Seeking truth using different methods

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Feb. 16, 2015

Abstract: Mathematics and physics are entirely different subjects, both in their methodology and results. The differences are illustrated with how they view randomness, enumeration, and truth. In physics, randomness can be an unknown, an observational error, an unpredictable event, or a quantum mystery. In math it is just part of probability theory. Mathematics has subtle properties of infinities that physics does not directly observe. Logical positivism is a philosophy that explains how mathematics and physics search for truth in different ways, but it has been rejected by modern philosophers, leaving them with deeply flawed views of what math and physics are all about.

Submitted to the FQXi.org 2015 Essay Contest answering: “Trick or Truth: the Mysterious Connection Between Physics and Mathematics”

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Introduction

Mathematics and physics have one big thing in common. They both search for objective truths. Beyond that, they have little in common. Math uses the methodology of logic and proof. Physics uses observation and experiment.

Mathematics and physics have many formal similarities. Both use numbers and formulas, and use them in ways that seem unintelligible to outsiders. The Newton-Leibniz discovery of differential and integral calculus was simultaneously a huge advance for both fields.

This essay emphasizes the differences. To illustrate the differences, random and infinite numbers are examined to show that they are viewed very differently in each field. Finally, these are related to philosophical foundations for science.

By *mathematics*, I mean rigorously proving theorems from axioms, as in typical math journals. By *physics*, I mean explaining the fundamental causes of nature, such as energy, motion, and force. They both use numbers.

Different views of randomness

The differences between math and science are well illustrated in their different views of randomness. In many sciences, especially the social and medical sciences, much of the quantitative part of research is devoted to distinguishing causal influences from random influences.

For example, a drug company will do a randomized clinical trial to try to get approval for a new drug. Patients will be randomly divided into two groups, with one getting the new drug and the other getting placebo pills. Assuming that the drug group does better, a statistical analysis must be done to decide whether the result was caused by the new drug, or could be explained by just random sampling.

Just about every experiment in science does something similar. When the discovery of the Higgs boson was announced, the first figures given were the mass and “5 sigma”. The latter is just a statistical way of saying that the result is much more likely to be a new particle than randomness.

In these examples, randomness can be seen as experimental error. It plays a more central role in quantum mechanics, where randomness and indeterminacy is usually considered an intrinsic part of nature. The canonical example is radioactive decay. Physics can predict the byproducts and the statistical half-life, but it cannot predict precisely when an individual atom will decay. Other examples involve observations of variables known to have a large variation from the Heisenberg uncertainty principle.

There are deterministic interpretations of quantum mechanics, so not everyone agrees with quantum randomness.

Mathematicians look at randomness completely differently. They do not care about radioactive decay or any physical uncertainty. For them, a random process is defined as part of probability theory that was axiomatized by Kolmogorov in the 1930s. A *random variable* is a function on a measure-one state space. That is, it is just a way of parameterizing outcomes based on some measurable set of samples. The probability of an event is the measure of the corresponding set of samples. Mathematics can then be used to formally prove the law of large numbers, and lots of other theorems.

The mathematics is air-tight, and not subject to debate. Statisticians have taken the task of interpreting probability theory so it can be applied to the real world. Interpretations range from probability being a physical entity like temperature and called *propensity*, to being just a subjective measure of how persuaded you are of something. The major interpretations are the *frequentist* and *Bayesian*.

To mathematicians, the interpretation is just metaphysical opinion of dubious value. Their proofs are valid regardless of how they are interpreted.

So what is randomness? Are the digits of pi random? Is a coin toss random? A mathematician would say that the question is ill-defined. A social scientist might say that they are random if they are uncorrelated with the other variables being considered. A physicist might say that they are random if current theory cannot predict them.

For the concept of randomness to be scientific, there needs to be some empirical test to decide whether some given data or process is or is not random. There is no such test that we can apply to radioactive decay. Maybe uranium U-238 and potassium K-40 decay according to a nuclear quark oscillation that was set in motion by a supernova several billion years ago, or maybe it is spontaneous and indeterministic.

