Generalized Relation of Unifying All Uncertainty Relations with Dimensions

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We propose the generalized relation to unify all the uncertainty relations (URs) with dimensions by the dimensional analysis. Here we find and prove the general expression of URs which the products of two non-commutative physical quantities (PQs) with dimensions are equivalent to the power products of the reduced Planck constant ħ, gravitational constant G, speed of light in vacuum c, Boltzmann constant κ and elementary charge e, and the basic relation that any physical quantity with dimension has a corresponding Planck scale. Many PQs have the same Planck scale because of same dimensions. All Planck scales are classified by two methods, one is the basic Planck scale and derived Planck scale, and another is Femi-Planck scale, Bose-Planck scale and Other-Planck scale. The corresponding Planck scale of any physical quantity is proved to be equivalent to the power products of the Planck length $\mathrm{L_P}$, Planck time $\mathrm{t_P}$, Planck mass $\mathrm{M_P}$, Planck temperature $\mathrm{T_P}$ and elementary charge $\mathrm{Q_e}$ (or Planck charge). The generalized relation is found and proved that the power products of non-commutative PQs are equivalent to the ones of corresponding Planck scales. We also find the Big Bang UR between its temperature and volume by the generalized relation, and the Schwarzschild black holes (SBH) UR between its mass and volume. These URs suggest no singularity at Big Bang and in SBH with the quantum effect. We show that the generalized relation is generalized, interesting and significant.

1. Introduction

The Heisenberg uncertainty principle [1] made great progress in the application [2, 3], development [4, 5] and experiment [6, 7]. These founded the firm foundation for it and extended its connotation. Now there are many uncertainty relations (URs) with dimensions:

$\Delta p\Delta r \geq \hbar$ [1]; $\Delta E\Delta t \geq \hbar$ [1]; $\delta t = \beta\, \mathrm{t_P}^{2/3} t^{1/3}$ [8]; $\eta / s \geq 4\pi\hbar / \kappa$ [9]; $\Delta T\Delta X \sim \mathrm{L_S^2} \sim \mathrm{L_P^2} / \mathrm{c}$ [10]; $\delta x\delta y\delta t \sim \mathrm{L_P^3} / \mathrm{c}$ [11]; $L_{\mu\nu} \sim \sqrt{\mathrm{L_P} L}$ [12]; $\varepsilon(Q)\eta(P) + \varepsilon(Q)\sigma(P) + \sigma(Q)\eta(P) \geq \hbar / 2$ [7]; $(\delta t)(\delta r)^3 \geq \pi r^2 \mathrm{L_P^2} / \mathrm{c}$ [13], etc.

where Δp is the momentum fluctuation, Δr is the position momentum, ħ is the reduced Planck constant; ΔE is the energy fluctuation, Δt is the time fluctuation; δt is the time fluctuation, β is an order one constant, $\mathrm{t_P} = \sqrt{\hbar \mathrm{G}/\mathrm{c}^5}$ is Planck time, G is the gravitational constant, c is the speed of light in vacuum, t is the time; η is the ratio of shear viscosity of a given fluid perfect, s is its volume density of entropy, κ is the Boltzmann constant; ΔT is the time-like, ΔX is its space-like, $\mathrm{L_S}$ is the string scale, $\mathrm{L_P} = \sqrt{\hbar \mathrm{G}/\mathrm{c}^3}$ is Planck length; δx, δy, δt are the position fluctuation and time fluctuation separately; $L_{\mu\nu}$ is the transverse length, L is the radial length; Q is the position of a mass, $\varepsilon(Q)$ is the root-mean-square error, P is its momentum, $\eta(P)$ is the root-mean-square disturbance, $\sigma(P)$ is the standard deviation; δt and δr are the sever space-time fluctuations of the constituents of the system at small scales, and r is the radius of globular computer.

So there are two problems: (i) Why hasn't G on some formulas rights hand? (ii) Whether has the unitive form for them? In this paper, we answer that G disappears because of being reduced fitly and the unitive form is the generalized relation. Moreover, for the origin and development of Planck length, Planck time, Planck mass $\mathrm{M_P} = \sqrt{\hbar \mathrm{c}/\mathrm{G}}$, Planck energy $\mathrm{E_P} = \sqrt{\hbar \mathrm{c}^5/\mathrm{G}}$ and Planck temperature $\mathrm{T_P} = \sqrt{\hbar \mathrm{c}^5/\kappa^2 \mathrm{G}}$, please refer to the literature [14-18].

This paper is organized as follows. In Sec. 2, we derive the general expression of URs and basic relation, and prove them. In Sec. 3, we obtain the Planck scales and classify them. In Sec. 4, we prove the corresponding Planck scale of any physical quantity being rewritten as the power products of basic Planck scales; find and prove the generalized relation, and prove the URs in Sec. 1. In Sec. 5, we find the Big Bang UR and SBH UR. We conclude in Sec. 6.

2. General expression of URs and Basic Relation

In this section, we discover the normal form of URs, derive the general expression of URs and basic relation, and prove them.

