The charged component of the vacuum field as the source of electric force in the modernized Le Sage's model

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The formula is derived for the electric force inside a uniformly charged spherical body, as well as for the Coulomb force between the charged bodies from the standpoint of the model of the vacuum field with charged particles. The parameters of the fluxes of charged particles are estimated, including the energy density, energy flux and cross section of interaction with the charged matter. The interaction cross section is almost exactly equal to the geometric cross section of nucleons and becomes equal to the cross section of interaction of gravitons with the matter, if it is assumed that the ratio of the energy density of graviton fluxes to the energy density of the charged particles in the vacuum field is equal to the ratio of masses of the proton and the electron. In this case, the energy density of gravitons in the Le Sage's gravitation model is expressed in terms of the strong gravitational constant, which establishes connection between the ordinary gravitation at the level of stars and the strong gravitation at the atomic-nucleon level of matter. The relation is derived, which connects the body charge and the rate of emission from the body of the charged particles of the vacuum field, which interacted with the matter and transferred their momentum to the body. The charge to mass ratio is determined for the charged particles that make up photons and the charged component of the gravitational field. These particles are identified as praons, while the praon level of matter is considered a lower level relative to the nucleon level of matter. Praons are related to nucleons the same way as nucleons are related to neutron stars. Based on the theory of infinite nesting of matter a conclusion is made that the charged particles of the vacuum field are generated at all levels of matter by the densest objects, such as nucleons and neutron stars.

The keywords: vacuum field, graviton field, electric force, praons, infinite nesting of matter.

1. Introduction

The similarity of Maxwell equations for the electromagnetic field, on the one hand, and the Heaviside equations for the gravitational field in the Lorentz-invariant theory of gravitation [1-2], on the other hand, as well as the similarity of formulas for the Coulomb force and the Newton force implies a large probability that the same physical mechanism is responsible for that. Earlier in [1] and [3], we derived the formula for the Newton's law of universal gravitation and the expression of the gravitational constant in terms of the graviton field parameters, using the modernized Le Sage's theory of gravitation. In addition, in [4] we found the expression for the body mass as the function of luminosity of the gravitons interacting with the body, as well as the expression for the strength of the gravitational field inside the body.

Now we intend to derive the formula for the Coulomb force between the charged bodies and to specify the parameters of the vacuum field, consisting of the graviton field and the field of charged particles. In the modernized Le Sage's theory of gravitation the allpermeating fluxes of the vacuum field particles consist of neutrinos, photons and charged particles, the properties of which are similar to high-energy cosmic rays. The presence of charged particles in the dynamic vacuum field allows us to describe the electrostatic forces and as a result to justify the electromagnetic phenomena.

2. The interaction picture

To understand the electric interaction of the bodies at a distance from each other, consider Figure 1 which shows the motion of small charged particles of the vacuum field near the two bodies, one of which is neutral and the other is positively charged. As can be seen, both positive and negative particles act symmetrically on the positively charged body, which does not result in emerging of any additional force in comparison with the force of gravitation. The same applies to the second neutral body.



Fig. 1. The lines of motion of the small particles of the vacuum field, which are a) positively charged, b) negatively charged, near two bodies one of which is neutral and the other is positively charged.

Figure 2 a) shows that the positive particles push the negatively charged body to the left, and Figure 2 b) shows that the negative particles push the positively charged body to the right (when the smallest particles pass through the body similarly to gravitons, they transfer their momentum to them). Consequently, both bodies will be attracted to each other.



Fig. 2. The lines of motion of the small particles of the vacuum field which are a) positively charged, b) negatively charged, near two bodies, one of which is negatively charged and the other is positively charged.

Figure 3 shows the lines of motion of the negative particles of the vacuum field near two positively charged bodies. Both bodies attract the negative particles and obtain an additional momentum from them, which leads to repulsion of bodies. The motion of the positive particles of the vacuum field in Figure 3 is not shown. It is assumed that they are repelled from the bodies and therefore their interaction with them is weak.



Fig. 3. The lines of motion of the small particles of the vacuum field, which are negatively charged, near two positively charged bodies.

For two negatively charged bodies the interaction is similar to the one shown in Figure 3, only it is necessary to replace the signs of all charges. This results in the repulsion of similarly charged bodies. The described above picture can be found in [5]. The common in all the Figures is the fact that depending on the sign of the charge of two bodies the number of charged particles falling on the body changes so that after calculating the momentum transferred from these particles the electric force with required direction emerges. Thus, we reduce the interaction between the charges at a distance to the interaction by means of the charged particles of the vacuum field.

3. The Coulomb force

To determine the expression for the electric force we use the approach applied in [3-4]. Let's assume that the fluence rate of the charged particles of the vacuum field is defined by idealized spherical distribution of the following form:

$$B_{0q} = \frac{dN_0}{dt \, d\alpha \, dA} \,. \tag{1}$$

According to (1) we suggest that some detector per unit time dt measures the charged particles of the vacuum field in the amount dN_0 that fall on the detector from the solid angle $d\alpha$ per unit surface area dA perpendicularly to this surface.

We will assume that inside the matter of each charged body an exponential change in the number of charged particles of the vacuum field takes place, as the flux of these particles travels some path x in this matter:

$$dB = -B \vartheta \eta \, dx \,, \qquad B = B_{0a} \exp(-\vartheta \eta \, x) \,, \qquad (2)$$

where \mathcal{G} is the cross section of interaction of the moving charged particles with the matter, η is the concentration of charges associated with the matter.

Denoting the positive elementary charge by e, for the absolute values of charges and the area A of ball segments in Figure 4 we obtain the following:

$$\left|Q_{1}\right| = e\eta_{1}x_{1}A, \qquad \left|Q_{2}\right| = e\eta_{2}x_{2}A, \qquad A = \alpha \cdot \left(\frac{R}{2}\right)^{2}. \qquad (3)$$



Fig.4. Charges Q_1 and Q_2 in the form of ball segments with different thickness and charge density, located at the distance *R* from each other.

The detector is located at point 0 in the middle between the two segments. For it, each segment is seen at the same solid angle α at the distance $\frac{R}{2}$, while the transverse areas of the segments are the same and equal A. It means that before we apply further arguments for the two large bodies, we should cut these bodies into segments and then calculate the total electric force between all the possible pairs of segments by means of vector summation of particular forces.

Let us first consider the case when the charge Q_1 is positive and the charge Q_2 is negative. Comparison with Figure 2 shows that interaction leads to attraction due to absorbing and scattering of charged particles falling on the charges and passing through them. As a first approximation we can assume that the main contribution is made by the flux of negatively charged particles falling on the charge Q_1 from the left and the flux of positively charged particles falling on the charge Q_2 from the right.

