Part I: The Units of Planck's Constant are J not J x s: Yes or No?

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Abstract: Yes.

Résumé: Oui.

1 Introduction

The standard model of cosmology (SM) is a theory that has been developed and popularized over the last 100 years in an attempt to explain the observable Universe in a consistent and elegant manner. As successful as SM has been, it has also faced many challenges. The success of a model is usually founded on its ability to make predictions, yet the SM failed to predict the accelerating expansion of the universe and completely missed dark matter and dark energy making up 95 percent of the mass/energy of the universe. Although SM was able to accommodate these surprising observations, one can't help but think "epicycles" when a model can be tweaked to such an extent. Many will agree that the standard model is incomplete. The lack of smallest measuring sticks for mass, charge and temperature is the elephant in the room of this particular discourse.

As a software designer of medical devices [1][2][3], I understand the importance of the calibration procedure. To properly calibrate a medical device we first need to measure the smallest measuring sticks or pixels of each component of the system. The relationships between these measuring sticks are then identified, calibrated and rigorously tested. This essay uses the logic of the calibration to study the system of constants and units associated with the standard model. Using this novel approach, a number of flaws were found in the language of SM calling into question, not the numerology of theoretical physics necessarily, but the language and interpretation of the equations themselves.

One of the motivations for this research was to develop a cosmology better suited for the fractal paradigm. Homogeneity is one of the main assumptions of the standard model [4] in opposition to the main postulate of fractal cosmology which assumes fractality at all knowable length scales. The quantum hall effect shows that fractals can and do appear at the quantum scale [5]. Large scale fractal structures seem to defy the laws of physics [6] yet the standard model is still holding out for homogeneity at some, yet to be determined, very large scale [7]. Although fractals seem to appear at most length scales [8, 9, 10, 11, 12, 13, 14], as well as in the time domain [15, 16], fractal cosmology is still considered fringe by the physics community. What do fractals have to do with calibrating the Universe? As Mandelbrot pointed out in his "How long is a coastline theory"[17], fractals are all about scaling the measuring sticks.

Borrowing from the logic of the calibration and the language of the fractal paradigm, we attempted to find a complete set of small scale measuring sticks (measure units) for each of time, space, mass, charge and temperature. We found that this was not possible unless we let the units of Planck's constant be Joule and not Joule x second. Following this correction, we were able to properly calculate a complete set of small scale measure units, calibrated to the scale of the cycle (ie. the quantum scale). We were also able to calculate a self-similar set of large scale measure units calibrated to the scale of the second (ie. the human scale). These measure units form a scalable (self-similar) relationship that is immutable.

We agree that it would be reckless to challenge the old paradigm without offering a new one to replace it. The approach presented in this essay replaces the standard paradigm with the fractal paradigm. As with all paradigm shifts, this new paradigm requires a new language, a new convention and a new set of tools, some of which are summarized in Appendix A. In short, we conclude that something is wrong with the standard model in terms of calibration. This, we argue, can be corrected by answering "yes" to the question in the title of this essay.

"... in order to learn you must desire to learn, and in so desiring not be satisfied with what you already incline to think, there follows one corollary which itself deserves to be inscribed upon every wall of the city of philosophy: Do not block the way of inquiry."
Charles Sanders Peirce

1.1 Methods

In this section, we briefly outline the steps followed in a standard calibration procedure:

Identify the main components of the system.
Assign reference units for each component.
Identify and record previously measured parameters of the system.
Measure the smallest measuring sticks for each component of the system.
Identify and correct any problems found in previous steps.
Report the results.

2 Identify the components of the system and select reference units.

2.1 Component list and associated reference units

In this section we identify the main components of the system and select our reference measuring sticks. See Table 3 in Appendix A.