2.1 General expression of URs

Observing these URs, we can discover the physical constants such as ħ, G, c and κ on the right hand and the physical quantities

(PQs) on left hand. We rewrite them as

$\Delta p\Delta r \geq \hbar^1$; $\Delta E\Delta t \geq \hbar^1$; δt / $\beta t^{1/3}$ = $t_P^{2/3}$= $\hbar^{1/3}G^{1/3}c^{-5/3}$; η / $4\pi s \geq \hbar\kappa^{-1}$; $\Delta T\Delta X \sim L_S^2 \sim L_P^2$ / c = $\hbar Gc^{-4}$; $\delta x\delta y\delta t \sim L_P^3$ / c = $\hbar^{3/2}G^{3/2}c^{-11/2}$; $L_{\mu\nu}$ / $\sqrt{L}\sim\sqrt{L_P}$ = $\hbar^{1/4}G^{1/4}c^{-3/4}$; $2[\varepsilon(Q)\eta(P)+\varepsilon(Q)\sigma(P)+\sigma(Q)\eta(P)] \geq \hbar^1$; $(\delta t)(\delta r)^3$ / $\pi r^2 \geq L_P^2$ / c = $\hbar Gc^{-4}$, etc.

Therefore the physical constants appear power products on the right hand. These are their normal form. Considering two non-commutative PQs with dimensions, we obtain the general expression of URs

$$AB \sim \hbar^x G^y c^z \kappa^w e^u \tag{1}$$

where A and B are non-commutative PQs, x, y, z, w and u are the unknown number, and e is the elementary charge. Applying the dimensional analysis [19] (here we use the LMtTQ units [20][1]), the dimensions of A and B are expressed as

$$[A] = [L]^{\alpha_1}[M]^{\beta_1}[t]^{\gamma_1}[T]^{\delta_1}[Q]^{\varepsilon_1}$$
$$[B] = [L]^{\alpha_2}[M]^{\beta_2}[t]^{\gamma_2}[T]^{\delta_2}[Q]^{\varepsilon_2} \tag{2}$$

where L, M, t, T and Q are the dimensions of length, mass, time, temperature and electric charge separately, α_1, α_2, β_1, β_2, γ_1, γ_2, δ_1, δ_2, ε_1 and ε_2 are the known real number. The dimensions of $\hbar^x G^y c^z \kappa^w e^u$ are

$$[\hbar^x G^y c^z \kappa^w e^u] = \{[L^2][M][t^{-1}]\}^x\{[L^3][M^{-1}][t^{-2}]\}^y\{[L][t^{-1}]\}^z \times\{[L^2][M][t^{-2}][T^{-1}]\}^w\{[Q]\}^u \tag{3}$$

By the dimensional analysis, we obtain

$$[L]^{\alpha_1}[M]^{\beta_1}[t]^{\gamma_1}[T]^{\delta_1}[Q]^{\varepsilon_1}\ [L]^{\alpha_2}[M]^{\beta_2}[t]^{\gamma_2}[T]^{\delta_2}[Q]^{\varepsilon_2}$$
$$= \{[L^2][M][t^{-1}]\}^x\{[L^3][M^{-1}][t^{-2}]\}^y\{[L][t^{-1}]\}^z$$
$$\times\{[L^2][M][t^{-2}][T^{-1}]\}^w\{[Q]\}^u \tag{4}$$

Solving the equation (4), we gain

$$x = [(\alpha_1+\alpha_2)+(\beta_1+\beta_2)+(\gamma_1+\gamma_2)+(\delta_1+\delta_2)] / 2,$$
$$y = [(\alpha_1+\alpha_2)-(\beta_1+\beta_2)+(\gamma_1+\gamma_2)-(\delta_1+\delta_2)] / 2$$
$$z = -[3(\alpha_1+\alpha_2)-(\beta_1+\beta_2)+5(\gamma_1+\gamma_2)-5(\delta_1+\delta_2)] / 2,$$
$$w = -(\delta_1+\delta_2),\ u = (\varepsilon_1+\varepsilon_2) \tag{5}$$

Thus we find the general expression of URs of two PQs

$$AB \sim[\hbar^{((\alpha_1+\alpha_2)+(\beta_1+\beta_2)+(\gamma_1+\gamma_2)+(\delta_1+\delta_2))}]^{\frac{1}{2}}$$
$$\times[G^{((\alpha_1+\alpha_2)-(\beta_1+\beta_2)+(\gamma_1+\gamma_2)-(\delta_1+\delta_2))}]^{\frac{1}{2}}$$
$$\times[c^{-(3(\alpha_1+\alpha_2)-(\beta_1+\beta_2)+5(\gamma_1+\gamma_2)-5(\delta_1+\delta_2))}]^{\frac{1}{2}}$$
$$\times\kappa^{-(\delta_1+\delta_2)}e^{(\varepsilon_1+\varepsilon_2)} \tag{6}$$

It shows that the products of two non-commutative PQs with dimensions are equivalent to the power products of the reduced Planck constant, gravitational constant, speed of light in vacuum, Boltzmann constant and elementary charge.

2.2 Basic relation

Ordering $\alpha_1=\alpha_2=\alpha$, $\beta_1=\beta_2=\beta$, $\gamma_1=\gamma_2=\gamma$, $\delta_1=\delta_2=\delta$, and $\varepsilon_1=\varepsilon_2=\varepsilon$ in the general expression of URs (6), that is A and B having the same dimensions

$$[A] = [B] = [L]^{\alpha}[M]^{\beta}[t]^{\gamma}[T]^{\delta}[Q]^{\varepsilon} \tag{7}$$

We obtain

$$\hbar^{(\alpha+\beta+\gamma+\delta)}G^{(\alpha-\beta+\gamma-\delta)}c^{-(3\alpha-\beta+5\gamma-5\delta)}\kappa^{-2\delta}e^{2\varepsilon}$$
$$= A_P B_P = A_P^2 = B_P^2 \tag{8}$$

where A_P and B_P are the corresponding Planck scale of A and B separately. Extracting the square root, we find the basic relation

$$A \sim A_P = [\hbar^{(\alpha+\beta+\gamma+\delta)}G^{(\alpha-\beta+\gamma-\delta)}c^{-(3\alpha-\beta+5\gamma-5\delta)}\kappa^{-2\delta}e^{2\varepsilon}]^{\frac{1}{2}} \tag{9}$$

The basic relation (9) shows that any physical quantity with dimension has a corresponding Planck scale which is equivalent to the power products of ħ, G, c, κ and e.