Decrease of the flux of charged particles on the left side after passing the first segment in Figure 4 according to (2) depends on the thickness of this segment and on the concentration of charge:

$$B_1 = B_{0q} \exp\left(-\mathcal{G}\eta_1 x_1\right) \,.$$

After that the flux of charged particles passes through the second segment with further decrease of the flux:

$$B_2 = B_1 \exp\left(-\vartheta \eta_2 x_2\right).$$

If p_q is the mean momentum of one charged particle in the flux of particles, then the force acting on the second segment from the left, taking into account (1) is equal to:

$$F_1 = p_q \alpha A(B_1 - B_2) = p_q \alpha A [1 - \exp(-\vartheta \eta_2 x_2)] B_{0q} \exp[-\vartheta \eta_1 x_1].$$

Decrease of the flux of charged particles, passing through the second segment from the right side, and the force from this side are, respectively:

$$B_3 = B_{0q} \exp(-\vartheta \eta_2 x_2), \qquad F_2 = p_q \alpha A(B_{0q} - B_3) = p_q \alpha A[1 - \exp(-\vartheta \eta_2 x_2)] B_{0q}.$$

For the force of electrical action on the second segment we find a symmetrical expression, which is equal by its absolute value to the force of electrical action on the first segment:

$$F_q = F_2 - F_1 = p_q \alpha A [1 - \exp(-\vartheta \eta_2 x_2)] B_{0q} [1 - \exp(-\vartheta \eta_1 x_1)].$$

Expanding the exponents in the linear approximation by the rule $\exp(-\kappa) \approx 1 - \kappa$, taking into account (3), we obtain for the force of attraction between two oppositely charged segments the following:

$$F_{q} = p_{q}B_{0q}\alpha A \mathcal{P}^{2}\eta_{2}x_{2}\eta_{1}x_{1} = \frac{4p_{q}B_{0q}\mathcal{P}^{2}}{e^{2}}\frac{|Q_{1}||Q_{2}|}{R^{2}}, \qquad \mathbf{F}_{q} = \frac{4p_{q}B_{0q}\mathcal{P}^{2}}{e^{2}}\frac{Q_{1}Q_{2}\mathbf{R}}{R^{3}}.$$
 (4)

In (4) the force \mathbf{F}_q is directed oppositely to the vector \mathbf{R} of the distance from the first segment to the second segment, since the charge Q_2 is negative.

According to the Coulomb's law, the formula for the electric force between two charged bodies is as follows:

$$\mathbf{F}_{q} = \frac{Q_{1}Q_{2}\mathbf{R}}{4\pi\varepsilon_{0}R^{3}}.$$
(5)

Comparing the values of the forces in (4) and (5), we arrive at the expression for the vacuum permittivity in terms of the parameters of charged particles fluxes in case of idealized spherical distribution:

$$\varepsilon_0 = \frac{e^2}{16\pi p_q B_{0q} \mathcal{P}^2} \,. \tag{6}$$

The vacuum permittivity in (6) depends on the cross section \mathcal{G} of interaction of charged particles fluxes with the matter, on the average momentum of one charged particle p_q , on the fluence rate B_{0q} and on the elementary charge e.

From the expression for the force we determine the electric field strength of one charge at the place of the second charge:

$$\mathbf{E} = \frac{\mathbf{F}_q}{Q_2} = \frac{Q_1 \mathbf{R}}{4\pi\varepsilon_0 R^3} \,. \tag{7}$$

We will assume now that the charge Q_2 in Figure 4 is positive like the charge Q_1 . This situation corresponds to Figure 3, from which it follows that after the passing the charge Q_1 the flux of charged particles effectively increases before falling on the charge Q_2 . For the flux of particles moving from the charge Q_2 and falling on the charge Q_1 the situation is symmetric. In order to take into account the effect of increasing of the flux of charged particles, we will introduce an additional coefficient ξ . Then the flux of charged particles from the left side after passing the first segment in Figure 4, taking into account (2), changes to the value:

$$B_1 = B_{0q} \exp\left[(\xi - \vartheta)\eta_1 x_1\right].$$

When passing through the second segment the flux decreases:

$$B_2 = B_1 \exp\left(-\vartheta \eta_2 x_2\right).$$

The force acting on the second segment from the left, taking into account (1), is equal to:

$$F_1 = p_q \alpha A(B_1 - B_2) = p_q \alpha A [1 - \exp(-\vartheta \eta_2 x_2)] B_{0q} \exp[(\xi - \vartheta) \eta_1 x_1].$$

For the flux of charged particles passing through the second segment from the right and the force from this side we obtain, respectively:

$$B_3 = B_{0q} \exp(-\vartheta \eta_2 x_2), \qquad F_2 = p_q \alpha A(B_{0q} - B_3) = p_q \alpha A[1 - \exp(-\vartheta \eta_2 x_2)] B_{0q}$$

For the force of electrical action on the second segment we obtain:

$$F_{q} = F_{1} - F_{2} = p_{q} \alpha A [1 - \exp(-\vartheta \eta_{2} x_{2})] B_{0q} [\exp[(\xi - \vartheta) \eta_{1} x_{1}] - 1].$$

In this expression, we will expand the exponent and use (3):

$$F_{q} = p_{q} \alpha A B_{0q} \vartheta \eta_{2} x_{2} (\xi - \vartheta) \eta_{1} x_{1} = \frac{4 p_{q} B_{0q} \vartheta (\xi - \vartheta)}{e^{2}} \frac{|Q_{1}| |Q_{2}|}{R^{2}}.$$
(8)

The repulsion force (8) after changing of the sign of the charge Q_2 must be equal by its magnitude to the attraction force in (4). For this the following condition must hold: $\xi = 2\vartheta$. There is a way to prove this relation. To do this, we should consider the situation in Figure 3, estimate the fluxes of charged particles from all sides and their interaction with the charged bodies, so that we could determine how much these fluxes increase when falling on the bodies as compared to the situation in Figure 1. We will return to this issue again in Section 6.

In Figure (1) we see that if one of the bodies has no charge, then the charged particles of the vacuum field do not interact with this body electrically. They pass through it almost freely, except for the gravitational action. As a result, between the charged and uncharged bodies there will be only the force of gravitational attraction.

4. The electric field strength inside the ball

In order to estimate the field inside a uniform ball it is more convenient to proceed from spherical distribution (1) to cubic distribution in the form of a mixed derivative for the flux of charged particles of the vacuum field directed in one way:

$$D_{0q} = \frac{dN_0}{dtdA},\tag{9}$$

where the fluence rate D_{0q} indicates the number of charged particles dN_0 , that during time dt fell on the area dA of one of the cube faces, limiting the volume under consideration, which is perpendicular to the flux.

Figure 5 shows the section of a uniform charged ball with a radius a, inside which there is a small test body in form of a ball with a radius b.



Fig.5. The small ball is at a distance r from the center of the large ball.

The fluxes of charged particles of the vacuum field move along the paths 1, 2, 3, as well as other paths, passing the section of the small ball, which is at a distance r from the center of the large ball. If we replace the small ball with the cube of the same size, then in case of idealized cubic distribution it is enough to consider the vertical fluxes along the path 2. The fluxes of charged particles passing through the other faces of the small cube will be symmetrical and will not influence the electric force. This means that with this approach we will take into account the fluxes along inclined paths 1 and 3 not directly, but indirectly. All these fluxes in case of vector summation will give the force, acting on the small ball and should be added to the force, calculated for path 2.

