Generalized Relation of Unifying All Uncertainty Relations with Dimension

Yong Bao

Postbox 777, 100 Renmin South Road, Luoding 527200, Guangdong, China
E-mail: baoyong9803@163.com

We investigate the relations between the physical quantities and the physical constants by the dimensional analysis. From normal form of uncertainty relations (URs), we propose an assumption which the physical quantities have the formal symmetry with the physical constants; and find three basic formulas. A physical quantity has a corresponding Planck scale and many physical quantities have the same Planck scales because of the same dimensions. All Planck scales can be classified by two methods, one is the basic Planck scale and derived Planck scale, and another is Femi-Planck scale, Bose-Planck scale and other-Planck scale. The basic formulas can be rewritten as the ones of corresponding Planck scales. We find the generalized relation which the power products of physical quantities are equivalent to the ones of corresponding Planck scales. Heisenberg UR is part of the generalized relation when the reduced Planck constant ħ appears. We also find the Big Bang UR between its temperature and volume by the generalized relation, and the Schwarzschild black holes (SBH) UR between its mass and volume. These suggest no singularity at Big Bang and in SBH with the quantum effect. We show that the generalized relation is interesting and significant.

1. Introduction

It has the great effect of Heisenberg uncertainty principle [1], its application, development and experiment made great progress, please refer to the literature [2-7]. These founded the hard foundation for Heisenberg uncertainty principle and extended its connotation.

Now there are many uncertainty relations (URs) with dimension: $\Delta p\Delta r \geq \hbar$ [1]; $\Delta E\Delta t \geq \hbar$ [1]; $\delta t = \beta\, t_P^{2/3} t^{1/3}$ [8]; $\eta / s \geq 4\pi\hbar / \kappa$ [9]; $\Delta T\Delta X \sim L_S^2 \sim L_P^2$ [10]; $L_{\mu\nu} \sim \sqrt{L_P L}$ [11]; $\varepsilon(Q)\eta(P) + \varepsilon(Q)\sigma(P) + \sigma(Q)\eta(P) \geq \hbar / 2$ [7]; $(\delta t)(\delta r)^3 \geq \pi r^2 L_P^2$ [12], etc.

where Δp is the momentum fluctuation, Δr is the position momentum, ħ is the reduced Planck constant; ΔE is the energy fluctuation, Δt is the time fluctuation; δt is the time fluctuation, β is an order one constant, $t_P = \sqrt{\hbar G/c^5}$ is Planck time, G is the gravitational constant, c is the speed of light, t is the time; η is the ratio of shear viscosity of a given fluid perfect, s is its volume density of entropy, κ is the Boltzmann constant; ΔT is the time-like, ΔX is its space-like, L_S is the string scale, $L_P = \sqrt{\hbar G/c^3}$ is Planck length; $L_{\mu\nu}$ is the transverse length, L is the radial length; Q is the position of a mass, $\varepsilon(Q)$ is the root-mean-square error, P is its momentum, $\eta(P)$ is the root-mean-square disturbance, $\sigma(P)$ is the standard deviation; δt and δr are the sever space-time fluctuations of the constituents of the system at small scales, and r is the radius of globular computer.

So there are two problems: (i) Why hasn't G on some formulas right hand? (ii) Whether has the unitive form for them? In this paper, we solve these problems. Moreover, for the origin and development of Planck length, Planck time, Planck mass $M_P = \sqrt{\hbar c/G}$, Planck energy $E_P = \sqrt{\hbar c^5/G}$ and Planck temperature $T_P = \sqrt{\hbar c^5/\kappa^2 G}$, please see [13-17].

This paper is organized as follows. In Sec. 2, we propose an assumption, and derive three basic formulas. In Sec. 3, we obtain the Planck scales and classify them. In Sec. 4, we prove the basic formulas being rewritten as the one of corresponding Planck scales, and find the generalized relation. In Sec. 5, we find the Big Bang UR and SBH UR. We conclude in Sec. 6.

2. An Assumption and Three Basic Formulas

In this section, we propose an assumption, and derive three basic formulas.