Or instructing $\alpha_1=\alpha$, $\beta_1=\beta$, $\gamma_1=\gamma$, $\delta_1=\delta$, $\varepsilon_1=\varepsilon$ and $\alpha_2=\beta_2=\gamma_2=\delta_2=\varepsilon_2=0$ in (6), we gain (9) again.

2.3 Proving Basic relation

We prove the basic relation (9) now. Considering n non-commutative PQs with dimensions, we have

$$\prod_{i=1}^n A_i \sim \hbar^x G^y c^z \kappa^w e^u,\ i = 1, 2, 3\ldots n \tag{10}$$

where A_i is the physical quantity, A_i and A_{i+1} are non-commutative. The dimensions of $\prod_{i=1}^n A_i$ are

$$[\prod_{i=1}^n A_i] = [L]^{\sum_i^n\alpha_i}[M]^{\sum_i^n\beta_i}[t]^{\sum_i^n\gamma_i}[T]^{\sum_i^n\delta_i}[Q]^{\sum_i^n\varepsilon_i} \tag{11}$$

where α_i, β_i, γ_i, δ_i and ε_i are known real number. By the dimensional analysis, we obtain

$$[L]^{\sum_i^n\alpha_i}[M]^{\sum_i^n\beta_i}[t]^{\sum_i^n\gamma_i}[T]^{\sum_i^n\delta_i}[Q]^{\sum_i^n\varepsilon_i}$$
$$= \{[L^2][M][t^{-1}]\}^x\{[L^3][M^{-1}][t^{-2}]\}^y\{[L][t^{-1}]\}^z$$
$$\times\{[L^2][M][t^{-2}][T^{-1}]\}^w\{[Q]\}^u \tag{12}$$

Solving (12), we gain

$$x = [(\textstyle\sum_i^n\alpha_i)+(\sum_i^n\beta_i)+(\sum_i^n\gamma_i)+(\sum_i^n\delta_i)] / 2,$$
$$y = [(\textstyle\sum_i^n\alpha_i)-(\sum_i^n\beta_i)+(\sum_i^n\gamma_i)-(\sum_i^n\delta_i)] / 2,$$
$$z = -[3(\textstyle\sum_i^n\alpha_i)-(\sum_i^n\beta_i)+5(\sum_i^n\gamma_i)-5(\sum_i^n\delta_i)] / 2,$$
$$w = -(\textstyle\sum_i^n\delta_i),\ u = \sum_i^n\varepsilon_i \tag{13}$$

So we find the general expression of URs of n PQs

$$\prod_{i=1}^n A_i \sim [\hbar^{((\sum_i^n\alpha_i)+(\sum_i^n\beta_i)+(\sum_i^n\gamma_i)+(\sum_i^n\delta_i))}]^{\frac{1}{2}}$$
$$\times[G^{((\sum_i^n\alpha_i)-(\sum_i^n\beta_i)+(\sum_i^n\gamma_i)-(\sum_i^n\delta_i))}]^{\frac{1}{2}}$$
$$\times[c^{-(3(\sum_i^n\alpha_i)-(\sum_i^n\beta_i)+5(\sum_i^n\gamma_i)-5(\sum_i^n\delta_i))}]^{\frac{1}{2}}$$
$$\times\kappa^{-(\sum_i^n\delta_i)}e^{(\sum_i^n\varepsilon_i)} \tag{14}$$

Certainly when n = 2, it become the general expression of URs (6); when n = 1, we obtain (9). Ordering $\alpha_i=\alpha_{i+1}=\alpha$, $\beta_i=\beta_{i+1}=\beta$, $\gamma_i=\gamma_{i+1}=\gamma$, $\delta_i=\delta_{i+1}=\delta$ and $\varepsilon_i=\varepsilon_{i+1}=\varepsilon$ in (14), A_i and A_{i+1} having the same dimensions, we obtain

$$[\hbar^{n(\alpha+\beta+\gamma+\delta)}]^{\frac{1}{2}}[G^{n(\alpha-\beta+\gamma-\delta)}]^{\frac{1}{2}}[c^{-n(3\alpha-\beta+5\gamma-5\delta)}]^{\frac{1}{2}}\kappa^{-n\delta}e^{n\varepsilon}$$
$$= A_P^n \tag{15}$$

Extracting the nth-root, we gain (9) again.

3. Planck Scales

In this section, we obtain the Planck scales, and classify them.

[1] Chien Wei-Zang used L, M, T, θ and Q indicated the dimensions of length, mass, time, temperature and electric charge separately in [20].