Let the volume of the small ball be equal to the volume of some cube. Then for the volume of a cube with an edge s and for the absolute value of charge |q| of this cube we obtain the relations:

$$s^{3} = \frac{4\pi b^{3}}{3}, \qquad |q| = e\eta_{b}s^{3}, \qquad (10)$$

where η_b is the concentration of charge in the small ball.

Distribution (9) replaces the actual distribution of the charged particles of the vacuum field in space with the idealized cubic distribution, when only six fluxes of charged particles fall on the given cubic volume perpendicularly to the faces of the cube.

By analogy with (2) we can write the dependence of the fluence rate of the charged particles of the vacuum field on the distance traveled in the matter:

$$dD = -D \vartheta \eta \, dx, \qquad D = D_{0a} \exp(-\vartheta \eta \, x). \tag{11}$$

Let us first assume the charge q of the small ball in Figure 5 as negative and the charge of the large ball as positive.

The flux of charged particles falling from above travels the path $a-r-\frac{s}{2}$ in the large ball with the concentration of charge η_a in its matter, and reaches the small cube, with which we replaced the small ball. According to (11) at this point the fluence rate decreases to the value:

$$D_1 = D_{0q} \exp\left[-\vartheta \eta_a \left(a - r - \frac{s}{2}\right)\right].$$

Then the flux passes through the small cube with concentration of charge η_b and decreases again:

$$D_2 = D_1 \exp\left(-\vartheta \eta_h s\right).$$

The force from this flux of charged particles is proportional to the square of the face of the small cube and to the number of charged particles, which transferred their momentum per time unit to the cube matter:

$$F_{1} = p_{q} s^{2} (D_{1} - D_{2}) = p_{q} s^{2} \left[1 - \exp(-\vartheta \eta_{b} s) \right] D_{0q} \exp\left[-\vartheta \eta_{a} \left(a - r - \frac{s}{2} \right) \right].$$
(12)

On the lower side of the large ball the flux of charged particles first passes the path $a+r-\frac{s}{2}$ to a small cube and then passes through the cube:

$$D_3 = D_{0q} \exp\left[-\vartheta \eta_a \left(a + r - \frac{s}{2}\right)\right], \qquad D_4 = D_3 \exp\left(-\vartheta \eta_b s\right).$$

The force acting on the small cube from this side equals:

$$F_{2} = p_{q}s^{2}(D_{3} - D_{4}) = p_{q}s^{2}\left[1 - \exp(-\vartheta\eta_{b}s)\right]D_{0q}\exp\left[-\vartheta\eta_{a}\left(a + r - \frac{s}{2}\right)\right].$$
 (13)

The total force is the difference between the forces (12) and (13):

$$F_q = F_1 - F_2 = p_q s^2 \left[1 - \exp\left(-\vartheta \eta_b s\right) \right] D_{0q} \left\{ \exp\left[-\vartheta \eta_a \left(a - r - \frac{s}{2}\right)\right] - \exp\left[-\vartheta \eta_a \left(a + r - \frac{s}{2}\right)\right] \right\}.$$

Since exponents in this expression are small enough, the exponents can be expanded in the small parameter by the rule: $\exp(-\kappa) \approx 1 - \kappa$. With this in mind, we obtain:

$$F_q = 2p_q D_{0q} \vartheta^2 s^3 \eta_b \eta_a r$$

In this expression, we will take into account that the charge density of the large ball is given by the formula: $\rho_q = e\eta_a$, and will use (10):

$$F_q = \frac{2|q|p_q D_{0q} \vartheta^2 \rho_q r}{e^2}, \qquad \mathbf{F}_q = \frac{2q p_q D_{0q} \vartheta^2 \rho_q \mathbf{r}}{e^2}$$

The force \mathbf{F}_q acts on the small ball with the negative charge q in Figure 5 so that the force is directed toward the center of the large ball and oppositely to the radius vector \mathbf{r} from the center of the large ball to the small ball. By definition, the electric field strength is the ratio of the force, acting on the test body, to the charge of the test body. Then the vector of the electric field strength inside the large ball will be:

$$\mathbf{E} = \frac{\mathbf{F}_q}{q} = \frac{2p_q D_{0q} \,\vartheta^2 \,\rho_q \,\mathbf{r}}{e^2} \,. \tag{14}$$

In electrostatics, the vector of the electric field strength inside a uniform charged ball is determined by the formula:

$$\mathbf{E} = \frac{\rho_q \,\mathbf{r}}{3\varepsilon_0} \,. \tag{15}$$

From comparison of (14) and (15) we find the expression of the vacuum permittivity in terms of the parameters of charged particles fluxes in the cubic distribution approximation:

$$\varepsilon_0 = \frac{e^2}{6p_q D_{0q} \vartheta^2} \,. \tag{16}$$

The difference between the used cubic (9) and spherical (1) distributions leads to the fact that the formulas for vacuum permittivity (16) and (6) differ by a numerical factor.

If the small ball in Figure 5 has not a negative charge but a positive charge q, then its interaction with the charge of the large ball should be considered in view of Figure 3 for the interaction of two positive charges. It means that it is necessary to introduce an additional coefficient ξ in order to take into account the effect of increasing of the flux of charged particles.

As a result, the fluence rates D_1 and D_2 and the force (12) from the flux of charged particles falling from above on the small cube, by which we replaced the small ball, will change and be equal to:

$$D_1 = D_{0q} \exp\left[\left(\xi - \vartheta\right)\eta_a \left(a - r - \frac{s}{2}\right)\right], \qquad D_2 = D_1 \exp\left(-\vartheta \eta_b s\right),$$

$$F_{1} = p_{q}s^{2}(D_{1} - D_{2}) = p_{q}s^{2} \left[1 - \exp(-\vartheta\eta_{b}s)\right] D_{0q} \exp\left[(\xi - \vartheta)\eta_{a}\left(a - r - \frac{s}{2}\right)\right].$$
 (17)

Similarly, at the lower side of the large ball for the fluence rate and the force, instead of (13), we have:

$$D_3 = D_{0q} \exp\left[\left(\zeta - \vartheta\right)\eta_a \left(a + r - \frac{s}{2}\right)\right], \qquad D_4 = D_3 \exp\left(-\vartheta \eta_b s\right),$$

$$F_{2} = p_{q}s^{2}(D_{3} - D_{4}) = p_{q}s^{2}\left[1 - \exp(-\vartheta\eta_{b}s)\right]D_{0q}\exp\left[(\xi - \vartheta)\eta_{a}\left(a + r - \frac{s}{2}\right)\right].$$
 (18)

The total force equals the difference between the forces (18) and (17):

$$F_{q} = F_{2} - F_{1} =$$

$$= p_{q} s^{2} \left[1 - \exp\left(-\vartheta \eta_{b} s\right) \right] D_{0q} \left\{ \exp\left[(\xi - \vartheta) \eta_{a} \left(a + r - \frac{s}{2} \right) \right] - \exp\left[(\xi - \vartheta) \eta_{a} \left(a - r - \frac{s}{2} \right) \right] \right\}.$$

Expanding the exponents by the rule: $\exp(\kappa) \approx 1 + \kappa$, we find:

$$F_q = 2p_q D_{0q} \mathcal{P}(\xi - \mathcal{P})s^3 \eta_b \eta_a r.$$

Let us assume that the charge density of the large ball is given by the formula: $\rho_q = e\eta_a$, and for the coefficient ξ the relation holds: $\xi = 2\vartheta$, which was found in the previous section. Then, with regard to (10), we obtain:

$$F_{q} = 2p_{q}D_{0q}\mathcal{G}^{2}s^{3}\eta_{b}\eta_{a}r = \frac{2p_{q}D_{0q}\mathcal{G}^{2}q\rho_{q}r}{e^{2}}.$$

The force F_q is directed radially from the center of the large ball, and the expression for this force after dividing by the charge q leads to the electric field strength (14).