One of the trinity of biological forces is random

Here is a scientist describing recent research claiming that two-thirds of adult cancer incidence across tissues can be explained primarily by bad luck:

> All cancers are caused by a combination of bad luck, the environment and heredity, and we've created a model that may help quantify how much of these three factors contribute to cancer development. [Tomasetti]

Not everyone agrees with these conclusions, but they nicely illustrate a common scientific view of luck.

The cancer researchers have an explanation for how you can get those bad luck cancers. Sometimes DNA mutations occur during cell division, and bad DNA can be cancerous. To the patient, a cancer seems like bad luck whether it came from genes, smoking, or mutation. All three factors presumably have completely causal explanations, if cancer were better understood. The reason they call the mutation bad luck is just that they have no good way of modeling it as a causal factor.

Attributing cancers to randomness has a psychological effect, they say. Some patients are relieved to be told that the cancer cannot be blamed on their parents or their lifestyles. Others are terrified that cancer could strike anyone at any time.

Here is another paper breaking biological causal factors into three forces:

> If our genes and environment govern our actions, does this mean that our behavior is deterministic? Not necessarily. Rather, there is a trinity of forces — genes, environment, and stochasticism (GES) — that governs all of biology including behavior, with the stochastic component referring to the inherent uncertainty of the physical properties of matter. [Cashmore]

To reductionist physicists, this all-encompassing trinity of forces is bizarre. Dividing between genes and environment is a little like dividing between the mind and body, or between computer software and hardware. The genes are the programs for the body. But what is stochasticism, and what does it do?

Randomness does not explain anything

Mathematically, nothing is ever explained by randomness or luck. Randomness is just a euphemism for possible causal factors not being modeled. In the cancer example, the scientists have two causal factors that they can isolate, environment and heredity. The random or luck factor is whatever is unexplained by those two.

Physicists have different views about randomness. A common view is that quantum mechanics describes true randomness, while all the other sciences have a pseudo randomness that results from complexity or uncertainty.

Physics also has pseudo randomness, such as in the study of statistical mechanics. An ideal gas might be considered to be random particles. That does not mean that they are truly random; they could be deterministic classical particles as long as their motions are uncorrelated.

Quantum mechanics is often interpreted to be a theory for assigning probabilities to truly random processes in nature. Bell's Theorem showed that quantum randomness cannot be simply a random sample of deterministic local hidden variables.

Regardless of the branch of science, attributing causality to randomness does not explain it at all. Randomness is just a buzz word to replace an admission that the some causal factors are unidentified.

When a uranium atom decays, it is often assumed that it is a spontaneous event that is independent of the history of the atom. But we have no proof of that. We can collect a large number of apparently similar atoms, and notice that they decay at much different times, but we do not truly know that the atoms are all in the same quantum state.

Likewise, the above cancer research did not find any evidence that the mutations were truly spontaneous. It found a correlation between cell divisions and cancer, suggesting that the cancer might be caused by a spontaneous and unpredictable mutation occurring during cell division. But saying “unpredictable” only means that the researchers have no way of predicting it, except to count cell divisions.

Thus for both uranium decay and cell division, science can give statistical estimates on how often events take place, but cannot prove that the events are truly spontaneous.

Philosophical view of randomness and causality

Randomness is even more confusing to philosophers, because it upsets their ideas about causality. Explaining the world with causality has been central to natural philosophy since Aristotle.

Religious scholars have also struggled with randomness and causality. Calvinists believe in predestination. Thomas Aquinas gave several arguments for randomness making a more perfect world. [Scarani] Many theologians have argued that man has a soul distinct from the body, and the mind has the free will to cause the body to do things.

Modern philosophers reject this mind-body dualism. They agree that causality is crucial to all the sciences except physics, but the big majority say that causality has no role in fundamental physics.

One philosophy book says, with puzzlement, that physicists adhere to “causal fundamentalism, which claims that the job of all of physics is to uncover the prevalent causal relations in nature.” [Galavotti] The popular electromagnetism textbook by Griffiths says that the principle of causality is “the most sacred tenet in all of physics”.

Thus there is a sharp split between physicists and philosophers. Physicists believe that causality is fundamental to physics, and philosophers reject that idea. It appears that they do not talk to each other.