3.1 Basic Planck scale

Ordering α = 1, $\beta = \gamma = \delta = \varepsilon$ = 0 in (7) and (9), we obtain Planck length immediately

$$L_P = \sqrt{\hbar G/c^3}$$

Instructing γ =1, $\alpha = \beta = \delta = \varepsilon$ = 0, obtain Planck time

$$t_P = \sqrt{\hbar G/c^5}$$

Ordering β = 1, $\alpha = \gamma = \delta = \varepsilon$ = 0, obtain Planck mass

$$M_P = \sqrt{\hbar c/G}$$

Instructing δ = 1, $\alpha = \beta = \gamma = \varepsilon$ = 0, obtain Planck temperature

$$T_P = \sqrt{\hbar c^5/\kappa^2 G}$$

Ordering ε = 1, $\alpha = \beta = \gamma = \delta$ = 0, obtain elementary charge (or Planck charge)

$$Q_P = Q_e = e$$

If using $[Q]^2 = [L]^3[M][T]^{-2}$, obtain

$$Q_P = \sqrt{\hbar c} \sim e$$

These are the basic Planck scale [14].

3.2 Derived Planck scale

From (7) and (9), we gain the derived Planck scale [14] which except the basic one. For example

Planck energy E_P

$$[E_P] = [L^2][M][T^{-2}],\ E_P = \sqrt{\hbar c^5/G}$$

Planck momentum P_P

$$[P_P] = [L][M][T^{-1}],\ P_P = \sqrt{\hbar c^3/G}$$

Planck curvature tensor $R_{\mu\nu P}$

$$[R_{\mu\nu P}] = [L^{-2}],\ R_{\mu\nu P} = c^3 / \hbar G$$

Because many PQs have the same dimensions, they have the same Planck scale, for example

Planck energy density ρ_P

$$[\rho_P] = [L^{-1}][M][T^{-2}],\ \rho_P = c^7 / \hbar G^2$$

Planck pressure p_P

$$[p_P] = [L^{-1}][M][T^{-2}],\ p_P = c^7 / \hbar G^2$$

Planck force per unit area f_P

$$[f_P] = [L^{-1}][M][T^{-2}],\ f_P = c^7 / \hbar G^2$$

Planck energy- momentum tensor $T_{\mu\nu P}$

$$[T_{\mu\nu P}] = [L^{-1}][M][T^{-2}],\ T_{\mu\nu P} = c^7 / \hbar G^2$$

Etc.

3.3 Classifications

We classify all the Planck scales by two methods. First are basic Planck scale and derived Planck scale [14]. Second are that One's power is the half integer, call it Femi-Planck scale, such as L_P, t_P, M_P, T_P, E_P, P_P, etc; another is the integer, call it Bose-Planck scale, such as Q_e, ρ_P, p_P, f_P, $R_{\mu\nu P}$, $T_{\mu\nu P}$, etc; others call Other-Planck scale, such as the Planck wave function ψ_P

$$[\psi_P] = [L^{-3/2}],\ \psi_P = (\hbar G / c^3)^{-3/4}$$

4. Generalized Relation

In this section, we prove that basic relation (9) is rewritten as the power products of basic Planck scales; find and prove the generalized relation, and prove the URs in Sec. 1,.

4.1 Proof

The basic relation (9) can be rewritten as

$$A_P = L_P^{\alpha} M_P^{\beta} t_P^{\gamma} T_P^{\delta} Q_e^{\varepsilon} \tag{16}$$

From (9), we obtain

$$\begin{aligned} A_P &= [\hbar^{\alpha} G^{\alpha} c^{-3\alpha}]^{\frac{1}{2}} [\hbar^{\beta} G^{-\beta} c^{\beta}]^{\frac{1}{2}} [\hbar^{\gamma} G^{\gamma} c^{-5\gamma}]^{\frac{1}{2}} [\hbar^{\delta} G^{-\delta} c^{5\delta}]^{\frac{1}{2}} \kappa^{-\delta} e^{\varepsilon} \\ &= [\sqrt{\hbar G/c^3}]^{\alpha} [\sqrt{\hbar c/G}]^{\beta} [\sqrt{\hbar G/c^5}]^{\gamma} [\sqrt{\hbar c^5/\kappa^2 G}]^{\delta} e^{\varepsilon} \\ &= L_P^{\alpha} M_P^{\beta} t_P^{\gamma} T_P^{\delta} Q_e^{\varepsilon} \end{aligned}$$

Therefore the corresponding Planck scale of any physical quantity is equivalent to the power products of Planck length, Planck mass, Planck time, Planck temperature and elementary charge.

4.2 Generalized relation

Considering all the non-commutative PQs, we find the generalized relation

$$\prod_{i=1}^{n} A_i^{a_i} \sim \prod_{i=1}^{n} A_{iP}^{a_i}; \quad i = 1, 2, 3 \ldots n \tag{17}$$

where A_i is the physical quantity, A_i and A_{i+1} are non-commutative, a_i is the real number, and A_{iP} is the corresponding Planck scale of A_i. It shows that the power products of non-commutative PQs are equivalent to the ones of corresponding Planck scales.

