All that glitters is not gold: Zero-point energy in the Johnson noise of resistors

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The thermal noise (Johnson noise) in resistors was discovered [1] by Johnson and explained [2] by Nyquist in 1927, a year later than the foundations of quantum physics were completed. The Johnson-Nyquist formula states that

$$S_u(f) = 4RhfN(f,T) \qquad (1)$$

where $S_u(f)$ is the power density spectrum of the voltage noise on the open-ended resistor of resistance *R* (replaced by the real part of the impedance if impedance is used); and *h* is the Planck constant. The Planck number $N(f,T)$ is the mean number of hf energy quanta in a linear harmonic oscillator with resonance frequency *f* , at temperature *T*

$$N(f,T) = \left[\exp(hf/kT) - 1\right]^{-1} \quad , \qquad (2)$$

which is $N(f,T) = kT/(hf)$ for the classical physical range $kT >> hf$. Eq. 2 results in an exponential cut-off of the Johnson noise in the quantum range $f > f_P = kT/h$, in accordance with Planck's thermal radiation formula. In the deeply classical (low-frequency) limit, $f << f_P = kT/h$, Eqs. 1-2 yield the familiar form used in electrical engineering

$$S_u(f) = 4kTR \qquad (3)$$

where the Planck cut-off frequency f_P is about 6000 GHz at room temperature, well-beyond the reach of today's electronics.

The quantum theoretical, generalized treatment of thermal noise was given only 24 years later by Callen and Welton [3] (often called Fluctuation-Dissipation Theorem (FDT). The quantum version [3] of the Johnson-Nyquist formula has an additive 0.5 to the Planck number, corresponding to the zero-point energy of linear harmonic oscillators:

$$S_u(f) = 4Rhf\left[N(f,T) + 0.5\right] \ . \qquad (4)$$

Thus the quantum correction of Eq. 1 is a temperature-independent additive term in Eq. 2:

$$S_{u,ZP}(f) = 2hfR \quad , \qquad (5)$$

which linearly depends on the frequency and it exists for any $f > 0$ frequency, even in the deeply classical, $f << f_P = kT/h$, frequency regime, and even at zero temperature. The zero-point term described by Eq. 5 has acquired a wide theoretical support during the years, e.g. [4-6] .

However, there have also been contra-arguments and debates. MacDonald [8] and Harris [9] argued that extracting energy/power from the zero-point energy is impossible thus Eq. 5 should not exist.

Grau and Kleen [10] (similarly to the original treatment of Nyquist [2]), argued that the Johnson noise of a resistor connected to an antenna, see Figure 1, must satisfy Planck's thermal radiation formula thus the noise must be zero at zero temperature, which would imply that Eq. 5 is invalid. It should be emphasize that it is a hard experimental fact that the zero-point term does not exist in the thermal radiation. This is obvious even by naked-eye observations: at 6000 K temperature, at 600 nm (orange color), the Planck number $N = 0.0164$. *Thus the zero-point term (0.5) is 30 times greater, implying that, if it would be present in the radiation, looking into a dark room instead of the sun, the light intensity at this wavelength would decrease only about a negligible 3%.*

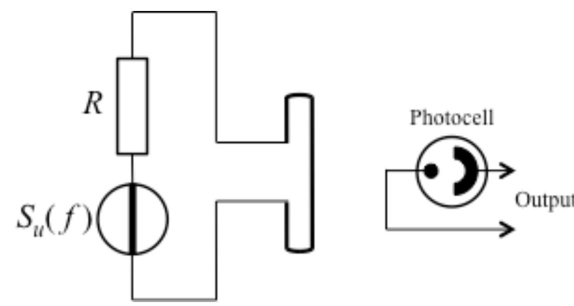

Figure 1. Measurement scheme based on an antenna and a photon counter, which does not show the zero-point term (Eq. 5) at its output.

Kish [11] showed that the existence of the zero-point term, which has and "*f*"-noise implies a 1/f noise and related logarithmic divergence of the energy of a shunt capacitor in the high-frequency limit. While this does not disprove the existence of the term, it may indicate that the problem is a renormalization problem, a mathematical artifact, which is not actually present at measurements.

Recently, Reggiani, et al. [12] objected the derivation [3-6] of Eq. 4 but did not show what the results avoiding their criticism should be.

Yet, on the contrary of all the criticisms above, the experimental test by Koch, van Harlingen and Clarke [13] *fully* confirmed the theoretical result Eq. 4 by measurements on resistively shunted Josephson-junctions.

However, Haus [14] and Kleen [15] stated that the zero-point term (Eq. 5) in Eq. 4 is the consequence of the uncertainty principle at phase-sensitive amplitude measurements, see Figure 2, which the linear voltage amplifiers measuring Johnson noise represent. Nevertheless, the uncertainly principle argument *cannot disprove* Eqs. 4,5. The claimed zero-point term in the noise voltage may still exist and satisfy the uncertainty principle instead of being solely an experimental artifact.

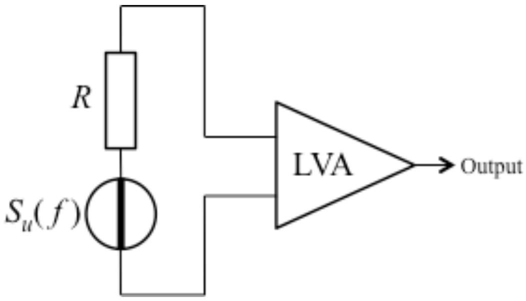

Figure 2. Measurement scheme based on a linear amplifier (LVA) system indicating the existence of the zero-point term or its uncertainty relation based artifact.

To claim the existence or non-existence of the debated zero-point noise term in the voltage is a serious matter because of its implications of the related current and energy flow.

Thus we devote our talk to the following question:

Is the zero-point voltage noise term (Eq. 5) and the power/energy flow it implies actually present in the wire connected to a resistor?

Abbott, et al.,[16] write in their education-article on thermal noise: "Until further evidence, the quantum zero- field should be regarded as a conservative field as far as the extraction of energy is concerned."

In this talk, we address this comment and close this issue by serving evidence [17] that the zero-point voltage component cannot exist in the wire otherwise at least two different types of perpetual motion machines can be built and both the energy conservation law and the second law of thermodynamics are violated [17].

The remaining *unsolved problem of noise* after our treatment is:

What is the proper general formula or formulas of Johnson noise in the voltage of resistors? The formula(s) that can reflect on the type of measurement that we use to characterize the Johnson noise of a resistor?

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