2.1 An assumption and basic formula I

Observing these URs, we can discover the physical constants such as ħ, G, c and κ on the right hand and the physical quantities on left hand. These are their normal form. Applying the π law [18], N physical quantities can be expressed the power of basic ones. First we consider the mechanics, using the Gauss units, we obtain

$$X \sim r^{\alpha} m^{\beta} t^{\gamma} \tag{1}$$

where X is N physical quantities, r, m and t are the length, mass and time separately, α, β and γ are the real number. From the normal form of above URs, we can assume

$$r^{\alpha} m^{\beta} t^{\gamma} \sim \hbar^{x} G^{y} c^{z} \tag{2}$$

where x, y and z are the unknown number. (2) shows that the physical quantities have the beautiful formal symmetry with the physical constants. By the dimensional analysis [18], we obtain

$$[L]^\alpha[M]^\beta[T]^\gamma \sim \{[L^2][M][T^{-1}]\}^x\{[L^3][M^{-1}][T^{-2}]\}^y\{[L][T^{-1}]\}^z \quad (3)$$

Solving (3) we find

$$x = (\alpha+\beta+\gamma) / 2,\ y = (\alpha-\beta+\gamma) / 2,\ z = (3\alpha-\beta+5\gamma) / 2$$

$$r^\alpha m^\beta t^\gamma \sim [\hbar^{(\alpha+\beta+\gamma)}G^{(\alpha-\beta+\gamma)}c^{(3\alpha-\beta+5\gamma)}]^{1/2} \quad (4)$$

This is the basic formula I.

2.2 Basic formula II and III

The formula I doesn't include the temperature T and electric charge Q; we can find the basic formula II and III by the same method.

$$r^\alpha m^\beta t^\gamma\ T^\delta \sim [\hbar^{(\alpha+\beta+\gamma+\delta)}G^{(\alpha-\beta+\gamma-\delta)}c^{(3\alpha-\beta+5\gamma-5\delta)}\kappa^{-2\delta}]^{1/2} \quad (5)$$

$$r^\alpha m^\beta t^\gamma T^\delta Q^\varepsilon \sim [\hbar^{(\alpha+\beta+\gamma+\delta)}G^{(\alpha-\beta+\gamma-\delta)}c^{(3\alpha-\beta+5\gamma-5\delta)}\kappa^{-2\delta}e^{2\varepsilon}]^{1/2} \quad (6)$$

where δ and ε are the real number, and e is the elementary charge. We can look I and II as III the special examples. Note that (5) and (6) are used the five units.

3. Planck Scales

In this section, we obtain the Planck scales, and classify them.

3.1 Basic Planck scales

Ordering $\alpha = 1$, $\beta = \gamma = \delta = \varepsilon = 0$ in (6), we obtain Planck length immediately

$$r_P = L_P = \sqrt{\hbar G/c^3}$$

Instructing $\gamma = 1$, $\alpha = \beta = \delta = \varepsilon = 0$, obtain Planck time

$$t_P = \sqrt{\hbar G/c^5}$$

Ordering $\beta = 1$, $\alpha = \gamma = \delta = \varepsilon = 0$, obtain Planck mass

$$m_P = M_P = \sqrt{\hbar c/G}$$

Instructing $\delta = 1$, $\alpha = \beta = \gamma = \varepsilon = 0$, obtain Planck temperature

$$T_P = \sqrt{\hbar c^5/\kappa^2 G}$$

Ordering $\varepsilon = 1$, $\alpha = \beta = \gamma = \delta = 0$, obtain elementary charge (or Planck charge)

$$Q_P = Q_e = e$$

These are the basic Planck scales.

3.2 Derived Planck scales

From (1) and (6), a physical quantity has a corresponding Planck scale. For example

Planck energy E_P

$$[E_P] = [L^2][M][T^{-2}],\ E_P = \sqrt{\hbar c^5/G}$$

Planck momentum P_P

$$[P_P] = [L][M][T^{-1}],\ P_P = \sqrt{\hbar c^3/G}$$

Because many physical quantities have the same dimensions, they have the same Planck scales, for example

Planck energy density ρ_P

$$[\rho_P] = [L^{-1}][M][T^{-2}],\ \rho_P = c^7 / \hbar G^2$$

Planck pressure p_P

$$[p_P] = [L^{-1}][M][T^{-2}],\ p_P = c^7 / \hbar G^2$$

Planck force per unit area f_P

$$[f_P] = [L^{-1}][M][T^{-2}],\ f_P = c^7 / \hbar G^2$$

Etc. These are belonging to the derived Planck scales.

