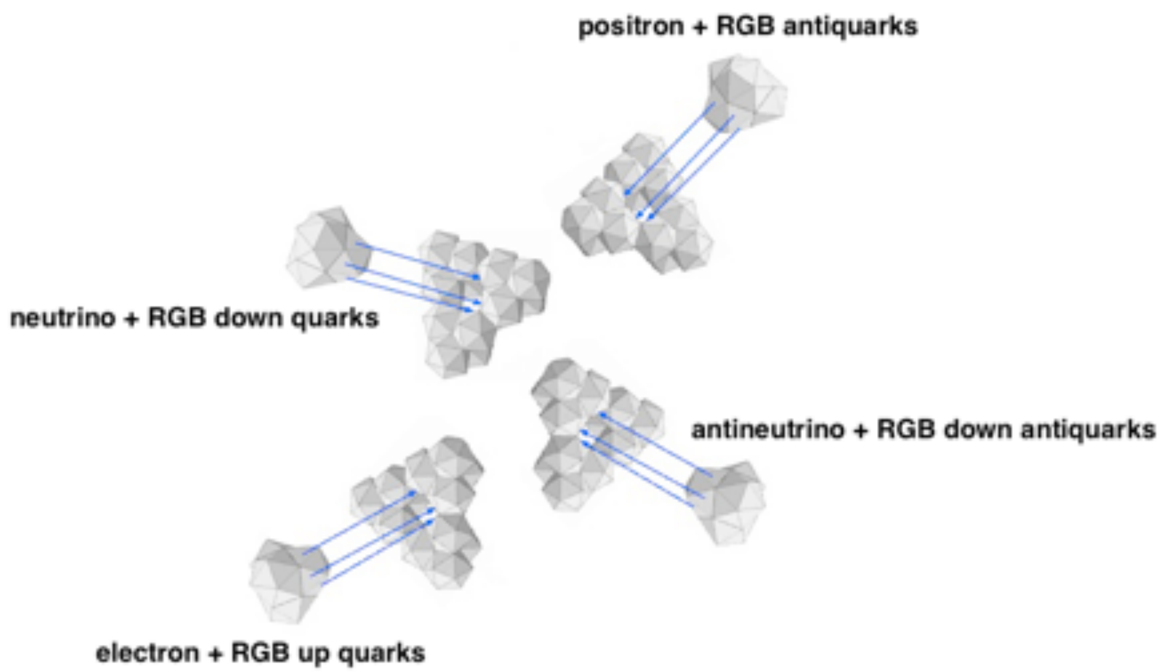
Each triangular face is made up of 3 Pearce 81G clusters in face contact,
for a total of 3x81 = 243 tetrahedra.

Consider a triangular face. You can insert one 57G into its center hole.

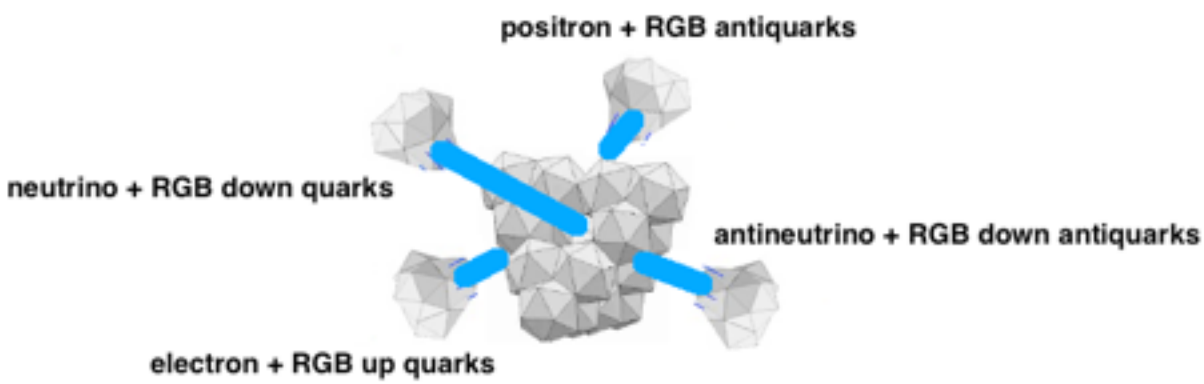


the Pearce Triangle face has 3x81 = 243 tetrahedra and 3x38 - 3x3 = 105 vertices.
Since the 57G TetraJJ Nucleus has 57 tetrahedra and 26 vertices,
and since fitting it into the center of the Pearce Triangle effectively cancels 9 vertices in forming the combined 57G + Triangle configuration,
the 57G + Triangle has 243 + 57 = 300 tetrahedra and 105 + 26 - 9 = 122 vertices.

Now consider 4 copies of 57G + Triangular Face



Taken together the 4 copies of 57G represent all 4x4 = 16 first-generation fermions, both particles and antiparticles, so that when you put them all together by superposition of Pearce 81G to make



you have a 3-dim diamond network basic unit (DNBU) containing
4x57 + 4x3x81 = 228 + 972 = 1200 tetrahedra
and
a 3-dim diamond network that is made up of tetrahedral-shaped DNBU connected to each other by sharing corner icosahedra.

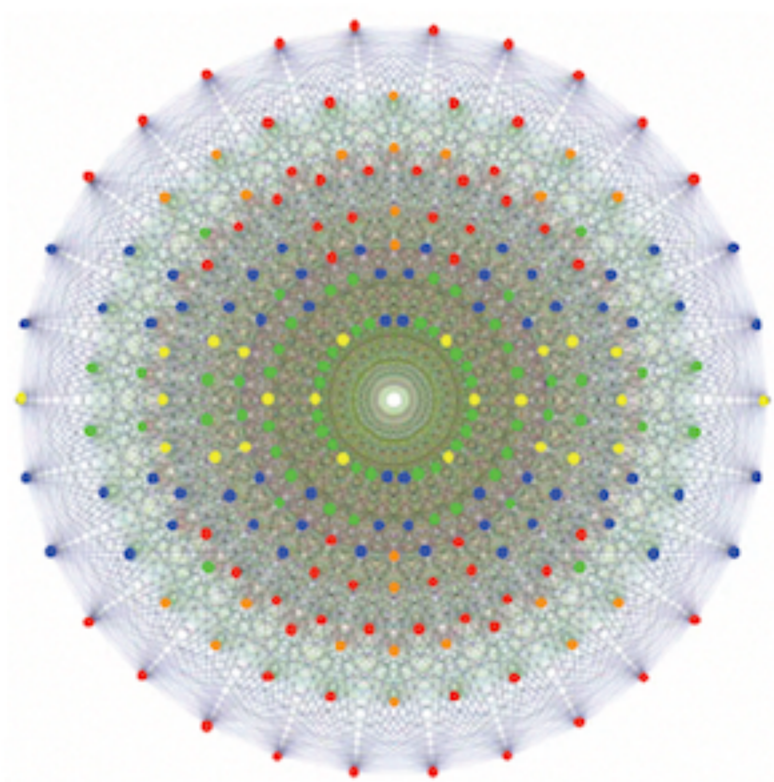
Each DNBU of the 3-dim diamond network contains

4 real 57G which together represent all 16 first-generation fermion particles/antiparticles
and
4 virtual quantum superpositions, each of 3 virtual 81G
so that
the 3-dim diamond network is a superposition of 3 virtual diamond networks,
corresponding to the 3 Imaginary Quaternions {i,j,k}

which is analogous to the Cl(16)-E8 physics 8-dim lattice that is a superposition of 8 E8 lattices including the 7 independent Integral Domains corresponding to the 7 Imaginary Octonions {i,j,k,E,I,J,K}.