4.3 Proving generalized relation

We prove the generalized relation by the same way in 2.3. Considering n non-commutative PQs with a_i power, we have

$$\prod_{i=1}^{n} A_i^{a_i} \sim \hbar^x G^y c^z \kappa^w e^u \tag{18}$$

The dimensions of $\prod_{i=1}^{n} A_i^{a_i}$ are

$$[\prod_{i=1}^{n} A_i^{a_i}] = [L]^{\sum_i^n a_i\alpha_i}[M]^{\sum_i^n a_i\beta_i}[t]^{\sum_i^n a_i\gamma_i}[T]^{\sum_i^n a_i\delta_i}[Q]^{\sum_i^n a_i\varepsilon_i} \tag{19}$$

Using the dimensional analysis, we obtain

$$\begin{aligned} &[L]^{\sum_i^n a_i\alpha_i}[M]^{\sum_i^n a_i\beta_i}[t]^{\sum_i^n a_i\gamma_i}[T]^{\sum_i^n a_i\delta_i}[Q]^{\sum_i^n a_i\varepsilon_i} \\ &= \{[L^2][M][t^{-1}]\}^x \{[L^3][M^{-1}][t^{-2}]\}^y \{[L][t^{-1}]\}^z \\ &\times \{[L^2][M][t^{-2}][T^{-1}]\}^w \{[Q]\}^u \end{aligned} \tag{20}$$

Solving (20), we have

$$\begin{aligned} x &= [(\textstyle\sum_i^n a_i\alpha_i)+(\sum_i^n a_i\beta_i)+(\sum_i^n a_i\gamma_i)+(\sum_i^n a_i\delta_i)] / 2, \\ y &= [(\textstyle\sum_i^n a_i\alpha_i)-(\sum_i^n a_i\beta_i)+(\sum_i^n a_i\gamma_i)-(\sum_i^n a_i\delta_i)] / 2, \\ z &= -[3(\textstyle\sum_i^n a_i\alpha_i)-(\sum_i^n a_i\beta_i)+5(\sum_i^n a_i\gamma_i)-5(\sum_i^n a_i\delta_i)] / 2, \\ w &= -(\textstyle\sum_i^n a_i\delta_i),\ u = \sum_i^n a_i\varepsilon_i \end{aligned} \tag{21}$$

Thus we gain the general expression of URs of n PQs with a_i power

$$\prod_{i=1}^{n} A_i^{a_i} \sim [\hbar^{\left((\sum_i^n a_i\alpha_i)+(\sum_i^n a_i\beta_i)+(\sum_i^n a_i\gamma_i)+(\sum_i^n a_i\delta_i)\right)}]^{\frac{1}{2}}$$

$$\times [G^{\left((\sum_i^n a_i\alpha_i) - (\sum_i^n a_i\beta_i) + (\sum_i^n a_i\gamma_i) - (\sum_i^n a_i\delta_i)\right)}]^{\frac{1}{2}}$$
$$\times [c^{-\left(3(\sum_i^n a_i\alpha_i) - (\sum_i^n a_i\beta_i) + 5(\sum_i^n a_i\gamma_i) - 5(\sum_i^n a_i\delta_i)\right)}]^{\frac{1}{2}}$$
$$\times \kappa^{-(\sum_i^n a_i\delta_i)} e^{(\sum_i^n a_i\varepsilon_i)}$$
$$= [\sqrt{\hbar G/c^3}]^{\sum_i^n a_i\alpha_i} [\sqrt{\hbar c/G}]^{\sum_i^n a_i\beta_i} [\sqrt{\hbar G/c^5}]^{\sum_i^n a_i\gamma_i}$$
$$\times [\sqrt{\hbar c^5/\kappa^2 G}]^{\sum_i^n a_i\delta_i} e^{\sum_i^n a_i\varepsilon_i}$$
$$= L_P^{\sum_i^n a_i\alpha_i} M_P^{\sum_i^n a_i\beta_i} t_P^{\sum_i^n a_i\gamma_i} T_P^{\sum_i^n a_i\delta_i} Q_e^{\sum_i^n a_i\varepsilon_i}$$
$$= \prod_{i=1}^n L_P^{a_i\alpha_i} M_P^{a_i\beta_i} t_P^{a_i\gamma_i} T_P^{a_i\delta_i} Q_e^{a_i\varepsilon_i} = \prod_{i=1}^n A_{iP}^{a_i} \qquad (22)$$

where $A_{iP} = L_P^{\alpha_i} M_P^{\beta_i} t_P^{\gamma_i} T_P^{\delta_i} Q_e^{\varepsilon_i}$.

4.4 Proving URs

Applying the generalized relation (17), we can prove the URs in Sec.1.

$\Delta p \Delta r \sim P_P L_P = \sqrt{\hbar c^3/G}\sqrt{\hbar G/c^3} = \hbar$; $\Delta E \Delta t \sim E_P t_P = \sqrt{\hbar c^5/G}\sqrt{\hbar G/c^5} = \hbar$; $\delta t / t^{1/3} \sim t_P / t_P^{1/3} = t_P^{2/3}$; $\eta / s \sim \eta_P / s_P = \sqrt{c^9/\hbar G^3} / \sqrt{c^9\kappa^2/\hbar^3 G^3} = \hbar / \kappa$; $\Delta T \Delta X \sim t_P L_P \sim \hbar G / c^4 = L_P^2 / c \sim L_S^2$; $\delta x \delta y \delta t \sim L_P^2 t_P = L_P^3 / c$; $L_{\mu\nu} / \sqrt{L} \sim L_P / \sqrt{L_P} = \sqrt{L_P}$; $\varepsilon(Q)\eta(P) + \varepsilon(Q)\sigma(P) + \sigma(Q)\eta(P) \sim \sqrt{\hbar G/c^3}\sqrt{\hbar c^3/G} = \hbar$; $(\delta t)(\delta r)^3 / r^2 \sim t_P L_P^3 / L_P^2 = L_P^2 / c$, etc.

where $\eta_P = \sqrt{c^9/\hbar G^3}$ is the Planck ratio of shear viscosity of a given fluid perfect, and $s_P = \sqrt{c^9\kappa^2/\hbar^3 G^3}$ is its Planck volume density of entropy (from basic relation (9)). Thus we find that there hasn't G on some formulas right hand because it is reduced fitly.