2.2 Discussion

From the perspective of the calibration, understanding the difference between standard units and measure units is imperative. It is easy to mix these two up as you will see. For example, a common definition for unit is as follows: "A unit of measurement is a definite magnitude of a physical quantity, defined and adopted by convention or by law that is used as a standard for measurement of the same physical quantity. Any other value of the physical quantity can be expressed as a simple multiple of the unit of measurement." In this definition, the first sentence is referring to standard units, but the second sentence is referring to measure units. This can be very confusing. To improve the language of the standard, and to prevent any mistakes in interpretation, we use the term "reference unit" in place of "standard unit" since the second, meter, kilogram, coulomb and kelvin are merely reference measurements against which all other measurements are made. They are not really "units" in the truest sense of the word since these quantities are divisible, ie. we can reference half a second. For the new standard, we explicitly define our terms:

Reference Unit: This term is analogous to the term "standard unit" only we refer to them as reference measurements not unit measurements. All other measurements are made relative to these magnitudes but need not be simple multiples of these magnitudes. Table 3 shows the set of (arbitrary) reference units calibrated to the human scale. Reference units in unit analysis are merely place holders for the actual measurements once they are known.

Measure Unit: This term is used in terms of the smallest measuring sticks or pixels of the system. Measure units are measured relative to the "reference units". Any other value can be expressed as a simple multiple of measure units (not reference units). Measure unit sets are scalable (ie. if you desire greater precision you simply shrink your measuring sticks). Table 5 in Section 5 contains the measure units calibrated to the scale of the cycle, and Table 6 contains a self-similar set calibrated to the scale of the second. **Measure units are only scalable when reference units are fixed.**

The scalability of measure units is a requirement of the fractal paradigm. In the next two sections, we show how we calculated the measure units found in Table 5 and 6.

3 Identify and record previously measured parameters of the system.

3.1 Measured constants of nature and associated reference units.

In Appendix A Table 4 we present the main constants of nature used throughout theoretical physics along with the reference units chosen by convention. These values can be verified by experimentation and correspond to physical properties of the Universe.

3.2 Discussion

Table 4 is a list of the most important measured constants of nature along with their associated reference units. Unit analysis describes the unit "relationships" between two or more of the components of the system. The values of these constants were taken from the NIST standard [18]. The reference units in this table were chosen by convention and are used throughout the physics community. Some of these relationships are called into question in this research, in particular, the unit relationship of Planck's constant which we argue is Joule and not Joule x second. The the logic behind this change will be outlined in Section (5.1). In the next section, we use the above constants to calibrate the measure units of time, space, mass, charge and temperature.

4 Measure the smallest measuring sticks for each component of the system.

In this section, we attempt to identify and measure the smallest measuring sticks (pixels) for each component of the system, specifically for time, space, mass, charge and temperature. Traditionally, the measured constants from Table 4 along with unit analysis are used to calculate Planck units. All values are calculated to 16 digits of precision corresponding to the digits of precision of the 32-bit computer used to calculate these values.

4.1 Calibrating Time

Using unit analysis and the measured constants of nature from Table 4, Planck time is calculated using the following formula:

$$t_p = \sqrt{\frac{G\hbar}{c^5}} = 5.3910604239631400 \times 10^{-44}[s] \tag{1}$$

In our model, Planck time is seen as the pixel of time or quantum of time, $\mathrm{Q}_{\mathrm{time}}$. It is considered a measure unit and is measured relative to the reference unit of the second. It is analogous to the time for one clock cycle of a computer CPU and, as we show, all measure units of the system are defined by the clock. In the new language, $\mathrm{Q}_{\mathrm{time}}$ is defined as the unit of measure-time. All time measurements can be expressed as simple multiples of this value.

4.2 Calibrating Space

Using unit analysis, Planck length is calculated using the following formula:

$$\lambda_p = \sqrt{\frac{G\hbar}{c^3}} = 1.6161992557264318 \times 10^{-35}[m] \tag{2}$$

In our model, Planck length is seen as the pixel of space or quantum of space, Q_{space}. It is considered a measure unit and is measured relative to the reference unit of the meter. It also corresponds to the distance light travels in one measure unit of time. In the new language, Q_{space} is defined as the unit of measure-space. All spatial measurements can be expressed as simple multiples of this value.

4.3 Calibrating Mass

Using unit analysis, Planck mass is calculated using the following formula:

$$m_p = \sqrt{\frac{\hbar c}{G}} = 2.1765092524764168 \times 10^{-8}[kg] \tag{3}$$

Although this value is much smaller than the reference unit of the kilogram, it cannot correspond to the pixel of mass since the masses of both the electron and the proton are much smaller than Planck mass. This herein lies the problem. Planck mass does not correspond to the smallest measuring stick of mass and therefore, cannot be considered as a pixel of the system. From the perspective of a calibration procedure, this is an issue that will need to be addressed.