The 4x57 + 4x3x81 = 1200 tetrahedra and the icosahedra formed by them
are not exactly regular in flat 3-dim space
but
they can be made regular by going into 4-dim (equivalent to curving 3-dim)
where they form two 600-cells (circles 1,3,4,5 and 2,6,7,8 in the image below)
and
the 3-dim diamond packing becomes a 4-dim hyperdiamond lattice.

Using Golden Ratio to get 600-cells of two sizes
and forming a 4+4 = 8-dim space
produces E8 lattices with 240-vertex polytopes (projected to 8 circles of 30 in the image below)
whose vertices are the 120+120 vertices of the two 600-cells.



Two {3,3,5} 600-cell sets of 20 vertices form a 240 Polytope in 4 dimensions
and if 4-dimensional space is extended to 8-dimensional space by considering Golden Ratio Irrational Algebraic Coordinates to be independent,
the 240 vertices of the 240 Polytope form the Root Vectors of the E8 Lie Algebra.
Jean-Francois Sadoc and Remy Mosseri in their book
"Geometric Frustration" (Cambridge 2006) said:
"... **The polytope 240** ...[is]... not a regular polytope in the Coxeter sense ... but ... **an ordered structure on a hypersphere ... S3** ...
the diamond crystalline structure can be obtained by starting from an f.c.c. structure and adding a second replica of the f.c.c. structure, translated by (1/4,1/4,1/4,1/4) with respect to the first one.
Similarly,
polytope 240 is generated by adding two replicas of the {3,3,5}, displaced along a screw axis of S3. ...
Each vertex of the first {3,3,5} replica is surrounded by four vertices from the second replica. ... The local configuration is perfectly tetrahedral ...

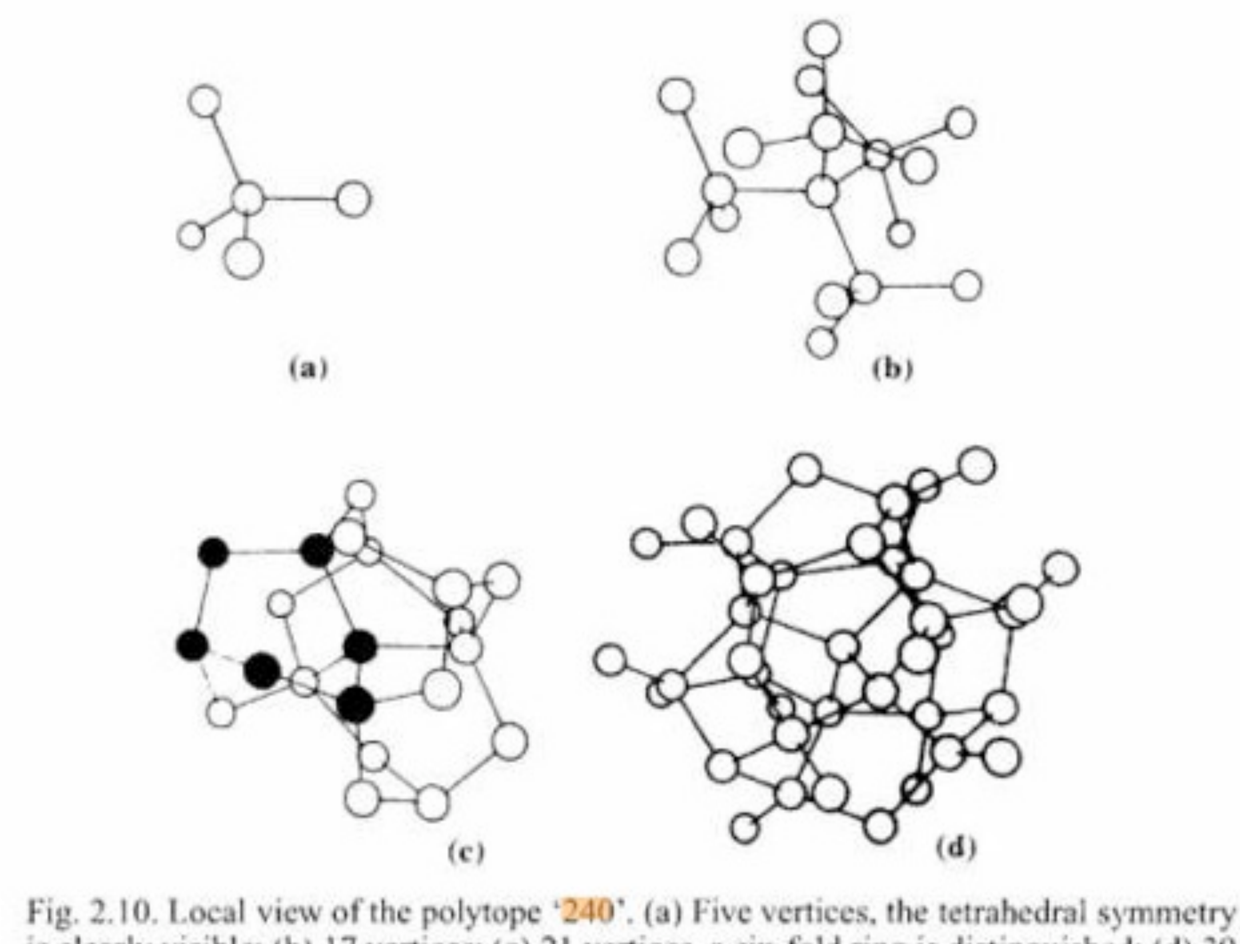
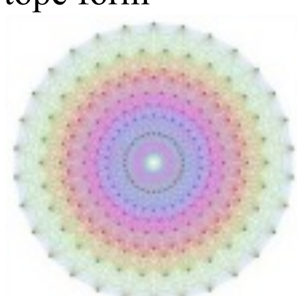


Fig. 2.10. Local view of the polytope '240'. (a) Five vertices, the tetrahedral symmetry is clearly visible; (b) 17 vertices; (c) 21 vertices, a six-fold ring is distinguished; (d) 39 vertices.

which shows orthogonal mapping of subsets of increasing size of the polytope.
... polytope 240 is locally denser than the diamond structure. This is exactly the corollary of what was said ... for the {3,3,5} compared to the f.c.c. dense structure ...
The direct symmetry group of ... polytope 240 ... is ... G240 = Y' x O' / Z2
where ... Y' ... is the binary icosahedral group ...[and]...
O' ... is the binary ... octahedral group ...
Note ... The direct symmetry group of the {3,3,5} polytope is a sub-group of SO(4) which reads G' = Y' x Y' / Z2 ... Since the order of Y' is 120, the quotient by Z2 implies that the order of G' is 7200. ... The total symmetry group G also includes indirect orthogonal transformations, analagous to reflections ... This adds 7200 new elements and gives the full group G of order 14400. ...
O' is not a subgroup of Y' ... polytope 240, while sharing some of the {3,3,5} symmetries, also has new symmetries, in particular a 40-fold screw axis ...
Another way to describe the 240 ... is to ... follow a building rule similar to that which leads to the diamond structure starting from the f.c.c. structure:
a new vertex is placed at the centre of some tetrahedral cells of the compact structure. In the f.c.c. case, one tetrahedron over two is centered, while in the present case, one tetrahedron over five will be centered, which has the consequence of breaking the five-fold symmetry of the polytope {3,3,5}, only a tenfold screw axis being preserved.
One gets a regular structure with 240 vertices, called polytope 240, which is chiral; it cannot be superimposed on its mirror image.
The polytope 240 with opposite chirality has ... O' x Y' / Z2 as its symmetry group. ...".