5. The parameters of the fluxes of charged particle of the vacuum field

We will estimate the energy density for cubic distribution of charged particles fluxes of vacuum field in space. Suppose there is a cube with an edge s, into which charged particles

fly from six sides perpendicularly to the faces of the cube. The speed of charged particles is assumed to be equal to the speed of light, so that in the time $\frac{s}{c}$ the cube will be completely filled. In view of distribution (9) the number of charged particles in the cube will be: $N_c = \frac{6s^3 D_0}{c}$. If the energy of one charged particle is $E_q = p_q c$, then with the help of (16) for the energy density of charged particles of vacuum field we find:

$$\varepsilon_{cq} = \frac{E_q N_c}{s^3} = 6p_q D_{0q} = \frac{e^2}{\varepsilon_0 \mathcal{G}^2}.$$
(19)

Now we will use the spherical distribution (1) to estimate the energy density of charged particles of vacuum field. An empty sphere with radius R can be filled with charged particles in the time $\frac{2R}{c}$, if the graviton fluxes are directed radially and correspond to the full solid angle 4π . The number of charged particles inside the sphere will equal $N_s = \frac{8\pi ARB_{0q}}{c}$. Multiplying this number by the energy of one charged particle and dividing by the sphere's volume we can find the energy density. In view of (6) and the condition $A = 4\pi R^2$, we obtain:

$$\varepsilon_{sq} = \frac{3E_q N_s}{4\pi R^3} = 24\pi p_q B_{0q} = \frac{3e^2}{2\varepsilon_0 g^2}.$$
 (20)

The energy density (20) with spherical distribution is 3/2 times greater than with cubic distribution (19), which emphasizes that our estimates are approximate due to the use of two idealized distributions.

Earlier in [4] we have applied the concept of the graviton field to calculate the Newton's gravitational force between two bodies and the gravitational constant. This allowed us to estimate the energy density of the graviton field for cubic distribution and the rate of the energy flux of the graviton field in one direction:

$$\varepsilon_c = \frac{E_g N_c}{s^3} = 6p_g D_0 = \frac{4\pi G M_n^2}{\sigma^2} = 7.4 \cdot 10^{35} \text{ J/m}^3, \qquad (21)$$

$$P_f = E_g D_0 = \frac{2\pi c G M_n^2}{3\sigma^2} = \frac{c\varepsilon_c}{6} = 3.7 \cdot 10^{43} \text{ W/m}^2,$$

here $E_g = p_g c$ is the average energy of one graviton, p_g is the average momentum of one graviton, D_0 is the number of gravitons falling per unit time on unit area from one of the six spatial directions in cubic distribution, $N_c = \frac{6s^3D_0}{c}$, G is the gravitational constant, M_n is the mass of one nucleon of the matter, $\sigma = 5.6 \cdot 10^{-50} \text{ m}^2$ is the cross section of interaction of gravitons and the matter.

The energy density ε_c in (21) is associated with the gravitational constant *G* and with gravitation at the level of nucleons. Similarly, the energy density of the charged particles of the vacuum field ε_{cq} in (19) is associated with the electromagnetic action of the field on each elementary charge *e* of the matter.

Further we will need the similarity coefficients, with the help of which in the theory of infinite nesting of matter [1], [5], [6] we will calculate the physical quantities inherent in each particular level of matter. As the typical parameters of a neutron star we will take the mass equal to 1.35 Solar mass or $M_s = 2.7 \cdot 10^{30}$ kg and the stellar radius equal to $R_s = 12$ km.

Dividing the mass of the neutron star by the proton mass M_p , we find the coefficient of

similarity in mass: $\Phi = \frac{M_s}{M_p} = 1.62 \cdot 10^{57}$. Similarly, we calculate the coefficient of similarity

in size as the ratio of the stellar radius to the proton radius: $P = \frac{R_s}{R_p} = 1.4 \cdot 10^{19}$, here the

quantity $R_p = 8.73 \cdot 10^{-16}$ m in the self-consistent model of the proton [7] was used.

The coefficient of similarity in speed equals the ratio of the characteristic speeds of the matter inside the star and the proton, respectively. For the star the characteristic speed C_s is calculated from the energy equality from the standpoint of the general principle of equivalence of mass and energy, generalized with respect to the absolute value of the total energy to any space objects:

$$M_s C_s^2 = \frac{kGM_s^2}{2R_s}$$
, $C_s = \sqrt{\frac{kGM_s}{2R_s}} = 6.8 \cdot 10^7 \text{ m/s}.$

Similarly, we find for the proton the equality of the characteristic speed of its matter and the speed of light:

$$C_p = \sqrt{\frac{k \Gamma M_p}{2R_p}} = c = 2.99 \cdot 10^8 \text{ m/s},$$

while $\Gamma = \frac{e^2}{4\pi\varepsilon_0 M_p M_e} = 1.514 \cdot 10^{29} \,\mathrm{m}^3 \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^{-2}$ is the strong gravitational constant,

calculated from the equality of electric and gravitational forces in the hydrogen atom, ε_0 is

the vacuum permittivity, M_e is the electron mass and according to [7] for the proton

k = 0.62. Hence, the coefficient of similarity in speed is equal to: $S = \frac{C_s}{C_p} = 0.23$.

As it was shown in [1], the ratio of the absolute value of strong gravitation energy density to the electromagnetic energy density of the proton is equal to the ratio of the proton mass to the electron mass $\frac{M_p}{M_a}$. Indeed, for the energy of the fields and their ratios, in view of the

definition of the strong gravitational constant Γ , we have: $|E_g| = \frac{k\Gamma M_p^2}{R}$, $E_e = \frac{ke^2}{4\pi\varepsilon_0 R}$,

 $\frac{\left|E_{g}\right|}{E_{e}} = \frac{4\pi\varepsilon_{0}\Gamma M_{p}^{2}}{e^{2}} = \frac{M_{p}}{M_{e}}.$ We believe that the same ratio exists for the energy densities of graviton field and charged particles of the vacuum field, which allows us to estimate the energy density of charged particles of the vacuum field:

$$\varepsilon_{cq} = \varepsilon_c \frac{M_e}{M_p} = 4 \cdot 10^{32} \text{ J/m}^3.$$
(22)