4.4 Calibrating Charge

Using unit analysis, Planck charge is calculated using the following formula:

$$q_p = \sqrt{4\pi\epsilon_0\hbar c} = 1.8755459562019361 \times 10^{-18}[C] \tag{4}$$

Here again we calculate a value that is much smaller than the Coulomb at 6.241 x 10^{18} C but is still bigger than elementary charge at 1.602 x 10^{-19} C. Like Planck mass, Planck charge cannot be the pixel of charge.

4.5 Calibrating Temperature

Using unit analysis, Planck temperature is calculated using the following formula:

$$T_p = \sqrt{\frac{\hbar c^5}{G k_B^2}} = 1.4168331310842985 \times 10^{32}[K] \tag{5}$$

Here, it is obvious that Planck temperature does not correspond to the pixel of temperature.

4.6 Discussion

In this section, we attempted to calibrate the smallest measuring sticks or pixels of time, space, mass, charge and temperature. Although we agree that Planck time and Planck meter could correspond to the smallest measuring sticks for the domains of time and space, Planck mass, Planck charge and Planck temperature cannot and do not correspond to the smallest measuring sticks of their respective domains. From the perspective of the calibration, this represents a huge problem and needs to be investigated further.

5 Identify and correct any problems with previous step.

5.1 Identifying the Problem

In the previous section, we discovered that Planck mass, Planck charge and Planck temperature do not correspond to the smallest measuring sticks or pixels of the system. As an experienced calibrator, when a measurement problem is found with a system, the first thing we do is study the system from first principles (and with a fresh set of eyes). We begin by studying the two most famous energy equations.

$$E = mc^2 \ [J = kg\frac{m^2}{s^2}] \tag{6}$$

The body of equation (6) expresses the relationship between energy and mass (presumably of a particle). In this equation, c^2 is constant and energy varies with the mass. The units of this equation on the right show that the unit of energy (Joule), forms a relationship between mass, space and time. This relationship, we argue, is immutable.

$$E = hf \ [J \times s \times \frac{1}{s}] \tag{7}$$

Equation (7) expresses the relationship between energy and frequency (presumably of a photon). In this equation, h is constant and energy varies with frequency. The unit section on the right, however, needs a bit of explaining. Historically (by convention) the reference unit of frequency was defined as 1/second, and, since it was well known that the unit of energy is Joule, in order to balance the unit equation, we had no choice but to conclude that the units for h were Joule x second.

However, what Planck et el didn't understand was that this energy equation, unlike equation (6) represents an experiment. For clarity, we expand the above equation as follows:

$$E = hf = \ MeasureTime \times \ \frac{h\,n}{MeasureTime} \ [J \times T \times \frac{1}{T}] \tag{8}$$

This equation reads: "for the duration of the experiment (measure-time), count n oscillations, multiply each n by h, then divide by measure-time when the experiment is complete". Although dividing by measure-time gets the result back in terms of reference time, measure-time we argue, is not the same as reference time. In the unit section, we assign a different label for measure-time (T). In the new standard, we define cycle (Cy) as the unit or pixel of measure-time. The main difference between this equation and equation (7) is that all the terms in the unit section are now accounted for in the body of the equation. In the above analysis, the unit of h is Joule. However, this is not exactly right because the term cycle (Cy) is still not in the unit section. When we put the term cycle back into the language, the unit section looks more like this:

$$E = hf\ [\frac{J}{Cy} \times T \times \frac{Cy}{T}] \tag{9}$$

Technically, the units of Planck's constant are J/Cy. In other words, h is the energy associated with one cycle. Planck's constant is not an action constant, it is an energy constant. The real action constant will be revealed in Part II of this essay. Because this constant has different units than the standard units, we give it a different label, $\mathbf{Q_{energy}}$.

5.2 Correcting the Problem

Now that we have identified a problem with the system, let's see if this fixes the calibration issues identified in Section 4. For the sake of comparison with the standard model, we will be using the numerical value of $\hbar$ in all our calculations.