The problem is rooted in philosophers’ misunderstandings about randomness. To them, causality is coupled with the imprecision of everyday experiences and expectations. They might say that cloudy skies cause rain. Some random error is understood.

The theories of fundamental physics are different. There are differential equations that make precise predictions, given precise initial value data. Those systems are completely deterministic. And there is the quantum mechanical observation that is just the opposite, and intrinsically random.

Neither of these fit into the philosophers’ preconception of what causality ought to be.

They refuse to use the term causality for something that is understood, somewhat like those who refuse to call something artificial intelligence once it is understood.

As a result, it is common for philosophers to reject the notion of free will as contrary to fundamental physics. Some say that we have an illusion of free will that is compatible with physical determinism, but they nearly all reject the libertarian free will that most people (outside philosophy) think that they have.

Different views applied to free will

If there were some experiment to distinguish different views of randomness, the matter would have been settled long ago. So the differences are most striking when applied to philosophical issues, like free will.

Physicist Seth Lloyd writes in a recent paper on free will:

> Although quantum mechanics implies that events are intrinsically unpredictable, the 'pure stochasticity' of quantum mechanics adds only randomness to decision making processes, not freedom. [Lloyd]

This is a very common opinion in the philosophy of free will, but it depends on a narrow view of randomness. To some people, randomness just means unpredictability, and that is also what free will is – the ability to make a choice other than what is predicted. That is, free will is the ability to make choices that appear random to others.

The above Cashmore paper attempts to define free will:

> Having now introduced the three forces that govern behavior, it is appropriate, at this rather late stage, to define what is meant by "free will." Searle has described free will as the belief "that we could often have done otherwise than we in fact did" (15). A difficulty with this definition is that it does not distinguish free will from the variability associated with stochasticism. For this reason, I believe that free will is better defined as a belief that there is a component to biological behavior that is something more than the unavoidable consequences of the genetic and environmental history of the individual and the possible stochastic laws of nature. [Cashmore]

By defining free will as something beyond the trinity of forces that seems to govern all behavior, he concludes that it is a scientific impossibility.

This view is common among philosophers and atheist intellectuals. Sam Harris writes:

> If determinism is true, the future is set — and this includes all our future states of mind and our subsequent behavior. And to the extent that the law of cause and effect is subject to indeterminism — quantum or otherwise — we can take no credit for what happens. There is no combination of these truths that seems compatible with the popular notion of free will. [Harris]

There is a popular view that quantum randomness is a physical thing that has been proved by XX century physics. Some say that the decisive proof was Bell's Theorem, and the experimental verifications of it. Others cite radioactive decay, or various optical experiments. Mathematically this corresponds to an interpretation of probability as objective, like propensity.

But other views are possible. Physicist N. David Mermin argues in favor of a Bayesian/psi-epistemic view, where the wave function represents our knowledge, as was common among the founders of quantum mechanics 80 years ago. He says that the probability shows our uncertainty about what is happening, but may or may not reflect some intrinsic randomness. [Mermin]

Cosmologist Max Tegmark has another view. In the March 12, 2014 episode of the TV show *Through the Wormhole,* he uses multiple universes to deny randomness:

> Luck and randomness aren't real. Some things feel random, but that's just how it subjectively feels whenever you get cloned. And you get cloned all the time. ... There is no luck, just cloning.

The idea is that in the many-worlds interpretation of quantum mechanics, every coin toss is actually a splitting of the world into one with heads, and one with tails. Likewise, every decision is a splitting, and free will is an illusion.

Different views of infinity

For another example of a difference between mathematics and physics, consider the use of infinite numbers. Mathematics uses them all the time, with the infinity of primes being one of the oldest theorems. But it is debatable whether any true infinities occur in science. Measurements always give finite values. Some physical entities are often thought to be infinite, such as the density of the center of a black hole, the unrenormalized charge of an electron, or the size of the universe. But none of these examples is very convincing, and many physicists are content to regard infinity as just some mathematical fiction.

More puzzling are larger infinities, such as the uncountability of the real numbers. It is an elementary theorem that the rational numbers can be enumerated, but any attempt to count the real numbers will leave some of them uncounted. These results were unsettling to people in the late 1800s, but it is hard to argue with a mathematical proof.