If 4-dimensional space is extended to 8-dimensional space by considering Golden Ratio Irrational Algebraic Coordinates to be independent,
the 240 vertices of the 240 Polytope form

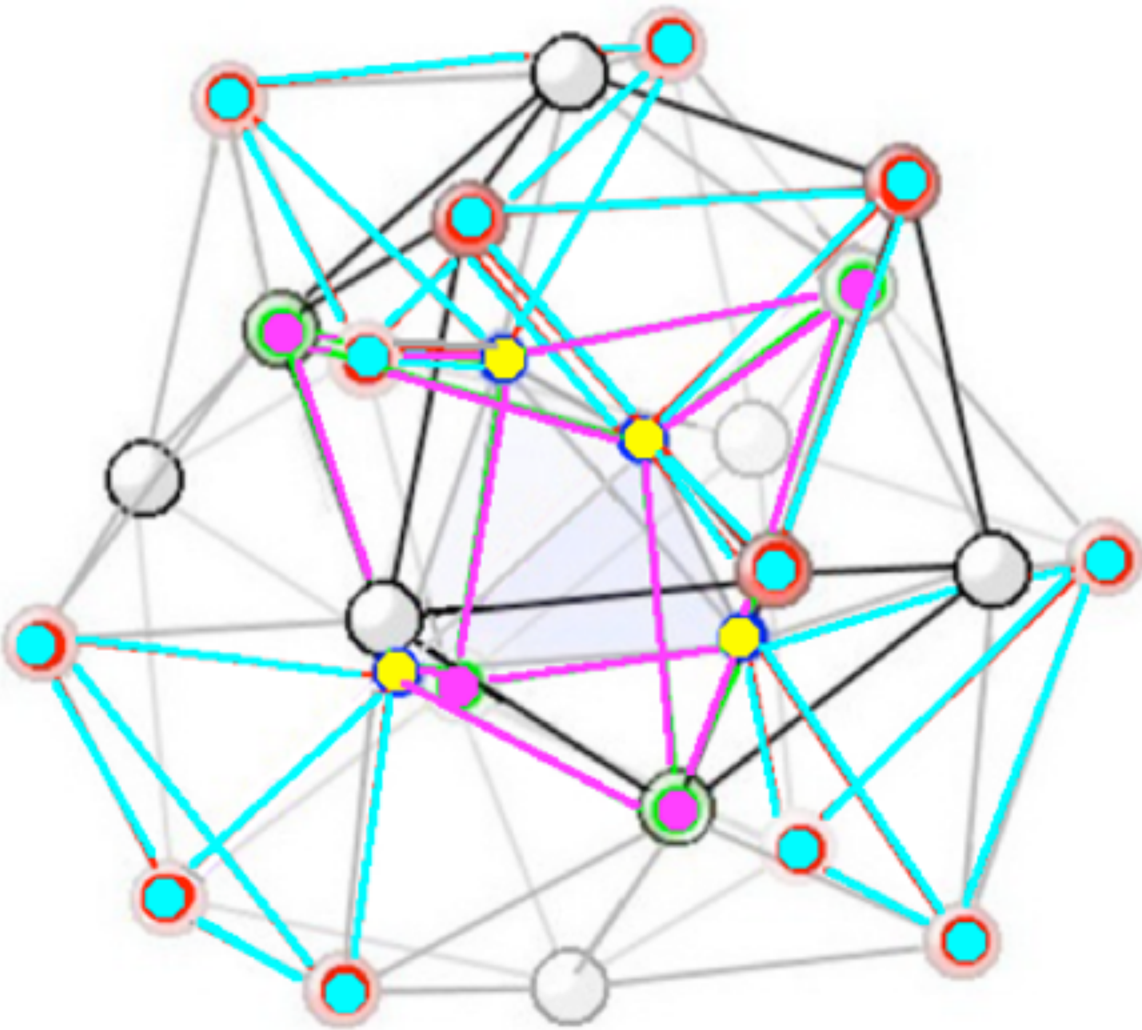


the 240 Root Vectors of the E8 Lie Algebra.
As to chirality, note that the Lie Algebra structure

248-dim E8 = 120-dim adjoint of D8 + 128-dim half-spinor of D8

does not include the other mirror image half-spinor of D8.

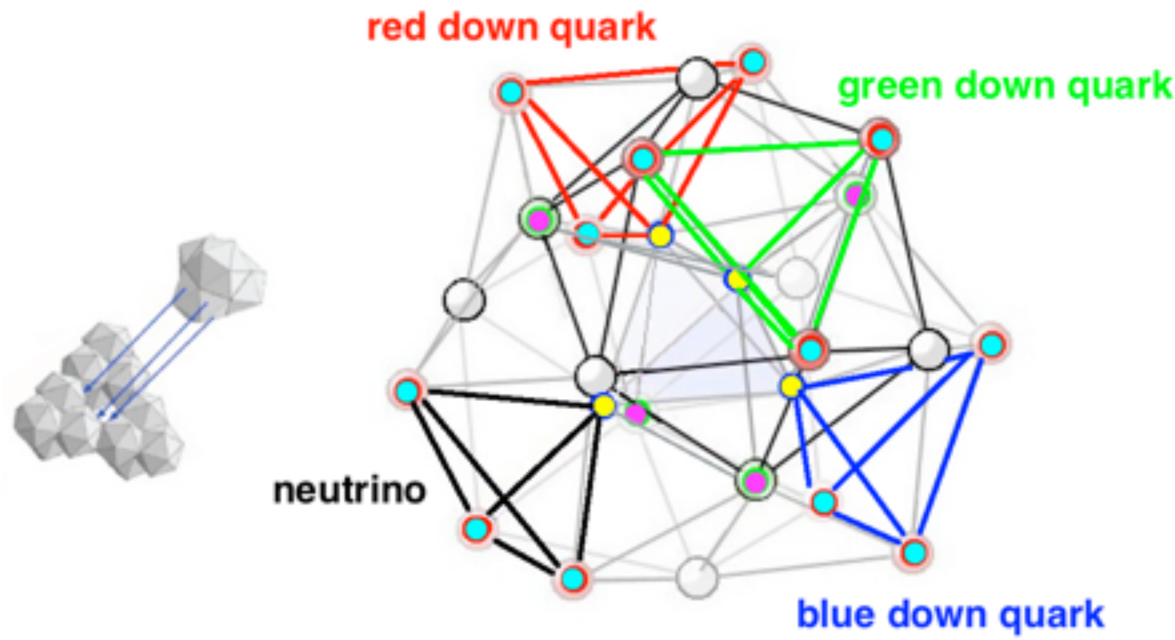
How does the 57G structure represent Physics ?