5.2.1 Correcting Mass

Since $\mathbf{Q_{energy}}$ has units in terms of J, we can now use the mass energy relationship in equation (6) to calculate the quantum of mass as follows:

$$Q_{mass} = \frac{Q_{energy}}{c^2} = 1.1733692893415208 \times 10^{-51}\ [\frac{kg}{Cy}] \tag{10}$$

As you can see, this mass is much smaller than the mass of the electron and the proton and all the other known particles with mass. $\mathrm{Q_{mass}}$ is considered the pixel of mass or quantum of mass and has units [kg/Cy].

What is Planck mass then? If we divide Planck mass by $\mathrm{Q_{mass}}$ we get the following:

$$\frac{Planck_{mass}}{Q_{mass}} = 1.8549226336900676 \times 10^{43} = f_p\ [\frac{Cy}{T}] \tag{11}$$

This of course is Planck frequency. Planck frequency in the new standard is analogous to the clock frequency of the CPU of a computer. Since it is a frequency, it has units [Cy/T] (cycles per measure-time). Again, we re-iterate that measure-time [T] is not the same as reference-time [s], thus, even when we set measure-time to 1 second, we argue it cannot be cancelled with [s] in the unit section of an equation. We now use $\mathrm{Q_{mass}}$ as a measuring stick to measure the electron and proton as follows:

$$\frac{Electron_{Mass}}{Q_{mass}} = 7.7634407110757659 \times 10^{20} = electronComptonFrequency\,[\frac{Cy}{T}] \quad (12)$$

$$\frac{Proton_{Mass}}{Q_{mass}} = 1.4254862430724202 \times 10^{24} = protonComptonFrequency\,[\frac{Cy}{T}] \quad (13)$$

$\mathrm{Q_{mass}}$ gives us a direct method of measuring the Compton frequency of physical particles with measurable mass. The above unit analysis does not work unless the units for mass are [kg/T]. This is an important feature of the new standard. Since all "phenomenon" is thought to be generated by the universal clock, all measurements, including charge and mass, have units in terms of time. Planck mass also has units [kg/T] as we saw above.

Although NIST does not report Compton frequency directly, it does report the Compton wavelengths of the proton and electron using the wavelength to frequency conversion formula $\nu_c = \mathrm{c}/f_c$. This, we argue, is an illegal move since they are attempting to use a measuring stick from the domain of space to make a measurement in the domain of mass, ie. they are mixing measuring sticks. Compton wavelength need not have any physical meaning in the new standard.

5.2.2 On Compton Scattering

Compton scattering, however, can be completely described in terms of the Compton frequency and de Broglie frequency without involking the term "wavelength" or wave-particle duality [19]. Here, we see that the moving electron (particle) merely "transports electromagnetic energy from a source to a sink". In other words, Compton scattering is the result of absorption-emmission events. The electron absorbs some of the energy (de Broglie frequency worth) from the incident X-ray and begins to move. When the electron comes to a stop, it releases the absorbed energy. The new standard avoids particle-wave duality by doing all mass and energy calculations in the frequency domain being careful to use the mass measuring stick (Q_{mass}) to measure mass and the energy measuring stick (Q_{energy}) to measure energy.

5.2.3 Correcting Charge

Here, we calculate the quantum of charge by dividing Planck charge by Planck frequency as follows:

$$Q_{charge} = \frac{q_p}{f_p} = 1.0111181577804362 \times 10^{-61}\,[\frac{C}{Cy}] \quad (14)$$

$\mathrm{Q_{charge}}$ is defined as the pixel of charge. It is related to the pixel of mass in the following manner:

$$\frac{Q_{mass} \times G}{Q_{charge}} = \frac{Q_{charge} \times K_e}{Q_{mass}} \tag{15}$$

This equation shows that the Coulomb constant, K_e and the gravitational constant, G, are merely scaling factors (calibration parameters) used to normalize the mass and charge measure units. Expanding on this idea we write the following:

$$\frac{Q_{mass}^2 \times G}{Q_{charge}^2 \times k_e} = 1.0000000000(34) \tag{16}$$

We refer to this as the unity equation as it seems to be saying that gravity and electrostatic charge are unified. In order for this equation to be true, all the units must cancel, and they do. It is interesting to note that most of the constants of nature specified in Table 4 are embedded in this equation, since they were originally used to calculate Planck mass and Planck charge in Section 4. This equation can be used to test the validity of these constants since, as we get more digits of precision of the constants, this equation should give us more zeros. This in itself is worthy of further investigation.