One thing that makes the results puzzling is that axiomatic set theories like ZFC have countable models. That is, you can have an entire mathematical universe, including the real numbers and everything else, in a countable collection. The catch is that while that model has everything necessary for mathematics in the sense of those axioms, it does not have the function that counts itself.

If that is not enough to make your head hurt, then there are some similar paradoxes associated to Goedel’s Incompleteness Theorem that are even more confusing.

On the other hand, physics has no interest in these paradoxes. The real numbers are just how we record measurements in the real world. Space and time form a continuum, or at least they appear to. The rational numbers might be good enough in practice, as floating point numbers on computers are rational, but physical numbers are usually thought of as just being like floating point numbers that could approximate anything.

There are some exceptions, like electric charge, which is always an integer multiple of the charge of an electron. Or if you include quarks, it must be a third of an integer multiple. But what about the fine structure constant that is approximately 1/137 and defines the strength of electromagnetism? It is not known to be determined by anything else, and might be a measure of something that happened in the very early universe.

A mathematician would want to know what kind of real number it is. Is it rational? Is it algebraic, like the root of a polynomial equation? Can its digits be enumerated by a Turing machine? Is it even in those countable models of ZFC?

That last question seems nonsensical, but only countably many real numbers have some natural definition in term of pure mathematics. If the fine structure constant does not, then the goal of reducing all of fundamental physics to mathematics seems doubtful. Not only might it be impossible to define a true theory of everything in ZFC, it might be impossible to define the fine structure constant in ZFC.

A physicist is not likely to see this as a problem at all. The real number line includes all real numbers, so of course it includes the fine structure constant. Whether it is rational or not is immaterial, as we could only measure it to finitely many decimal places anyway.

I am not trying to persuade you which view is more reasonable, the mathematician or physicist view. Each has its place. I am just illustrating how different the views are.

Logical Positivism

There was a philosophy of science in the 1930s called *logical positivism* that clarified the difference between mathematics and science. It is a variant of *positivism*, a view that is peculiar because it is widely accepted among scientists today, but widely rejected among philosophers of science. Wikipedia says logical positivism is “dead, or as dead as a philosophical movement ever becomes.”

The core of logical positivism is to classify statements based on how we can determine their truth or falsity. There are three types.

1. Math. This includes mathematical theorems, logic, tautologies, and statements that are true or false by definition.

2. Science. This includes observations, measurements, induction, approximations, hypothesis testing, and other empirical work.

3. All else. Mysticism, ethics, subjective beauty, opinion, religion, metaphysics, aesthetics, etc.

Mathematical truth is the highest kind of truth, and is given by logical proofs. It goes back to Euclid and the ancient Greeks, and is not known to have been independently discovered by anyone else. When a theorem is proved, you can be 100% sure of it.

You might think that mathematical knowledge is empirical, but it has not been for two millennia. You might also have been told that Goedel's work somehow undermined the axiomatic method. The truth is more nearly the opposite. Math means logical proof. If I say that the square root of two is irrational, that means that I have a proof.

Science arrives at somewhat less certain truths using empirical methods. A conclusion might be that the Sun will rise in the East tomorrow morning. This is backed up by many observations, as well as theories that have been tested in many independent ways. Science also makes predictions with lesser confidence. These are considered truths, even though they are not as certain as mathematical truths.

There are many other statements that are not amenable to mathematical proof or empirical verification. For example, I may be of the opinion that sunsets are beautiful. But I do not have any way of demonstrating it, either mathematically or empirically. If you tell me that you disagree, I would be indifferent because there is no true/false meaning attached to the statement. It is meaningless, in that sense.

So far, this is just a definition, and not in serious dispute. What makes logical positivists controversial is their strong emphasis on truth. They tend to be skeptical about aspects of scientific theories that have not been tested and verified, and they may even be contemptuous of unverifiable metaphysical beliefs. Others don't like it when their opinions are called meaningless.

Positivism has split physics and philosophy

Philosophers have turned against positivism, and especially logical positivism. They consider the subject dead. The most common criticism is to falsely attribute some metaphysical belief to logical positivists. Here is a very brief summary of alternate views.