Let us substitute (22) into (19), using the value of ε_c from (21), and take into account the proximity of the proton mass and the average mass of a nucleon $M_p \approx M_n$, as well as the definition of the strong gravitational constant in the form $\Gamma = \frac{e^2}{4\pi\varepsilon_0 M_p M_e}$. This gives an estimate of the cross section of interaction of the charged particles of the vacuum field with the charged matter:

$$\mathcal{G} = \frac{e}{\sqrt{\varepsilon_0 \varepsilon_{cq}}} = e \sqrt{\frac{M_p}{\varepsilon_0 \varepsilon_c M_e}} = e \sigma \sqrt{\frac{1}{4\pi \varepsilon_0 M_p M_e G}} = \sigma \sqrt{\frac{\Gamma}{G}} = 2.67 \cdot 10^{-30} \,\mathrm{m}^2 \,. \tag{23}$$

This cross section has a value that almost exactly coincides with the geometrical cross section of a nucleon and significantly exceeds the cross section $\sigma = 5.6 \cdot 10^{-50} \text{ m}^2$ of interaction of gravitons with the matter. In order to find the significant difference between ϑ and σ , we will express ε_c from (22), use ε_{cq} from (19) and take into account the definition of Γ :

$$\varepsilon_c = \varepsilon_{cq} \frac{M_p}{M_e} = \frac{e^2 M_p}{\varepsilon_0 \vartheta^2 M_e} = \frac{4\pi \Gamma M_p^2}{\vartheta^2}.$$
(24)

From comparison of (24) and (21), provided that $M_p \approx M_n$, it follows that if in (21) we pass from the cross section σ to the cross section ϑ , then at the same time it is necessary to substitute the gravitational constant G with the strong gravitational constant Γ . In (24) the energy density ε_c of the graviton field at the level of nucleons is fully expressed in terms of the parameters of the nucleon level of matter. Similarly, in (19) the energy density ε_{cq} of the charged particles of the vacuum field is expressed in terms of the parameters of the nucleon level of matter. In this case, both in (19) and (24) the same cross section ϑ of interaction of the vacuum field particles with the matter consisting of nucleons is used.

By analogy with (24) for the graviton field at the stellar level we can write:

$$\varepsilon_s = \frac{4\pi G M_s^2}{g_s^2}.$$

If in this expression we shall consider the following relations in accordance with the dimensional analysis, coefficients of similarity and (24):

$$G = \frac{PS^2}{\Phi} \Gamma$$
, $M_s = \Phi M_p$, $\vartheta_s = P^2 \vartheta$,

then we obtain the relation $\varepsilon_s = \frac{\Phi S^2}{P^3} \varepsilon_c = 2.3 \cdot 10^{34} \text{ J/m}^3$, in which the energy density of graviton field at the stellar level ε_s , needed to keep the matter in neutron stars, linked to the energy density ε_c . Since the energy density ε_c is required for the integrity of the nucleons in the field of strong gravitation, then $\varepsilon_c > \varepsilon_s$.

In view of (16), (19), (22) and the relation $E_q = p_q c$, for the rate of the energy flux of charged particles of the vacuum field in one direction we find:

$$P_{fq} = E_q D_{0q} = \frac{c e^2}{6\varepsilon_0 g^2} = \frac{c\varepsilon_{cq}}{6} = 2 \cdot 10^{40} \text{ W/m}^2.$$
(25)

Due to the fact that the above-mentioned energy density ε_{cq} of charged particles of the vacuum field is less than the energy density ε_c of graviton field in (21), the rate of the energy flux of charged particles of the vacuum field P_{fq} is less than the rate the energy flux of the graviton field P_f .

6. The estimates of forces and energies

In [1] and [5] the assumption is made that some neutron stars – magnetars can have a positive electric charge of up to $Q_s = eS\sqrt{\Phi P} = 5.5 \cdot 10^{18}$ C, where *e* is the elementary electric charge and the similarity coefficients are used in accordance with the dimensional analysis.

The proton electric energy on the surface of the charged magnetar will reach $E_{pe} = \frac{eQ_s}{4\pi\varepsilon_0 R_s} = 6.6 \cdot 10^5 \,\text{J} \quad \text{or} \quad 4 \cdot 10^{24} \,\text{eV}.$ The corresponding electric force will be equal to

 $F_{pe} = \frac{eQ_s}{4\pi\varepsilon_0 R_s^2} = 55$ N. It is assumed that it is precisely the electrical energy in the magnetar

field that is the energy source of high energy cosmic rays.

For the absolute value of the gravitational energy of the proton on the surface of the magnetar similarly we have: $|E_{pg}| = \frac{GM_pM_s}{R_c} = 2.5 \cdot 10^{-11}$ J. This energy and the gravitational

force, associated with it, are clearly not enough to keep the proton, on which the repulsive force is acting from the entire charge of the magnetar. However the magnetar looks like a huge atomic nucleus consisting of a number of closely-spaced nucleons. Between nucleons there is strong interaction, which holds them together. In the gravitational model of strong interaction [5] the idea of strong gravitation is used to describe the strong interaction. The nucleons in the atomic nuclei are attracted to each other by strong gravitation and repel from each other by means of the torsion field, which arises from the rapid rotation of the nucleons. According to the Lorentz-invariant theory of gravitation [1-2], the torsion field arises similarly to the magnetic field in electromagnetism, and in the general theory of relativity it corresponds to the gravitomagnetic field. The balance of attractive and repulsive forces, arising from strong gravitation, can be responsible for the integrity of the atomic nuclei, as well as for the integrity of the charged neutron star. We did the estimates of forces and energies in the atomic nuclei in [5] and [8]. For example, the nickel nucleus ${}^{62}_{28}$ Ni consists of A = 62 nucleons, among which there are 28 protons and 34 neutrons. The mass of this nucleus is $M_N = 1.028 \cdot 10^{-25}$ kg, and the radius is obtained from experiments on the scattering of electrons by the formula: $R_N = R_0 A^{1/3} = 4.9 \cdot 10^{-15}$ m, where $R_0 \approx 1.23 \cdot 10^{-15}$ m. Based on these data we will estimate the force, acting from the nucleus on the proton located on the nucleus surface, with the help of strong gravitation: $F_{pN} = \frac{\Gamma M_p M_N}{R_{..}^2} = 1 \cdot 10^6$ N.

The surface of the magnetar as a neutron star apparently consists of the nuclei of such elements as iron, nickel and heavier nuclei, since their binding energy per nucleon is maximum. If the proton was near one of these nuclei on the magnetar surface, the force F_{pN} would keep the proton, acting against the force of electrical repulsion $F_{pe} = 55$ N from the magnetar charge. But the concentration of nuclei on the stellar surface is such that the proton on the average will be located somewhere between the nuclei at a distance r from them.

To keep the proton the condition $F_p = \frac{\Gamma M_p M_N}{r^2} > F_{pe}$ must hold, which implies that $r < 7 \cdot 10^{-13}$ m. For a cube with the edge 2r, at the corners of which there are 8 nuclei $\frac{62}{28}$ Ni, and the proton is in the center of the cube, the matter density is equal to $\rho = \frac{M_N}{r^3} = 3 \cdot 10^{11} \text{ kg/m}^3$. The matter density on the magnetar surface must exceed this value, so that the condition of stability with respect to electric forces is satisfied. On the other hand, the estimates in [9] of the matter density in the crust of the neutron star imply that at a density of $2.7 \cdot 10^{11} \text{ kg/m}^3$ and more the nuclei $\frac{62}{28}$ Ni begin to decay. Consequently, heavier nuclei must prevail in the magnetar crust, in particular, a typical nucleus according to [9] is $\frac{105}{35}$ Br. From these calculations it follows that the magnetar charge is almost the maximum charge

that the star can have without loss of its integrity. And the main contribution into the stability of a star is made by not ordinary but strong gravitation, acting at the level of atomic nuclei.