5. No singularity at Big Bang and SBH

In this section, we find the Big Bang UR and SBH UR by the generalized relation.

5.1 Big Bang UR

S.W. Hawking and R. Penrose proved that the universe originated the Big Bang singularity [21]. Many literatures discussed no singularity at the Big Bang and black holes with the quantum effect, please refer to [18] [22-25]. The one of the characteristic of Big Bang singularity is zero volume and limitless high temperature.

Then we can find the relation of Big Bang temperature and its volume by the generalized relation (17)

$$T_B V_B \sim T_P V_P = T_P L_P^3 = \hbar^2 G / \kappa c^2 \qquad (23)$$

where T_B is the Big Bang temperature, V_B is its volume, and $V_P = L_P^3$ is the Planck volume. This is the Big Bang UR. It shows that it is impossible to measure the Big Bang temperature and its volume simultaneously. When $\hbar \to 0$, we obtain

$$T_B V_B \sim 0 \qquad (24)$$

Because $T_B > 0$, we gain $V_B \sim 0$, the Big Bang volume is zero, thus the Big Bang singularity appears without the quantum effect. We suggest no singularity at the Big Bang with quantum effect. Substituting $a = c\kappa T / 2\pi\hbar$ [26] into (23), we obtain

$$a_B V_B \sim a_p V_p = \hbar G / 2\pi c \qquad (25)$$

where a_B is the Big Bang acceleration, and $a_p = \sqrt{c^7/\hbar G}$ is the Planck acceleration. It is the UR between Big Bang acceleration and its volume.

5.2 SBH UR

Similarly considering the mass and volume of SBH, we find

$$M_H V_H \sim M_P V_P = M_P L_P^3 = \hbar^2 G / c^4 \qquad (26)$$

where M_H is the SBH mass, and V_H is its volume. It is the SBH UR. Also it is impossible to measure the SBH mass and volume simultaneously. When $\hbar \to 0$, we obtain

$$M_H V_H \sim 0 \qquad (27)$$

Because $M_H > 0$, we have $V_H \sim 0$, the volume is zero, the SBH singularity appears without quantum effect also. We also suggest no singularity in SBH with quantum effect. Taking $M = \rho V$ to (26), we gain

$$M_H^2 / \rho_H \sim \hbar^2 G / c^4, \ \rho_H V_H^2 \sim \hbar^2 G / c^4 \qquad (28)$$

where ρ_H is the mass density of SBH. These are the URs between the mass density of SBH and its mass or volume.

6. Conclusion

In this paper, we investigate the URs with dimensions by the dimensional analysis. We find the following results.

1) The normal form of URs is discovered. The PQs are on the left hand of URs, and the physical constants such as the reduced Planck constant ħ, gravitational constant G, speed of light in vacuum c and Boltzmann constant κ are on the right hand. These physical constants which are rewritten appear the power products.

2) The general expression of URs are found and proved. It shows that the products of two non-commutative PQs with dimensions are equivalent to the power products of ħ, G, c, κ and elementary charge e.

3) The basic relation is found and proved. Any physical quantity with dimension has a corresponding Planck scale which is equivalent to the power products of ħ, G, c, κ and e.

4) The Planck length L_P, Planck time t_P, Planck mass M_P, Planck temperature T_P, elementary charge Q_e (or Planck charge), Planck energy E_P, Planck momentum P_P, Planck curvature tensor $R_{\mu\nu P}$, Planck energy density ρ_P, Planck pressure p_P, Planck force per unit area f_P, Planck energy-momentum tensor $T_{\mu\nu P}$ etc are obtained again. Many PQs have the same Planck scale because of the same dimensions such as ρ_P, p_P, f_P and $T_{\mu\nu P}$.

5) All the Planck scales are classified by two methods. First are the basic Planck scale including L_P, t_P, M_P, T_P and Q_e, and derived Planck scale such as E_P, P_P, ρ_P, p_P, f_P, $R_{\mu\nu P}$, $T_{\mu\nu P}$, Planck wave function ψ_P etc. Second are the Femi-Planck scale which power is the half integer such as L_P, t_P, M_P, T_P, E_P, P_P,

etc, the Bose-Planck scale which power is the integer such as Q_e, ρ_P, p_P, f_P, $R_{\mu\nu P}$, $T_{\mu\nu P}$, etc and the Other-Planck scale which power is others such as ψ_P.

6) The corresponding Planck scale of any physical quantity is proved to be equivalent to the power products of L_P, t_P, M_P, T_P and Q_e.

7) The generalized relation is found and proved. It shows that the power products of the non-commutative PQs are equivalent to the ones of corresponding Planck scales. The URs in Sec. 1 are proved by the generalized relation. G disappears on some URs because of being reduced fitly.

8) The Big Bang UR between its temperature T_B and volume V_B is found by the generalized relation. It suggests no singularity at the Big Bang with the quantum effect. The UR between Big Bang acceleration a_B and V_B is obtained. Similarly the SBH UR between its mass M_H and volume V_H is found; also no singularity is in SBH with quantum effect. The URs between the mass density ρ_H of SBH and M_H or V_H is gained.