The opposite of positivism is what I call *negativism*. In this view, there is no such thing as positive truth. Philosopher Karl Popper argued that there are only falsified theories, and theories that have not yet been falsified. In effect, he says you can only prove a negative, not a positive.

Another view is *paradigm shift* theory, modeled primarily on the example of the *Copernican revolution*. Philosopher Thomas S. Kuhn argued that Copernicus had no convincing rational or measurable advantages to his model, but other astronomers eventually joined the paradigm anyway.

Realism is a philosophy that argues that theories like quantum mechanics are too limited, because they do not tell you what is really going on. They prefer making the theory more realistic by adding hidden variables, Bohmian pilot waves, or many worlds.

All of these alternatives are much more popular than positivism among philosophers today. Some have argued that today’s split between physicists and philosophers is rooted in the latter’s rejection of positivism, in spite of the overwhelming success of positivism in XX century physics.

Relativity and quantum mechanics are regarded as great theoretical advances of the century, and positivist ideas were essential to both. Relativity discarded preconceptions about space, time, and mass, and replaced them with ideas more closely connected with what is measurable, and then relied heavily on experiment. The aether became a metaphysical idea whose existence was meaningless unless some way of measuring it were found.

Quantum mechanics, in the Copenhagen interpretation, made the observables the heart of the theory and openly declared the non-empirical questions to be meaningless. There is a long history of physicists and philosophers being unhappy with the positivism of this, and endorsing other interpretations that are supposedly more realist. Albert Einstein famously denounced quantum mechanics, and others have also sought theories with hidden variables, pilot waves, and parallel universes. What they have in common is to assert the reality of unobservables, and to fail to provide an empirical test of them.

The good work in quantum mechanics follows the slogans “Shut up and calculate” and “Unperformed experiments have no results.” These represent the positivist view that science is all about explaining and predicting what is observable, and avoiding speculation about what is not. The famous quantum paradoxes are all derived from anti-positivist and unnecessarily realist interpretations. [Fuchs-Peres]

Richard P. Feynman did not say either of those slogans, but his textbook did say something similarly positivist (and which philosophers would put down as anti-realist):

> Another thing that people have emphasized since quantum mechanics was developed is the idea that we should not speak about those things which we cannot measure. (Actually relativity theory also said this.) [Feynman]

A recent example of positivism is the 2011 Nobel Prize for the discovery of the dark energy that permeates the universe. The recipients and most other astrophysicists refuse to say what it is, except that the phrase is shorthand for the supernovae evidence for the accelerating expansion of the universe. When asked to speculate, one of them says “Reality is the set of ideas that predicts what you see.” [Schmidt] Again, a positivist view prevails.

Mathematical and physical realities are different

This essay contest included this question:

> What would it mean for something in the physical world to be NOT describable or model-able in terms of mathematics?

This question seems squarely aimed at me, as my 2012 FQXi essay argued:

> Much of modern theoretical physics assumes that the true nature of reality is mathematics. This is a great mistake. The assumption underlies most of the paradoxes of quantum mechanics, and has no empirical justification. Accepting that the assumption is wrong will allow physics and mathematics to progress as distinct disciplines. ...
>
> The 2011 FQXi essay contest asked, “Is Reality Digital or Analog?” The answers accepted the premise that reality had to be one or the other, and no one admitted the possibility that it might be neither because both are mathematical. [Schlafly]

I could be wrong, of course, but I seem to be the only one to have seriously considered the possibility that fundamental physics is not perfectly describable by mathematics.

I was influenced by a logical positivist philosophy that embraces a logical view of mathematics, and an empirical view of science. To me, these views are so different that it would be bizarre if one were completely reducible to the other.

Since then I have learned how thoroughly modern philosophers have rejected logical positivism, and how their reasons are entirely mistaken.

Conclusion

Logical positivism gives a coherent view of modern math and science. It more closely resembles the views of modern successful mathematicians and scientists than the alternatives that philosophers espouse. In short, scientists believe that they are finding truth, and philosophers deny that it is possible.

Mathematical truth is based on proofs from axioms, and scientific truth is based on empirical observation. While they both agree that 2+1=3, they are strikingly different for random and infinite numbers. These differences should be recognized and celebrated.

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