With the help of the similarity coefficients we can calculate the mass, radius and charge of the praon – the particle, which relates to the proton, as the proton relates to the magnetar:

$$m_{pr} = \frac{M_p}{\Phi} = 1 \cdot 10^{-84} \text{ kg}, \quad r_{pr} = \frac{R_p}{P} = 6.2 \cdot 10^{-35} \text{ m}, \quad q_{pr} = \frac{e}{S\sqrt{\Phi P}} = 4.6 \cdot 10^{-57} \text{ C}.$$
 If the praon is

located at the surface of the proton, its electrical energy and gravitational energy in the strong gravitational field will be equal: $E_{pre} = \frac{q_{pr}e}{4\pi\varepsilon_0 R_p} = 7.6 \cdot 10^{-51} \text{ J}, \ \left|E_{pg}\right| = \frac{\Gamma m_{pr}M_p}{R_p} = 2.9 \cdot 10^{-67} \text{ J}.$

The ratio of these energies is the same as the ratio of the electric energy of the proton at the surface of the magnetar to the gravitational energy of this proton in the gravitational field of the magnetar. In the substantial model of the proton and neutron, presented in [5], it is assumed that the nucleons consist of neutral and charged praons, just as neutron stars consist of nucleons. In addition, by analogy with the composition of cosmic rays, consisting mainly of relativistic protons, we can assume that the charged component of the vacuum field can consist of praons accelerated by positively charged atomic nuclei up to high energies.

At the present time cosmic rays are registered with energies up to $E_r = 6 \cdot 10^{19}$ eV or 9.6 J per nucleon. Assuming that this is the energy of the accelerated proton, we will divide it by the coefficient of similarity in energy and will find the corresponding energy of the praon: $E_{pr} = \frac{E_r}{\Phi S^2} = 1 \cdot 10^{-55}$ J. Equating this energy to the energy E_q of a charged particle of the vacuum field, we can estimate the concentration of these charged particles as the concentration of relativistically moving praons. In view of (19) and (22) we obtain:

$$n_{pr} = \frac{N_c}{s^3} = \frac{\varepsilon_{cq}}{E_q} = \frac{\varepsilon_{cq}}{E_{pr}} = 4 \cdot 10^{87} \,\mathrm{m}^{-3}$$

Multiplying this concentration of charged particles by the charge of one praon q_{pr} and the speed of light, we can estimate the density of the current in the vacuum in one direction, which arises from the flux of positively charged praons in one direction at cubic distribution:

$$j_{+} = n_{pr} q_{pr} c = 5.5 \cdot 10^{39} \text{ A/m}^2$$

Beside the current density j_+ , we should expect another similar current density j_- in the same direction, which arises from the flux of negatively charged praons. This should ensure a certain degree of vacuum electroneutrality and existence of electrical forces of repulsion and attraction.

Now we will consider the question of neutron star's matter permeability for gravitons and charged particles of the vacuum field, respectively. The fluence rates from a unit solid angle similarly to (2) have the form:

$$B = B_0 \exp(-\sigma n x), \qquad B_q = B_{0q} \exp(-\vartheta \eta x).$$

If the neutron star has a radius of 12 km and a mass of 1.35 solar masses, then the average concentration of nucleons will equal $n = \frac{3M_s}{4\pi M_n R_s^3} = 2.2 \cdot 10^{44} \text{ m}^{-3}$. The average concentration of the positive charge in the magnetar is $\eta = \frac{3Q_s}{4\pi e R_s^3} = 4.7 \cdot 10^{24} \text{ m}^{-3}$. Assuming that $x = 2R_s = 24 \text{ km}$, for the exponents in view of (21) and (23) we find: $\sigma nx = 0.3$, $\vartheta \eta x = 0.007$. It follows that if we put three neutron stars in the way of the flux of gravitons, the flux will reduce approximately by a factor of e_n , where $e_n = 2.71828$... is the base of the

natural logarithm. But for the flux of charged particles of the vacuum field in order to reduce it noticeably we need to put in a line about 140 magnetars.

This difference in fluxes allows us to explain the saturation effect of the specific binding energy, when the nuclear binding energy per nucleon, depending on the number of nucleons in nuclei, first increases, reaching a maximum of 8.79 MeV per nucleon for the nucleus ${}^{62}_{28}$ Ni , and then begins to decrease. For light nuclei the increase in the specific energy agrees well with the increase of the specific gravitational energy of the nucleus in the strong gravitational field, when the energy increases in direct proportion to the square of mass and in inverse proportion to the radius of the nucleus. The saturation effect comes into play in the range of 17 to 23 nucleons, forming the nucleus. Besides, adding a new nucleon to the nucleus increases the energy not proportionally to the square of mass, but to a lesser extent. This is due to the fact that gravitons of strong gravitation cannot permeate the nucleus with a lot of nucleons, as is evident from the exponent. Each new nucleon is simply pressed to the nucleus from the outside by the strong gravitation, until for the large nuclei this force reaches the maximum, conditioned by the pressure of the graviton flux. However, the charged particles of the vacuum field in these conditions have almost 50 times larger path length, and therefore the positive electrical energy of the nucleus' protons further decreases the negative gravitational energy of the nucleus, making the main contribution into the observed decrease in the specific binding energy of massive nuclei.

Earlier in [3] we estimated the maximum force between two stellar objects:

$$F_{\rm m} = \frac{c^4}{16k^2G} = 2 \cdot 10^{43} \,\,{\rm N},$$

where k = 0.6 for the case of uniform density of each object, and it is assumed that the graviton fluxes are fully retained by these objects, which are located close to each other.

A similar expression for the maximum force at the nucleon level of matter, after replacing the gravitational constant by the strong gravitational constant, in view of the coefficient of similarity in speed S = 0.23 has the form:

$$f_{\rm m} = \frac{c^4}{16k^2 S^4 \Gamma} = 5 \cdot 10^6 \,\,{\rm N}$$

We should note that the corresponding ratio of the gravitational energy and the force between two protons to their electrostatic energy and force is equal to the ratio of the proton mass to the electron mass. Indeed, for the forces and their ratios in view of the definition of

the strong gravitational constant
$$\Gamma = \frac{e^2}{4\pi\varepsilon_0 M_p M_e}$$
, we have: $F_g = \frac{\Gamma M_p^2}{R^2}$, $F_e = \frac{e^2}{4\pi\varepsilon_0 R^2}$,

 $\frac{F_g}{F_e} = \frac{4\pi\varepsilon_0 \Gamma M_p^2}{e^2} = \frac{M_p}{M_e}.$ We can explain this by the fact that in the expression for B_q the exponent for the flux of charged particles of the vacuum field in the magnetar and hence in the proton is less than the corresponding exponent for the flux of gravitons in the expression for B. The gravitons are retained in the proton matter more than the charged particles of the vacuum field, and therefore the gravitational force is greater than the electric force.