9) The generalized relation unifies all URs with dimensions. It is generalized, interesting and significant; any UR is its special case. Because depends on the dimensions, generalized relation can't obtain the factor and relation without dimensions.

References

[1] W. Heisenberg, *Z. Phys*. **43** (1927) 172; The Physical Principles of the Quantum Theory, University of Chicago Press 1930, Dover edition 1949; J.A. Wheeler, W.H. Zurek (Eds.), Quantum Theory and Measurement, Princeton Univ. Press, Princeton, NJ, 1983, P. 62, 84.

[2] S.W. Hawking, Commun. *Math. Phys*. **43** (1975) 199-220; Nature (London). **248** (1974) 30.

[3] Y-d. Zhang, J-w. Pan, H. Rauch, *Annals of the New York Academy of Sciences*, **755** 353 (1995), 353-360.

[4] D. Amati, M. Ciafaloni, and G. Veneziano, *Phys. Lett.* B **216**, 41 (1989); A. Kempf, G. Mangano, and R. B. Mann, *Phys. Rev.* D **52**, 1108 (1995); L. N. Chang, D. Minic, N. Okamura, and T. Takeuchi, *Phys.Rev.* D **65**, 125027 (2002); L. N. Chang, D. Minic, N. Okamura, and T. Takeuchi, *Phys. Rev.* D **65**, 125028 (2002); A. Tawfik and A. Diab, submitted to *Int. J. Mod. Phys*. D (2014); arXiv: gr-qc/1410.0206; J.L. Cortes and J. Gamboa, *Phys. Rev*. D **71**, 065015 (2005); J. Magueijo and L. Smolin, *Phys. Rev*. D **67**, 044017 (2003); G. Amelino-Camelia, *Int. J. Mod. Phys*. D **11**, 35 (2002); G. Amelino-Camelia, *Int. J. Mod. Phys*. D **11**, 1643 (2002); A. Tawfik, *JCAP* 1307, 040 (2013); A. F. Ali and A. Tawfik, Adv. *High Energy Phys*. 2013, 126528 (2013); A. F. Ali and A. Tawfik, *Int. J. Mod. Phys*. D **22**, 1350020 (2013); A. Tawfik, H. Magdy and A.Farag Ali, *Gen. Rel. Grav*. 45, 1227 (2013); A. Tawfik, H. Magdy and A.F. Ali, arXiv: physics.gen-ph/ 1205.5998; I. Elmashad, A.F. Ali, L.I. Abou-Salem, Jameel-Un Nabi and A. Tawfik, *Trans. Theor. Phys*, 1,106 (2014); A. Tawfik and A. Diab, "Black Hole Corrections due to Minimal Length and Modified Dispersion Relation", in press; A. F. Ali, S. Das and E. C. Vagenas, arXiv: hep-th/ 1001.2642. A. Na. TAWFIK, arXiv: gr-qc/1410.7966. M. Faizal, M. M. Khalil, S. Das, arXiv: physics.gen-ph/ 1501.03111; Jun-Li Li & Cong-Feng Qiao, *Sic. Rep*. **5**, 12708 (2015).

[5] S. Benczik, L. N. Chang, D. Minic, N. Okamura, S. Rayyan, and T. Takeuchi, *Phys. Rev*. D **66**, 026003 (2002); P. Dzierzak, J. Jezierski, P. Malkiewicz, and W. Piechocki, *Acta Phys. Polon*. B **41**, 717 (2010); L. J. Garay, *Int. J. Mod. Phys*. A **10**, 145 (1995); C. Bambi, F. R. Urban, *Class. Quantum Grav*. **25**, 095006 (2008); K. Nozari, *Phys. Lett*. B. **629**, 41 (2005); A. Kempf, G. Mangano and R. B. Mann, *Phys. Rev*. D **52**, 1108 (1995); A. Kempf, *J. Phys*. A **30**, 2093 (1997); S. Das, and E. C. Vagenas, *Phys. Rev. Lett*. **101**, 221301 (2008); S. Das, E. C. Vagenas and A. F. Ali, *Phys. Lett*. B **690**, 407 (2010); M.J.W. Hall, *Phys. Rev.* A **62**, (2000) 012107; *Phys. Rev.* A **64** (2001) 052103; M.J.W. Hall and M. Reginatto, *J. Phys. A: Math. Gen.* 35 3289 (2002); arXiv: quant-ph /0201084.

[6] ch-F. Li, J-Sh. Xu, X-Y. Xu, K. Li, G-c. Guo, *Nature. Phy*., **7**, 10 (2011) 752, 756.

[7] H. P. Robertson, *Phys. Rev*. **34**, 163-164 (1929); E. Arthurs, & M. S. Goodman, *Phys. Rev. Lett*. **60**, 2447–2449 (1988); S. Ishikawa, *Rep. Math. Phys*. **29**, 257-273 (1991); M. Ozawa, Quantum limits of measurements and uncertainty principle. pp 3-17 in Bendjaballah, C. *et al*. (eds) Quantum Aspects of Optical Communications. (Springer, Berlin, 1991); M. Ozawa, *Phys. Rev.* A **67**, 042105 (2003); M. Ozawa, *Ann. Phys*. **311**, 350-416 (2004); M. Ozawa, *Phys. Lett*. A **318**, 21-29 (2003); R. F. Werner, *Inf. Comput*. **4**, 546-562 (2004); M. Ozawa, *J. Opt.* B: Quantum Semiclass. Opt. 7, S672 (2005); J. Erhart, G. Sulyok, G. Badurek, M. Ozawa and Y. Hasegawa, *Nature Physics* DOI: 10.1038/NPHYS2194 (2012).