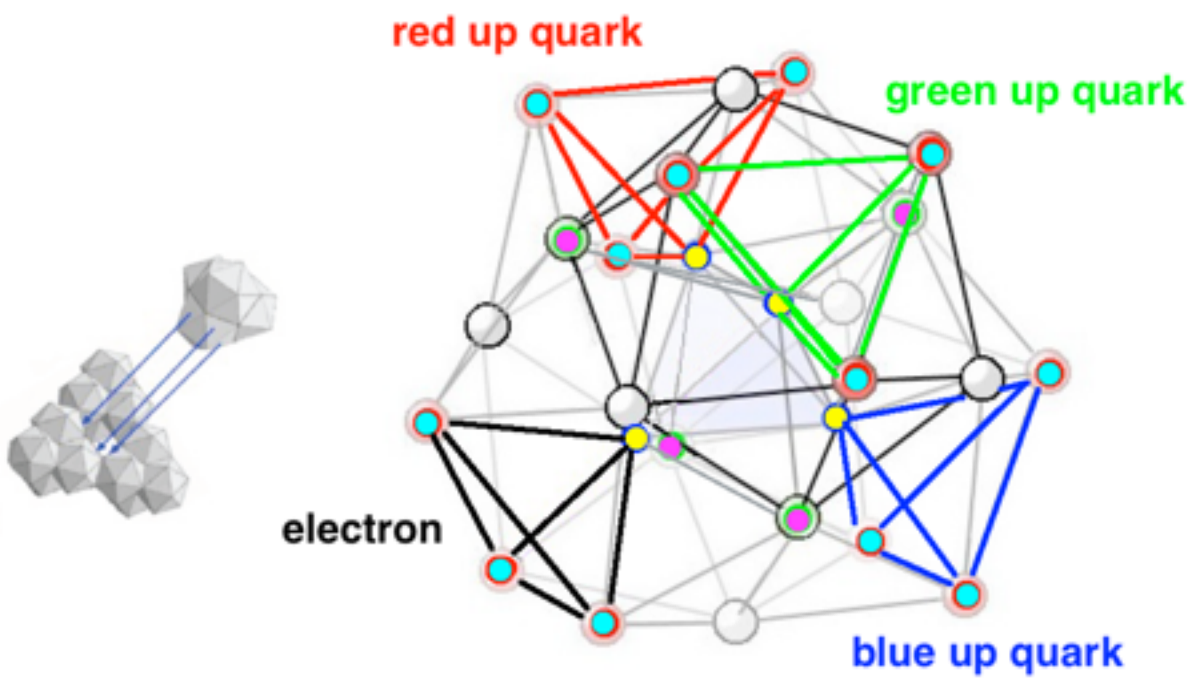
The 26 vertices and 57 tetrahedra (image from Lord et al does not show all interior edges) of a 57G are:
4 yellow vertices of the central tetrahedron
4 magenta vertices that add 4 tetrahedra in face contact with the central tetrahedron
4x3 = 12 cyan vertices adding 4 tetrahedra in vertex contact with the central tetrahedron
The 4 cyan-vertex tetrahedra represent a fundamental set: lepton + 3 RGB quarks

6 white vertices that each add 2 tetrahedra in edge contact with the central tetrahedron
and each add 6 tetrahedra in vertex contact with the central tetrahedron
for 6x8 = 48 new tetrahedra
The 48 new white-vertex tetrahedra represent Gauge Bosons

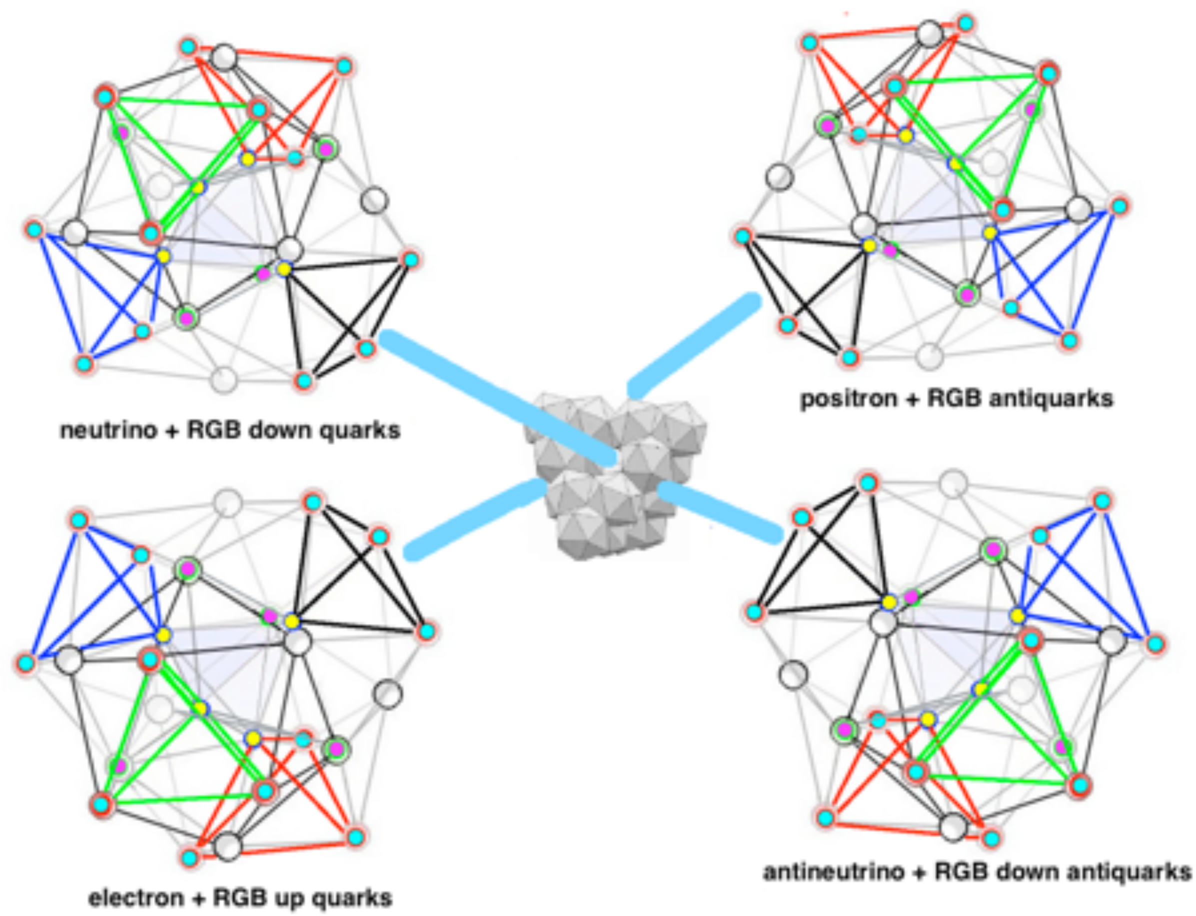
Consider the orientation of the 57G as it is inserted into the Pearce triangle and look at the representation of the neutrino + RGB down quarks



and of the electron + RGB up quarks



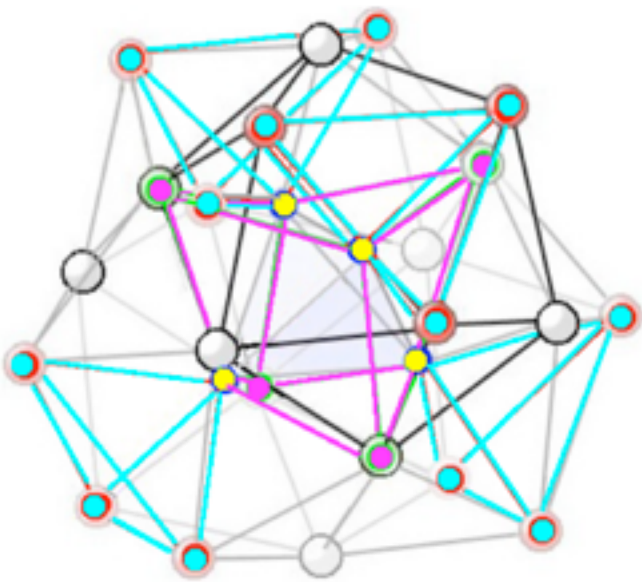
Their antiparticles are similarly represented, so four 57G account for all 16 first-generation fermion particles and antiparticles:

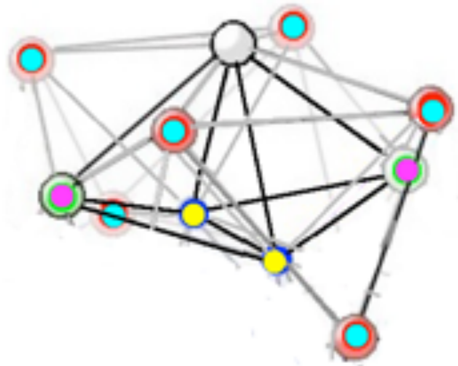


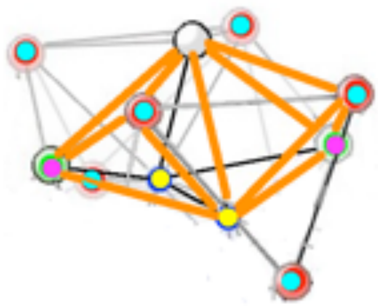
What about Gauge Bosons and Ghosts ?

