

New formula of the mobius function

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Theorem

$$\sum_{1 \leq n \leq x} \mu(n) \left[\frac{x}{n} \right] = 1$$

Proof)

$$x = 1 \Rightarrow 1 = 1$$

$$x = 2 \Rightarrow 2 - 1 = 1$$

$$x = 3 \Rightarrow 3 - 1 - 1 = 1$$

$x = 29$ and $x = 30$ cases

$1 - 1 - 1 - 1 + 1 + 1 + 1 - 1 = 0$ increase.

$$\sum_{1 \leq n \leq 30} \mu(n) \left[\frac{30}{n} \right] = 1$$

General cases are simillally.

Remark:

$$\sum_{1 \leq n \leq x} \mu(n) = O\left(\sum_{1 \leq n \leq x} \mu(n) \left[\frac{x}{n} \right]\right) = O(1)$$

is not true.