3.3 Classification

We can classify all the Planck scales by two methods. First are basic Planck scales and derived Planck scales. Second are that One's power is the half integer, call it Femi-Planck scale, such as L_P, t_P, M_P, T_P, E_P, P_P, etc; another is the integer, call it Bose-Planck scale, such as Q_e, ρ_P, p_P, f_P, etc; others call other-Planck scale, such as the Planck wave function ψ_P

$$[\psi_P] = [L^{-3/2}],\ \psi_P = (\hbar G / c^3)^{-3/4}$$

4. Generalized Relation

In this section, we prove that (6) can be rewritten as the one of corresponding Planck scales, find the generalized relation, and prove the URs in Sec. 2.

4.1 Proof

The basic formula III can be rewritten as the one of corresponding Planck scales

$$r^\alpha m^\beta t^\gamma T^\delta Q^\varepsilon \sim L_P^\alpha M_P^\beta t_P^\gamma T_P^\delta Q_e^\varepsilon \quad (7)$$

We prove (7) now. From (6), we obtain

$$r^\alpha m^\beta t^\gamma T^\delta Q^\varepsilon \sim$$
$$[\hbar^\alpha G^\alpha c^{-3\alpha}]^{1/2}[\hbar^\beta G^{-\beta} c^\beta]^{1/2}[\hbar^\gamma G^\gamma c^{-5\gamma}]^{1/2}[\hbar^\delta G^{-\delta} c^{-5\delta}]^{1/2}\kappa^{-\delta}e^\varepsilon$$
$$= [\hbar G / c^3]^{\alpha/2}[\hbar c / G]^{\beta/2}[\hbar G / c^5]^{\gamma/2}[\hbar c^5 / \kappa^2 G]^{\delta/2}e^\varepsilon$$
$$= L_P^\alpha M_P^\beta t_P^\gamma T_P^\delta Q_e^\varepsilon$$

So the basic formula III is equivalent to the one of corresponding Planck scales.

4.2 Generalized relation

Considering all the physical quantities, we find

$$\prod_{i=1}^{n} A_i^{\alpha_i} \sim \prod_{i=1}^{n} A_{iP}^{\alpha_i};\quad i = 1, 2, 3\ldots n \quad (8)$$

where A_i is the physical quantity, α_i is the real number, and A_{iP} is the corresponding Planck scale. This is the generalized relation. It shows that the power products of physical quantities are equivalent to the ones of corresponding Planck scales.

4.3 Proving URs

Distinctly when ħ appears on the generalized relation (8) right hand, it becomes the Heisenberg UR. For example

$\Delta p \Delta r \sim P_P L_P = \sqrt{\hbar c^3/G}\sqrt{\hbar G/c^3} = \hbar$; $\Delta E \Delta t \sim E_P t_P =$

$\sqrt{\hbar c^5/G}\sqrt{\hbar G/c^5} = \hbar$; $\delta t / t^{1/3} \sim t_P / t_P^{1/3} = t_P^{2/3}$; $\eta / s \sim \eta_P / s_P = \sqrt{c^9/\hbar G^3} / \sqrt{c^9\kappa^2/\hbar^3 G^3} = \hbar / \kappa$; $\Delta T\Delta X \sim t_P L_P \sim \hbar G / c^4 = c L_P^2 \sim L_S^2$; $L_{\mu\nu} / \sqrt{L} \sim L_P / \sqrt{L_P} = \sqrt{L_P}$; $\varepsilon(Q)\eta(P) + \varepsilon(Q)\sigma(P) + \sigma(Q)\eta(P) \sim \sqrt{\hbar G/c^3}\sqrt{\hbar c^3/G} = \hbar$; $(\delta t)(\delta r)^3 / r^2 \sim t_P L_P^3 / L_P^2 \sim L_P^2$, etc.

where $\eta_P = \sqrt{c^9/\hbar G^3}$ is the Planck ratio of shear viscosity of a given fluid perfect, and $s_P = \sqrt{c^9\kappa^2/\hbar^3 G^3}$ is its Planck volume density of entropy. Thus we find that there hasn't G on some formulas right hand because it is reduced fitly.