The 6 white vertices and their 6x8 = 48 new tetrahedra represent Gauge Bosons

To see how they are configured look more closely at the top white vertex





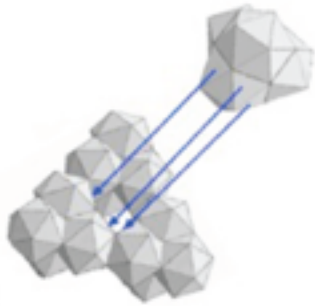


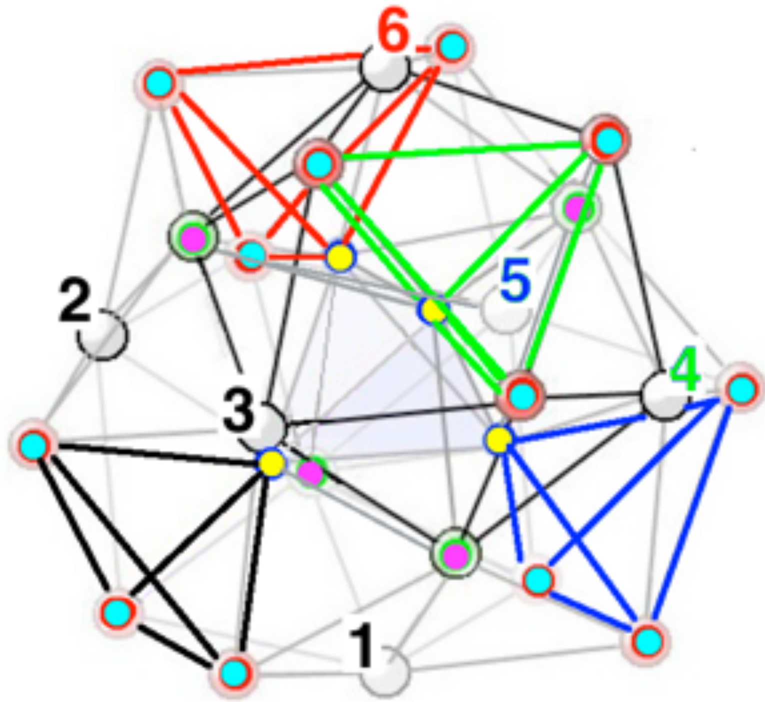
In the middle there are 2 black tetrahedra sharing a white vertex - central edge face and containing 2 magenta vertices

Then look at the front side of the middle where you see
2 orange tetrahedra with a white vertex,
connecting yellow and magenta vertices and a cyan vertex of a front cyan tetrahedron
and
1 purple tetrahedron with a white vertex, between the 2 orange tetrahedra,
connecting a yellow vertex with 2 cyan vertices of the front cyan tetrahedron

The back side of the middle also has 2+1 = 3 tetrahedra so
there are 2 + 2+1 + 2+1 = 8 new tetrahedra associated with each of the 6 white vertices.

Now look at the basic structure of 57G representation of a lepton and 3 RGB quarks:

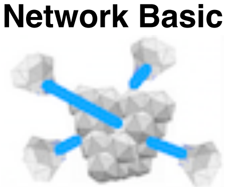




The lepton tetrahedron (black) is surrounded by 3 white vertices (labelled 1,2,3)
each of which is surrounded by 8 tetrahedra as described above so
the lepton set of 3 white vertices corresponds to 3x8 = 24 tetrahedra
which represent the 24 root vectors of the E8 subalgebra 28-dim D4
with 16 for Gauge Bosons of Gravity + Dark Energy and 12 for Standard Model Ghosts.
Upon insertion of 57G the (1,2,3) stuff is in close contact with the Pearce triangle hole.

The quark tetrahedron (green-blue-red) are surrounded by 3 white vertices (labelled 4,5,6)
each of which is surrounded by 8 tetrahedra as described above so
the quark set of 3 white vertices corresponds to 3x8 = 24 tetrahedra
which represent the 24 root vectors of the E8 subalgebra 28-dim D4
with 12 for Standard Model Gauge Bosons and 16 for Gravity + Dark Energy Ghosts.
The (4,5,6) stuff is interleaved with the quarks with Red, Green, Blue SU(3) color.

The central yellow-vertex tetrahedron of each 57G
and the 4 magenta-vertex tetrahedra in face contact with it act as
an organizing core for interactions among the 4+48 fermion/gauge boson tetrahedra.
Therefore each Diamond Network Basic Unit containing four 57G



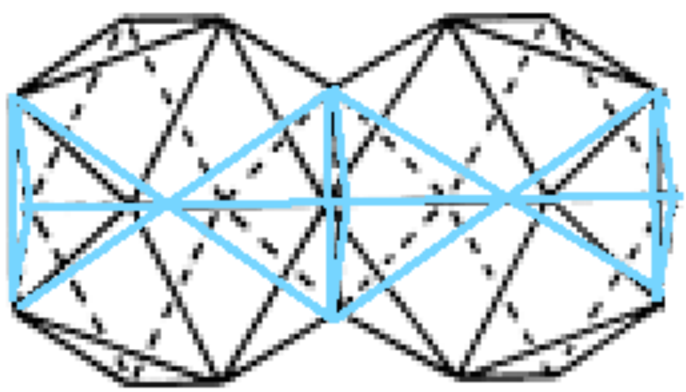
not only accounts for all 16 first-generation Fermion Particles and AntiParticles
but also accounts for the Standard Model and Gravity + Dark Energy
of the Cl(16)-E8 Physics model.

Now that we have **Fermion Particles and AntiParticles**
and **Gauge Bosons + Ghosts for the Standard Model and Gravity+ Dark Energy**

What About Spacetime ?

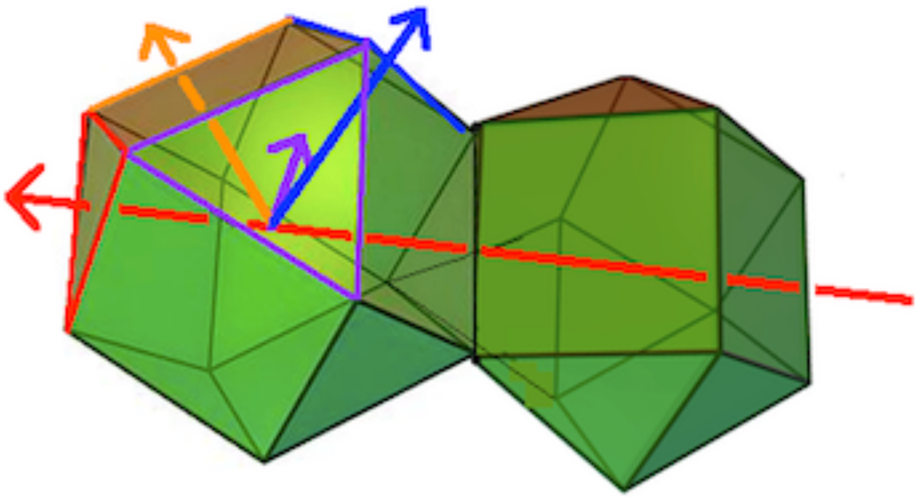
A path in the 3-dim Pearce-cluster Diamond network