[8] F. Karolyhazy, *Nuovo. Cim*, A **42** (1966) 390.

[9] P.K. Kovtun, D.T. Son and A. O. Starinets, *Phys. Rev. Lett*. **94**, 111601(2005).

[10] T. Yoneya, Duality and Indeterminacy Principle in String Theory in "Wandering in the Fields", eds. K. Kawarabayashi and A. Ukawa (World Scientific, 1987), P.419; see also

String Theory and Quantum Gravity in “uantum String Theory”, eds. N. Kawamoto and T. Kugo (Spring, 1988), P.23; T. Yoneya, *Mod. Phys. Lett*. A4, **1587** (1989); M. Li and T. Yoneya, *Journal of Chaos, Solitons and Fractals* on “Superstrings, M,F,S...Theory”, arXiv: hep-th/ 9806240.

[11] T. Yoneya, Mod. *Phys. Lett*. A **4**, 1587 (1989); *Prog. Theor. Phys*. **97**, 949 (1997); arXiv:hep-th/9707002; *Prog. Theor. Phys*. **103**, 1081 (2000); *Int. J. Mod. Phys*. A **16**, 945 (2001); M. Li and T. Yoneya, *Phys. Rev. Lett*. **78**, 1219 (1997); *Chaos Solitons Fractals* **10**, 423 (1999); A. Jevicki and T. Yoneya, *Nucl. Phys*. B **535**, 335 (1998); H. Awata, M. Li, D. Minic and T. Yoneya, *JHEP* 0102, 013 (2001); D. Minic, *Phys. Lett*. B **442**, 102 (1998); L. N. Chang, Z. Lewis, D. Minic, and T. Takeuchi, arXiv: hep-th/1106.0068.

[12] C.J. Hogan, arXiv: astro-ph/0703775; C.J. Hogan, arXiv: gr-qc/0706.1999; M. Li and Y. Wang, *Phys. Lett.* B **687**: 243-247 (2010).

[13] Y-X. Chen and Y. Xiao, *Phys. Lett.* B *666*: 371 (2008).

[14] M. Planck, Akad. Wiss. Berlin, Kl. *Math.-Phys. Tech.*,**5**: 440–480, 1899; M. Planck, Vorlesungen über die Theorie der Wärmestrahlung, P. 164. J.A. Barth, Leipzig, 1906.

[15] J. Magueijo, and L. Smolin, *Phys. Rev. Lett*. **88**, 190403 (2002); J. Magueijo, and L. Smolin, *Phys. Rev*. D **71**, 026010 (2005); J. L. Cortes and J. Gamboa, *Phys. Rev*. D **71**, 065015 (2005); M. Duff, L.B. Okun, G. Veneziano, *J. High Energy Phys.*, 3: 023 (2002); F. Wilczek, arXiv: physics.gen-ph/ 0708.4361; *Frank Wilczek* web site.

[16] S. Weinstein and D. Rickles, Quantum gravity, in Edward N. Zalta, editor, *The Stanford Ency-clopedia of Philosophy*. Spring 2011 edition, 2011.

[17] M. Tajmar, *Physics Essays* 25(3), 466-469 (2012).

[18] Z-Y. Shen, *Journal of Modern Physics*, **4** (2013), 1213-1380.

[19] E. Buckingham, *Physical Review*, 1914, 4(4): 345-376; P.W. Bridgman, Dimensional Analysis, New Haven: Yale University Press, 1922.

[20] Chien Wei-Zang, Applied Mathematics, Anhui Science and Technology Press, 1993, P. 154.

[21] S.W. Hawking and R. Penrose, *Proc. Roy. Soc. London*. A **314** (1970), 529, 48; S.W. Hawking, F.R. Ellis, The large scale structure of space-time, Cambridge University Press, 1973; J.K. Beem, and P.E. Ehrlich, Global Lorentzian Geometry, Marcel Dekker, New York, 1981.

[22] M.B. Green, J.H. Schwarz and E. Witten, Superstring Theory, Cambridge University Press, 1987; M. B. Green, J. H. Schwarz and E. Witten, Superstring Theory, Vol. I and Vol. II, Cambridge University Press, 1988; J. Polchinski, String Theory, Vol. I and Vol. II, Cambridge University Press (1998); K. Becker, M. Becker and J. H. Schwarz, String Theory and M-Theory: A Modern Introduction, Cambridge University Press, 2007.

[23] M. Bojowald, *Phys. Rev. Lett*. **86** (2001), 5227-5230; H. Viqar, W. Oliver, *Phys. Rev*. D **69** (2004), 084016; L. Modesto, *Phys. Rev*. D **70** (2004), 124009; LI ChangZhou, YU Guoxiang, XIE ZhiFang, *Acta*. *Physica*. *Sinica*., 59, **3** (2010).

[24] Y.J. Wang, Black Hole Physics, ChangSha: HuNan Normal University Press, 2000.4.

[25] A.F. Ali, S. Das, *Phys. Lett*. B **741** (2015) 276-279.

[26] W.G. Unruh and R.M. Wald, *Phys. Rev.* D **25**, 942; *Gen. Rel. Grav.*, **15** 195 (1983); *Phys. Rev.* D **27**, 2271 (1983).