5. No singularity at Big Bang and SBH

In this section, we find the Big Bang UR and SBH UR by the generalized relation.

5.1 Big Bang UR

S.W. Hawking and R. Penrose proved that the universe originated the Big Bang singularity [19]. Many literatures discussed no singularity at the Big Bang and black holes with the quantum effect, please refer to [17] [20-23]. The one of the characteristic of Big Bang singularity is zero volume and limitless high temperature.

Then we can find the relation of Big Bang temperature and its volume by the generalized relation

$$T_B V_B \sim T_P V_P = T_P L_P^3 = \hbar^2 G / \kappa c^2 \qquad (9)$$

where T_B is the Big Bang temperature, V_B is its volume, and $V_P = L_P^3$ is the Planck volume. This is the Big Bang UR. (9) shows that it is impossible to measure the Big Bang temperature and its volume simultaneously. When $\hbar \to 0$, we obtain

$$T_B V_B \sim 0 \qquad (10)$$

Because $T_B > 0$, we gain $V_B \sim 0$, the Big Bang volume is zero, thus the Big Bang singularity appears without the quantum effect. We suggest no singularity at the Big Bang with quantum effect. Substituting $a = c\kappa T / 2\pi\hbar$ [24] into (9), we obtain

$$a_B V_B \sim a_p V_p = \hbar G / 2\pi c \qquad (11)$$

where a_B is the Big Bang acceleration, and $a_p = \sqrt{c^7/\hbar G}$ is the Planck acceleration. It is the UR between Big Bang acceleration and its volume.

5.2 SBH UR

Similarly considering the mass and volume of SBH, we find

$$M_H V_H \sim M_P V_P = M_P L_P^3 = \hbar^2 G / c^4 \qquad (12)$$

where M_H is the SBH mass, and V_H is its volume. Also it is impossible to measure the SBH mass and volume simultaneously. When $\hbar \to 0$, we obtain

$$M_H V_H \sim 0 \qquad (13)$$

Because $M_H > 0$, we have $V_H \sim 0$, the volume is zero, the SBH singularity appears without quantum effect also. We also suggest no singularity in SBH with quantum effect. Taking $M = \rho V$ to (12), we gain

$$M_H^2 / \rho_H \sim \hbar^2 G / c^4, \quad \rho_H V_H^2 \sim \hbar^2 G / c^4 \qquad (14)$$

where ρ_H is the mass density of SBH. These are the URs between the mass density of SBH and its mass or volume.

6. Conclusion

In this paper, we investigate the relations between the physical quantities and the physical constants by the dimensional analysis. We find the following results.

1) An assumption is proposed. It shows that the physical quantities have the formal symmetry with the physical constants.

2) Three basic formulas are found. The power product of the length, mass, time, temperature and electric charge is equivalent to the one of the reduced Planck constant ħ, gravitational constant G, speed of light c, Boltzmann constant κ and elementary charge e.

3) A physical quantity has a corresponding Planck scale from the basic formulas. The Planck length, Planck time, Planck mass, Planck temperature, elementary charge (or Planck charge), Planck energy, Planck momentum, Planck energy density etc are found. Many physical quantities have the same Planck scales because of the same dimensions.

4) All the Planck scales are classified by two methods. First are the basic Planck scales and derived Planck scales. Second are the Femi-Planck scales which power being the half integer, the Bose-Planck scales which power being the integer, and the other-Planck scales which power being others.

5) The basic formulas can be rewritten as the one of corresponding Planck scales. The power product of the length, mass, time, temperature and electric charge is equivalent to the one of Planck length, Planck mass, Planck time, Planck temperature and elementary charge.

6) The generalized relation is found. It shows that the power products of the physical quantities are equivalent to the ones of corresponding Planck scales. The Heisenberg UR is part of the generalized relation when ħ appears. G disappears on some URs because of being reduced fitly.

7) The Big Bang UR between its temperature and volume is found by the generalized relation. It suggests no singularity at the Big Bang with the quantum effect. The UR between Big Bang acceleration and its volume is obtained. Similarly the SBH UR between its mass and volume is found; also no singularity is in SBH with quantum effect. The URs between the mass density of SBH and its mass or volume is gained.

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