After passing from dense and charged objects such as magnetars and protons to the bodies surrounding us the situation with the ratio of forces is changing. The gravitational force decreases rapidly with decreasing of the mass of bodies, and we can hardly influence it. However, by changing the charges of bodies we can change their electrical interaction, so that the electric force can be many times greater than the gravitational force between these bodies. This can be seen from the ratio of the electric and gravitational forces for two identical bodies with the mass m and charge q, which is proportional to the squared charge:

$$\frac{F_e}{F_g} = \frac{q^2}{4\pi\varepsilon_0 Gm^2} \,.$$

Let us take for example two iron balls with the radius r = 5 cm each. With the density of iron 7874 kg/m³ it gives the mass of each ball of approximately 4.1 kg. For the equality of the gravitational and electrical forces it is enough to charge the balls up to $q = m\sqrt{4\pi\varepsilon_0 G} = 3.5 \cdot 10^{-10}$ C, so that the potential of each ball reaches the value $\varphi = \frac{q}{4\pi\varepsilon_0 r} = 63$ V. Let us estimate the electrical energy of the praon, flying near the ball,

taking into account that above we estimated the charge of the praon with the value $q_{pr} = \frac{e}{S\sqrt{\Phi P}} = 4.6 \cdot 10^{-57} \text{ C}: E_{pre} = q_{pr} \varphi = 3 \cdot 10^{-55} \text{ J}.$ On the other hand, the energy of a praon,

regarded as a relativistic particle similar by its properties to cosmic rays, has been found above in the form: $E_{pr} = \frac{E_r}{\phi S^2} = 1 \cdot 10^{-55}$ J. Comparison of these two energies allows us to make the following conclusions. Firstly, even weakly charged bodies, which interact at the level of low gravitational force, can influence the motion of praons near them and deflect them aside. This substantiates the pattern of motion of the charged particles of the vacuum field near the charged bodies in Figures 1-3 and our calculations of the electric force. Secondly, if we decrease the charges and increase the sizes of bodies, there can be deviations from the Coulomb law. However, these deviations should be distinguished from the gravitational force, which in this case becomes greater than the electric force.

The last conclusion can be specified as follows. In order to find the deviations from the Coulomb law, it is desirable that the condition of small potentials is satisfied

$$\varphi = \frac{q}{4\pi\varepsilon_0 r} < \frac{E_{pr}}{q_{pr}} = 22$$
 V. To reduce the dependence on the gravitational force, there are the

following conditions $F_e > F_g$ or $q > m\sqrt{4\pi\varepsilon_0 G}$. Hence for the corresponding electrical

potential of one ball, we have:
$$\frac{m\sqrt{G}}{r\sqrt{4\pi\varepsilon_0}} < \phi < 22 \text{ V}$$
 or $\frac{m}{r} < 28.4 \text{ kg/m}$. For the iron balls it

gives r < 2.9 cm, m < 0.8 kg. Another complication in the experiments for finding deviations from the Coulomb law occurs due to the fact that in conductive bodies the uncompensated charges are located in the thin layer on the bodies' surface, with a thickness of the order of 1 or 2 atomic layers. Free electrons easily go out of the equilibrium position in the external electric field, either repelling or being attracted to the source of the external field, thereby changing their concentration on the body. Due to this, in two interacting charged metal balls additional electrical forces appear, which are usually calculated by the method of images.

7. Interaction of the body's charge with the vacuum field

The Coulomb law, due to the presence of charged particles in the vacuum field, can be explained with the help of Le Sage's model. However, not only the fluxes of charged particles influence the interaction of charged bodies, but the charges of bodies themselves influence the fluxes of charged particles around the bodies. One example of this influence is deflection of the charged particles from their trajectories, as it was described in the previous section. In addition, each charged body achieves a certain balance of energy and momentum during interaction with the vacuum field.

Let us consider the energy density of the charged particles of the vacuum field inside the charged body and near it. Suppose there is a body in the form of a cube with an edge s. The number of charged particles D per unit time through a unit area during particles' motion in the matter decreases according to formula (11). During time $\frac{s}{c}$ six fluxes of charged particles from each side will pass inside the cube through the faces with the area s^2 and will change up to the value:

$$D_1 = D_{0q} \exp(-\vartheta \eta s), \qquad N = \frac{6s^3 D_{0q}}{c} \exp(-\vartheta \eta s),$$

where N is the number of charged particles that passed through the cube.

If charged particles flew through the same empty volume, the number of charged particles coming out would be $N_0 = \frac{6s^3 D_{0q}}{c}$. Consequently, the number of charged particles, which

interacted with the matter of charged body during time $\frac{s}{c}$, equals:

$$\Delta N = N_0 - N = \frac{6s^3 D_{0q}}{c} \left[1 - \exp(-\vartheta \eta s) \right] \approx \frac{6\vartheta \eta s^4 D_{0q}}{c}$$

As it was shown in [4], almost all the energy of the graviton field, which interacts with the matter, is re-emitted back to the graviton field, without heating the bodies significantly. This also applies to the fluxes of charged particles the vacuum field, that transfer their momentum to the matter with return of the energy back to the vacuum field.

Let us estimate in view of (19) the energy density of those charged particles that interact with the bodies' matter:

$$\varepsilon_m = \frac{\Delta N E_q}{s^3} \approx \varepsilon_{cq} \, \vartheta \eta \, s \,. \tag{26}$$

From (26) we will calculate the luminosity of charged particles of a body in the form of a cube, multiplying \mathcal{E}_m by the volume s^3 and dividing by the time $\frac{s}{c}$. Expressing the charge concentration in terms of the charge, in view of (19) we have:

$$\eta = \frac{Q}{es^3}, \qquad P_q = \varepsilon_m s^2 c = \varepsilon_{cq} \,\vartheta\eta \, s^3 c = \varepsilon_{cq} \,\vartheta c \frac{Q}{e} = \frac{ceQ}{\varepsilon_0 \,\vartheta}. \tag{27}$$

From (27) it follows that the luminosity P_q of the charged particles, understood as the luminosity of those charged particles fluxes that interacted with the charged matter of body and gave their momentum to it, is proportional to body charge Q. This means that the body charge can be expressed in terms of the parameters of the charged particles fluxes interacting with the body.