is a chain of mirror-image pairs of face-sharing Icosahedra

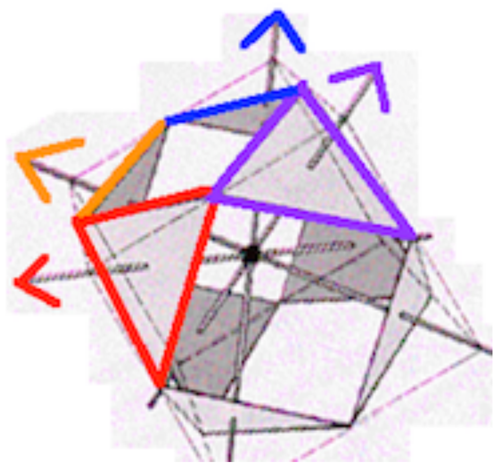


The core of each Icosahedron in the pair is a pair of vertex-sharing Tetrahedra.

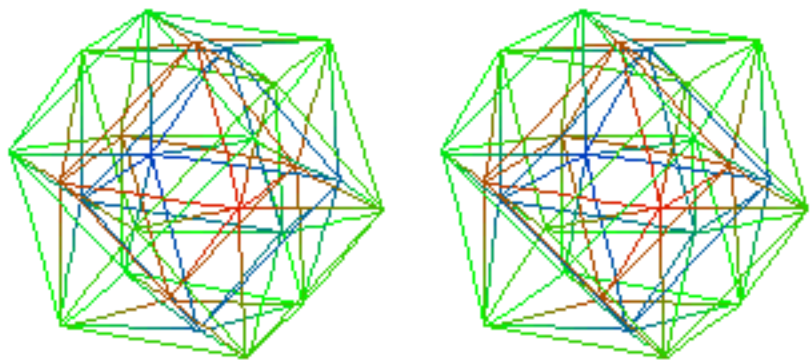
If you Jitterbug transform each Icosahedron of the pair into a Cuboctahedron you get a mirror-image pair of triangle-face-sharing Cuboctahedra



The core of of the Cuboctahedra-pair can be considered to be a 3-space axis running through triangle-faces.
If you consider Buckminster Fuller Vector Equilibrium (cuboctahedron) geometry



you see that in the Left Cuboctahedron are 3 other axes through triangle-faces that, although they plus the original axis are not independent in 3-dim space, they make 4 nice independent axes in a 4-dim space in which the cuboctahedron is the central figure of a 4-dim 24-cell



(the image is a 3-dim stereo pair with 4th dimension represented by colors red -> green -> blue)

Therefore,
the Left Cuboctahedron of the Jitterbug-transformed 3-dim Diamond network goes from 3-dim space to 4-dim space that can represent M4 Physical Spacetime
and
if you consider the Right Cuboctahedron of the mirror-pair to also have similar Buckminster Fuller Vector Equilibrium Geometry
you get another 4-dim space that can represent CP2 Internal Symmetry Space
so that

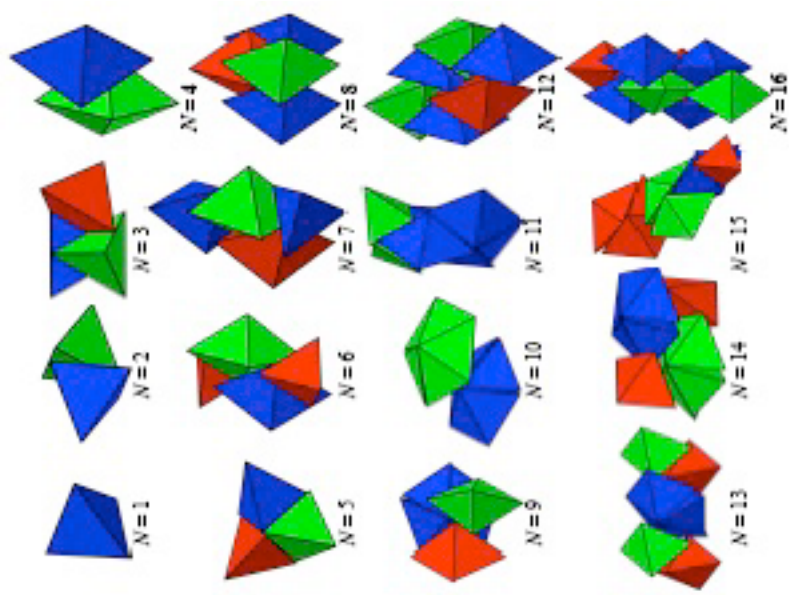
Tetrahedral 57G Physics acts in (4+4)-dim M4xCP2 Kaluza-Klein Spacetime
with 8 components of position and 8 components of momentum
corresponding to D8 / D4xD4 = 64 = 8x8 of Cl(16)-E8 Physics
and
Standard Model and Gravity + Dark Energy Gauge Bosons and Ghosts
corresponding to D4xD4 of Cl(16)-E8 Physics
and
Fundamental First-Generation Fermion 8 Particles and 8 AntiParticles
with 8-component structure from M4xCP2 Kaluza-Klein
corresponding to E8 / D8 = 128 = 8x8 + 8x8 of ClI16)-E8 Physics

Quantum Physics comes from Sum-Over-Histories Path Integral
corresponding to the Completion of Union of Tensor Products of Cl(16) AQFT of Cl(16)-E8 Physics

The 57G - 81G Pearce - 600-cell - 240 E8 construction with tetrahedra requires that the initial flat 3-dim space be curved

What happens if you require the 3-dim space to remain flat ?

If you construct with (exactly regular) tetrahedra in 3-dim space that remains flat that is like making a tetrahedral dense packing of flat 3-dim space.
The densest such packing now known is described by Chen, Engel, and Glotzer in arXiv 1001.0586 :
"... We present the densest known packing of regular tetrahedra with density Phi = 4000 / 4671 = 0.856347 ...



... The dimer structures are remarkable in the relative simplicity of the 4-tetrahedron unit cell as compared to the 82-tetrahedron unit cell of the quasicrystal approximant, whose density is only slightly less than that of the densest dimer packing.
The dodecagonal quasicrystal is the only ordered phase observed to form from random initial configurations of large collections of tetrahedra at moderate densities.
It is thus interesting to note that for some certain values of N, when the small systems do not form the dimer lattice packing, they instead prefer clusters (motifs) present in the quasicrystal and its approximant, predominantly pentagonal dipyramids. This suggests that the two types of packings - the dimer crystal and the quasicrystal/approximant - may compete, raising interesting questions about
the relative stability of the two very different structures at finite pressure. ...".