5.2.4 Correcting Temperature

Now, we calculate the quantum of temperature by dividing Planck temperature by Planck frequency as follows:

$$Q_{temp} = \frac{T_p}{f_p} = 7.6382330203483409 \times 10^{-12} \, [\frac{K}{Cy}] \tag{17}$$

Q_{temp} is defined as the quantum of temperature. This value is related to the quantum of energy in the following manner:

$$K_b \times Q_{temp} = 1.0545717253664311 \times 10^{-34} = Q_{energy} \tag{18}$$

This equation shows the relationship between the quantum of energy and the quantum of temperature. Boltzmann's constant, K_b, can now be seen as the scaling factor (calibration parameter) between the measure unit of temperature and the measure unit of energy.

5.3 Measure units calibrated to the cycle

We can now report a complete set of measuring sticks or measure units for time, space, mass, charge and temperature. See Appendix B Table 5 and 6. The prefix Q in Table 5 is given to each **measure unit** as a reminder that they are calibrated to the **quantum scale** of the cycle. This table is referred to as the **per-cycle table**. These measure units can be used both in unit analysis and as constants in the body of the equation as in equations (12 and 13). When used as a constant in the body of an equation, the per-cycle measure unit much be explicitly shown in the unit section (eg. Q_{mass} has units [kg/Cy]) as seen in Section (5.2.1). When used in unit analysis the CY term is implied by the square brackets. The per-cycle measure units are used for measuring small scale structures such as photons, electrons and protons.

5.4 Measure units calibrated to the second

A self-similar set of measure units, calibrated to the second, can now be calculated by multiplying each component from Table 5 by Planck frequency. The prefix ST in Table 6 is given to each measure unit as a reminder that they are calibrated to the standard reference unit of the second. This table is referred to as the **per-second table**. The per-second measure units are self-similar to the per-cycle measure units and are primarily used for measuring large scale structures. These measure units can be considered the "pixels" of the domain of the second. If you need more resolution to your measurements, then you just make your pixel smaller (by using the Q measure units).

5.5 Summary

In this section, we calibrated two measure unit sets: 1) the per-cycle unit set and 2) the per-second unit set. These unit sets are self-similar in accordance with the fractal paradigm. The per-cycle measure units are better suited for measuring small scale structures. The per-second measure units are better suited for measuring large scale structures. The second, meter, kilogram, Coulomb and Kelvin are merely reference measurements and are not "units" in the truest sense of the word. All measurements, including measure units, are made relative to these carefully calibrated references. Looking closely at equation (9), it appears that Planck et al inadvertently injected the per-second measure unit of one second into the reference units of Planck's constant, unwittingly handicapping our ability to properly calibrate the universe as we just did. From this, we argue that removing this anomalous second from the units of Planck's constant, is **the right thing to do**. Rather than modifying the old standard to accommodate these new ideas, we instead create a new standard so the two can be compared. The main components of this new standard are summarized in Appendix B.

6 Conclusion

In this essay, we argue that the units of Planck's constant are Joule and not Joule x second. To demonstrate this, we use the logic of the calibration and the language of the fractal paradigm to calibrate the pixels of time, space, mass, charge and temperature. During this process, we noticed that Planck mass, Planck charge and Planck temperature did not correspond to smallest measuring sticks or pixels of the system. By applying our correction to the units of Planck's constant, we were able to properly calibrate the smallest measuring sticks or pixels for each component of the system. We apply this calibration to two self-similar measure scales, the scale of the cycle and the scale of the second. The per-cycle measure units are best for measuring small scale structures and the per-second measure units are best for measuring large scale structures.

Given the ubiquitous use of Planck's constant throughout theoretical physics, it is imperative that we get this right. From our analysis we conclude that Planck et el made a mistake in terms of unit analysis and inadvertently injected 1 second worth of measure time into the units of h. This prevented us from properly calibrating the Universe for the last 100 years. The implications of this simple change to the units of h is far reaching making is almost impossible to repair the old standard. In Appendix B, we begin to define a new standard based on the information provided in the body of this essay.