If you regard a Tetrahedron as a pair of Binary Dipoles

then the Chen - Engel - Glotzer high (0.85+) density configurations have the same 8-periodicity property as the Real Clifford Algebras:

#Binary Dipoles M	Maximum Density Numerical, $\hat{\phi}$	Analytical, ϕ	Success Rate	Motifs, Structural Description
2	0.367346	18/49	100%	1 monomer [11]
4	0.719486	ϕ_2	100%	2 monomers, transitive [22]
6	0.666665	2/3	21%	3 monomers, three-fold symmetric
8	0.856347	4000/4671	80%	2 dimers (positive + negative)
10	0.748096	ϕ_5	22%	1 pentamer, asymmetric
12	0.764058	ϕ_6	11%	2 dimers + 2 monomers
14	0.749304	3500/4671	15%	2×2 dimers minus 1 monomer
16	0.856347	4000/4671	44%	2×2 dimers, identical to $N = 4$
18	0.766081		—	1 pentagonal dipyramid + 2 dimers
20	0.829282	ϕ_{10}	2%	2 pentagonal dipyramids
22	0.794604		—	1 nonamer + 2 monomers
24	0.856347	4000/4671	3%	3×2 dimers, identical to $N = 4$
26	0.788728		4%	1 pentagonal dipyramid + 4 dimers
28	0.816834		3%	2 pentagonal dipyramids + 2 dimers
30	0.788693		—	Disordered, non-optimal
32	0.856342	4000/4671	< 1%	4×2 dimers, identical to $N = 4$
⋮	⋮			⋮
164x8	0.850267			Quasicrystal approximant [21]

which is consistent with regarding the 4 vertices of a Tetrahedron as the 4 elements of the Cl(2) Real Clifford Algebra, isomorphic to the Quaternions, with graded strucure 1+2+1, and so 4 tetrahedra as Cl(4x2) = Cl(8).

The Large N Limit of 4N Tetra Clusters =

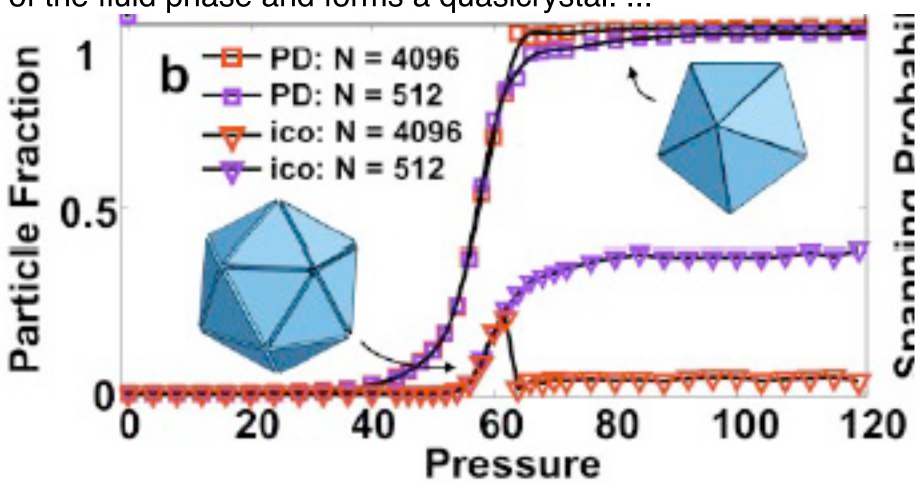
= **Completion of Union of All 4N Tetra Clusters** would correspond to

the same generalized Hyperfinite II1 von Neumann factor of Cl(16)-E8 Physics that gives a natural Algebraic Quantum Field Theory structure.

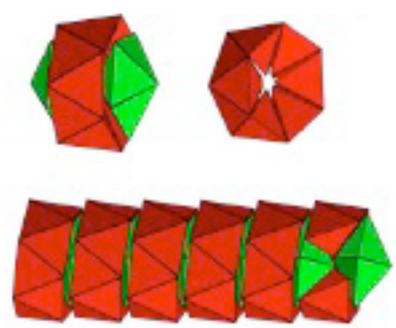
What about the QuasiCrystal / approximant in flat 3-dim space ?

Haji-Akbari1, Engel, Keys, Zheng, Petschek, Palffy-Muhoray, and Glotzer in arXiv 1012.5138 say: "... a fluid of hard tetrahedra undergoes a first-order phase transition to a dodecagonal quasicrystal,
which can be compressed to a packing fraction of ϕ = 0.8324.
By compressing a crystalline approximant of the quasicrystal,
the highest packing fraction we obtain is ϕ = 0.8503.
...

To obtain dense packings of hard regular tetrahedra, we carry out Monte-Carlo (MC) simulations ... of a small system with 512 tetrahedra and a large system with 4096 tetrahedra. ... The large system undergoes a first order transition on compression of the fluid phase and forms a quasicrystal. ...

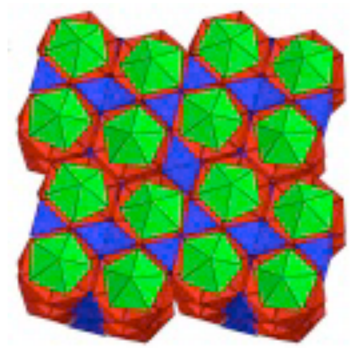


... the quasicrystal consists of a periodic stack of corrugated layers ...
Recurring motifs are rings of twelve tetrahedra that are stacked periodically to form "logs"...

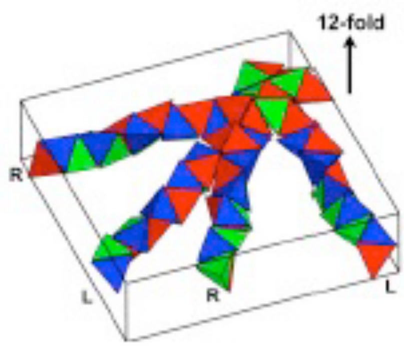


... Perfect quasicrystals are aperiodic while extending to infinity; they therefore cannot be realized in experiments or simulations, which are, by necessity, finite. ...
Quasicrystal approximants are periodic crystals with local tiling structure identical to that in the quasicrystal. Since they are closely related, and they are often observed in experiments, we consider them as candidates for dense packings.

The dodecagonal approximant with the smallest unit cell (space group) has 82 tetrahedra ...



... At each vertex we see the logs of twelve-member rings (shown in red) capped by single PDs (green). The logs pack well into squares and triangles with additional, intermediary tetrahedra (blue). The vertex configuration of the tiling is ...



...”.

The QuasiCrystal approximant is not as dense as the 4N Tetra Cluster packing,
so I do not regard it as being as useful for fundamental physics as the 4N Tetra packing.

The true QuasiCrystal is less dense than the QuasiCrystal approximant,
so I regard it as being less useful for fundamental physics. However,
as Sadoc and Mosseri say in their book “Geometrical Frustration” (Cambridge 2005)
“... quasiperiodic structures [can be] derived from the eight-dimensional lattice E8. ...
... using the cut and project method, it is possible to generate
a four-dimensional quasicrystal having the symmetry of the [600-cell] polytope {3,3,5} ...
a shell-by-shell analysis ...

Table A9.1. *Number of vertices on shells surrounding the origin in the E8 lattice. The first shell is a Gosset polytope in eight dimensions*

N	Squared radius r^2	Vertices on $E8$ shell
1	1/2	240
2	1	2160
3	3/2	6720

... recalls in some respects ... the Fibonacci chain ...