In Part II we do rigourous unit analysis using the measure units from both tables. Here we study all the relationships including space-time, mass-time charge-time, temperature-time. We also study the emission-absorption, momentum and energy unit relationships. All constants of nature from Table 2 can be derived using the new measure units. During this analysis, the true nature of alpha and the true action of energy is revealed. The new unit analysis leads to a new interpretation of physics that is more intuitive and visualizable. Although a lot more work needs to be done to vindicate this approach, it is the contention of the author that the units of Planck's constant are not J x s.

7 Appendix A

Component	Reference	Symbol
Time	Second	s
Space	Meter	m
Mass	Kilogram	kg
Charge	Coulomb	C
Temperature	Kelvin	K

Table 1: List of components of the system and associated reference units.

Constant	Label	Value	Measure Reference
Speed of Light	c	2.99792458 e+8	m s $^{-1}$
Planck's Constant	h	6.62606957 e-34	J x s = kg m ^{2} s $^{-2}$ s ???
Planck's Constant / 2π	$\hbar$	1.054571726 e-34	J x s = kg m ^{2} s $^{-2}$ s ???
Gravitational Constant	G	6.67384 e-11	m ^{3} kg $^{-1}$ s $^{-1}$ s $^{-1}$
Coulomb Constant	ke	8.987551787 e+9	N m ^{2} C $^{-2}$ = kg m s $^{-2}$ C $^{-2}$
Boltzmann Constant	kb	1.3806503 e-23	J K $^{-1}$ = kg m ^{2} s $^{-2}$ K $^{-1}$
Permittivity	e0	8.854187817 e-12	C^2 s^2 kg $^{-1}$ m $^{-3}$

Table 2: List of the measured constants of nature from the NIST standard and associated reference units.

8 Appendix B

Component	**Reference**	**Symbol**
Oscillation	Cycle	Cy
Time	Second	s
Space	Meter	m
Mass	Kilogram	kg
Charge	Coulomb	C
Temperature	Kelvin	K

Table 3: List of components of the system and associated reference units. Here we add the term Oscillation with units Cy.

Constant	**Label**	**Value**	**Measure Reference**
Speed of Light	c	2.99792458 e+8	m s $^{-1}$
Planck's Constant	h	6.62606957 e-34	J x s = kg m ^{2} s $^{-2}$
Planck's Constant / 2π	$\hbar$	1.054571726 e-34	J x s = kg m ^{2} s $^{-2}$
Gravitational Constant	G	6.67384 e-11	m ^{3} kg $^{-1}$ s $^{-1}$ T $^{-1}$
Coulomb Constant	ke	8.987551787 e+9	N m ^{2} C $^{-2}$ = kg m s $^{-1}$ T^{-1} C $^{-2}$
Boltzmann Constant	kb	1.3806503 e-23	J K $^{-1}$ = kg m ^{2} s $^{-2}$ K $^{-1}$
Permittivity	e0	8.854187817 e-12	C^{2} s^{2} kg $^{-1}$ m $^{-3}$

Table 4: List of the measured constants of nature from the NIST standard and associated reference units.

Label	**Domain**	**Value**	**Reference Unit**
Q_{time}	Time	5.3910604239631400 e-44	second, s
Q_{space}	Space	1.6161992557264318 e-35	meter, m
Q_{mass}	Mass	1.1733692893415208 e-51	kilogram, kg
Q_{charge}	Charge	1.0111181577804362 e-61	Coulomb, C
Q_{temp}	Temperature	7.6382330203483409 e-12	Kelvin, K
Q_{cycle}	Oscillation	1	Cycle, Cy

Table 5: Per-cycle table. This table shows the values of the measure units calibrated to the cycle. These are displayed with 16-digits of precision which corresponds to the limit to the digits of precision of the 32-bit computer used to calculate these values.

Label	**Domain**	**Value**	**Reference Unit**
ST_{time}	Time	1	second, s
ST_{space}	Space	2.99792458 e+8	meter, m
ST_{mass}	Mass	2.1765092524764168 e-8	kilogram, kg
ST_{charge}	Charge	1.8755459562019361 e-18	Coulomb, C
ST_{temp}	Temperature	1.4168331310842985 e+32	Kelvin, K
ST_{cycle}	Oscillation	7.4000706546312243 e+42	Cycle, Cy

Table 6: Per-second table. This table shows the values of the measure units calibrated to the second. These are displayed with 16-digits of precision corresponding to the limit to the digits of precision of the 32-bit computer used to calculate these values.

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