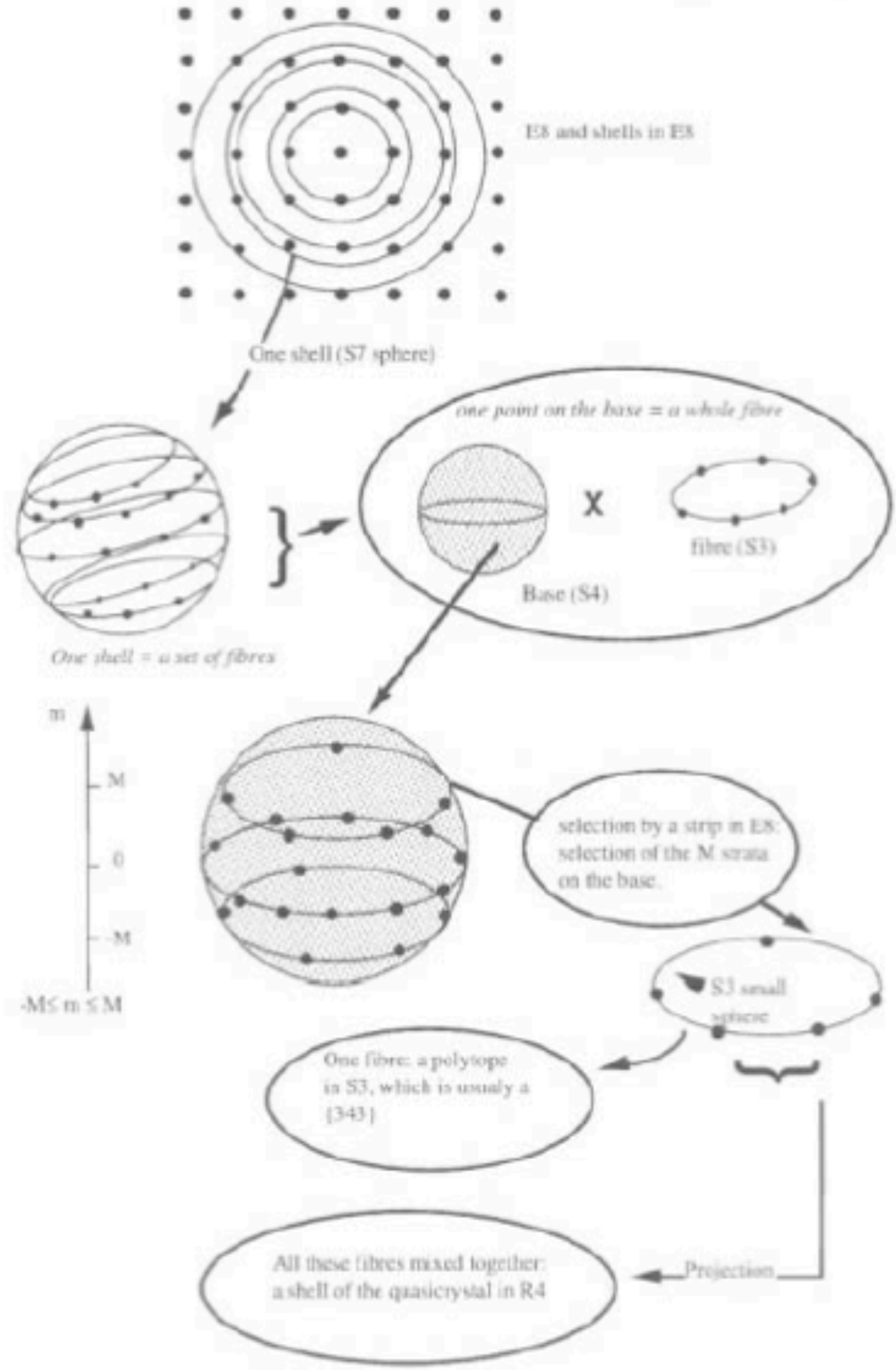


Fig. A9.1. Scheme summarizing the four-dimensional construction method: take an $E8$ shell, considered as a discrete fibration of S^7, select the fibres which map (H-map) onto a stratum M of the base of the fibration, and finally orthogonally map (O-map) the selected sites onto R^4.

…".

The relationship between QuasiCrystals and QuasiCrystal approximants is discussed by An Pang Tsai in an IOP review “Icosahedral clusters, icosahedral order and stability of quasicrystals - a view of metallurgy”:

“... we overview the stability of quasicrystals ... in relation to phason disorder ...
the phonon variable leads to long wavelength and low energy distortion of crystals, the phason variable in quasicrystals leads to a ... type of distortion ...
Let a two-dimensional lattice points sit at the corners of squares in a grid.
... a strip with a slope of an irrational number ... golden mean ... is ... a Fibonacci sequence and is exactly a one-dimensional quasicrystal ...
... [if] the slope of the strip is ... a rational number ...[it]... is a periodic sequence ... [and]... is called an approximant ...
in the approximant where the sequence changes by a flip ... This flip is called phason flip ... a flipping of tiles in two-dimensions or three-dimensions ...

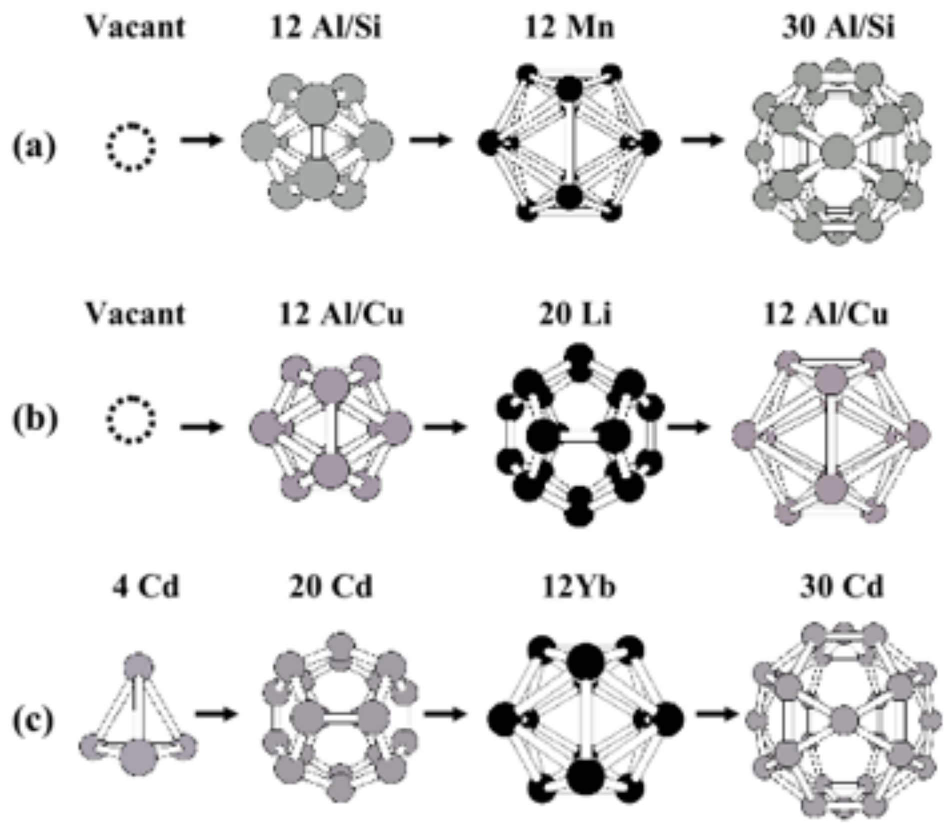


Figure 3. Concentric structures of three types of icosahedral clusters derived from three 1/1 approximants of quasicrystals. (a) The Al–Mn–Si class or Mackay icosahedral cluster: the center is vacant, the 1st shell is an Al/Si icosahedron, the 2nd shell is a Mn icosahedron, and the 3rd shell is an Al/Si icosidodecahedron. (b) The Zn–Mg–Al class or Bergman cluster: an example is R-AlLiCu: the center is vacant, the 1st shell is an Al/Cu icosahedron, the 2nd shell is a Li dodecahedron, the 3rd shell is a larger Al/Cu icosahedron. (c) The Cd–Yb class: the center is a Cd tetrahedron, the 1st shell is a Cd dodecahedron, the 2nd shell is a Yb icosahedron, and the 3rd shell is a Cd icosidodecahedron.

... ‘phason strain’ ... is the characteristic disorder for quasicrystals but does not exist in crystals ...
a fully annealed stable iQc [icosahedral quasicrystal]... is almost free of phason disorder ...”.