In (27) there is a product $n s^3$ equal to the number of uncompensated elementary charges in the body under consideration. Then the charged particles luminosity per one elementary charge, in view of (19), (22-23) will equal:

$$P_{1} = \frac{P_{q}}{\eta s^{3}} = \varepsilon_{cq} \vartheta c = 6p_{q} D_{0q} \vartheta c = 6E_{q} D_{0q} \vartheta = 3.2 \cdot 10^{11} \,\mathrm{W}.$$
 (28)

The ratio of the luminosity P_1 to the average energy of a charged particle $E_q = p_q c$ gives the number of charged particles that interact with one uncompensated elementary charge of matter per unit time and gave their momentum to it. According to (28), this number of charged particles is equal to the product $6D_{0q} \vartheta$, while the cross section ϑ characterizes the effective area of elementary charge's interaction with charged particles, and the coefficient 6 is associated with the six sides of cubic distribution of charged particles fluxes D_{0q} in (11). Expression (27) can be given a different meaning, if we assume that the area of the cube face is connected with the cross section \mathcal{G} by the following relation: $s^2 = k_1 N \mathcal{G}$, where k_1 is some numerical coefficient, N is the number of uncompensated elementary charges in the cube. Then under the condition eN = Q (27) can be rewritten as follows:

$$P_q = \frac{ck_1Q^2}{\varepsilon_0 s^2}.$$

This relation shows that the emission rate is proportional with accuracy to a coefficient k_1 to the electric energy of the charged body, derived from the body in the time $\frac{s}{c}$ of passing the body characteristic size by the charged particles.

We note one more aspect concerning the interaction between the electromagnetic and gravitational fields. The concept of the general field [10] shows that the vector fields, including the electromagnetic and gravitational fields, are the components of one general field. And in case if the theorem of equipartition of the energy is satisfied, the equations of particular fields no longer depend on each other and are similar in form to the Maxwell equations. If the fields interact with each other, then in the Hamiltonian it is manifested in the terms with the field energy, where the cross-terms with the products of different field strengths appear. This is possible, for example, in non-stationary processes in the systems that have not reached equilibrium. From the viewpoint of the vacuum field, it means that in stationary conditions the gravitons and charged particles of the vacuum field interact with the matter relatively independently, creating gravitational and electromagnetic forces. If there is no equilibrium in the system, then the kinetic energy of matter and the energies of some fields are transformed into the energy of other fields, and the exchange of energies between

gravitons and charged particles in the vacuum field is possible as well. This leads to the crossterms in the system's energy.

8. Photons and praons

In this section we will try to specify which particles can be responsible for electromagnetic phenomena. The charged particles of the vacuum field not only lead to the electric forces in the Coulomb law, but should be part of the photons, i.e. the electromagnetic quanta emitted by atoms. Let us consider, for example, a photon with the wavelength $\lambda = 1.21567 \cdot 10^{-7}$ m and the angular frequency $\omega = \frac{2\pi c}{\lambda} = 1.54946 \cdot 10^{16} \text{ s}^{-1}$, arising in the hydrogen atom in the transition of an electron from the second to the first level in the Lyman series. The probability of this transition equals $A_2 = 4.699 \cdot 10^8 \text{ s}^{-1}$ [11], which gives the average lifetime of an electron at the second level $\tau_2 = \frac{1}{A_2} = 2.1 \cdot 10^{-9} \text{ s}$, as a measure of duration of photon emission during the transition. In quantum mechanics [12] there is a formula for the oscillator's oscillations decay time in e_n times, where $e_n = 2.71828...$ is the base of the natural logarithm, with the help of which we obtain the following estimate:

$$\tau_o = \frac{12\pi cM_e}{\mu_0 \omega^2 e^2} = 1.3 \cdot 10^{-9} \, \text{s.}$$

where μ_0 is the vacuum permeability.

The duration of photon emission can be calculated directly within the Bohr model of a hydrogen atom. In this model, the electric force between a proton and an electron acts as a centripetal force in the electron's rotation around the nucleus in the form of a proton. In this

rotation, the electron must emit an electromagnetic wave, since it is constantly accelerated towards the nucleus. The formula for the charge emission rate during its rotation is well known, which allows us to relate the electron velocity and the effective force acting on the electron from emission. Moment of this force decreases the angular momentum of the electron, leading to a decrease in the radius of rotation. Hence we can derive the dependence of the radius on the time [5]. From this dependence we find the duration of photon emission as the time of transition of an electron from the second to the first level of energy. Given that the average radius of the electron rotation on the second level equals $r_2 = 4a_B$, and the average radius of the electron rotation on the first level is the Bohr radius $r_1 = a_B$, we have the following:

$$\tau = \frac{4\pi^2 \varepsilon_0^2 c^3 M_e^2}{e^4} \left(r_2^3 - r_1^3 \right) = 1.2 \cdot 10^{-9} \,\mathrm{s.}$$
⁽²⁹⁾

For the instantaneous power of electromagnetic emission we obtain the formula:

$$P_r = \frac{e^6}{96\pi^3 \varepsilon_0^3 c^3 M_e^2 r_e^4}$$

This implies a strong dependence of the emission rate on the current radius r_e of the electron rotation, which is inversely proportional to the fourth power of this radius. It turns out that the main photon energy is emitted when the electron approaches the lower energy level.

Knowing the emission duration we can find the length of the photon $\ell = c \tau$. To calculate the volume of the photon we also need its midsection. In the first approximation, we assume that the mean radius of the photon equals $r = 4a_B$, which is equal to r_2 . We note that in the substantial model of electron [5], it is considered as a thin disk that has on the main energy level the inner radius $0.5a_B$ and the outer edge $1.5a_B$, and the Bohr radius $r_1 = a_B$ is obtained as a certain characteristic radius of the disk and the average radius of the electron rotation. On the second level, the outer edge of the electron disk is greater than the average radius of the electron will electron rotation $r_2 = 4a_B$ on this level. With this in mind, the volume of the photon will equal: $V = \pi r^2 \ell = 16\pi a_B^2 c \tau$.

Further on we will use a simplified model of photon from [1], [13], according to which the photon consists of charged particles, the rotation of which around the photon's axis creates the angular momentum of the photon. In addition, inside the photon as well as in the electromagnetic wave there must be mutually-perpendicular periodically varying electric and magnetic fields. Electromagnetic energy of the photon consists of the equal electric and magnetic components, and for the total energy density we can write: $\varepsilon_e + \varepsilon_m = \frac{\varepsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2 = \frac{\varepsilon_0}{2}(E^2 + c^2B^2) = \varepsilon_0E^2$, since in the wave E = cB. The electric field strength E inside the photon will be characterized by the amplitude E_0 . The field inside the photon oscillates, varying from zero to the peak value, so for the average density of the electromagnetic energy of the photon, we assume that $\varepsilon_{em} = \frac{\varepsilon_0}{2} E_0^2$. We also assume that the photon energy is equally divided between the mechanical energy of the charged particles and the electromagnetic energy. The photon energy $W = \hbar \omega$ is proportional to the Planck constant \hbar and the angular frequency ω . Dividing the photon energy by the photon volume, we obtain the energy density, which can be equated to the doubled density of electromagnetic energy inside the photon:

$$\frac{W}{V} = 2\varepsilon_{em}, \qquad \qquad \frac{\hbar\omega}{\pi r^2 c \tau} = \varepsilon_0 E_0^2. \tag{30}$$

Substituting in (30) the photon angular frequency $\omega = 1.54946 \cdot 10^{16} \text{ s}^{-1}$, the duration of the photon emission τ from (29) and the photon radius $r = 4a_B$, we estimate the amplitude of the electric field strength inside the photon: $E_0 = 2.7 \cdot 10^6 \text{ V/m}$. For comparison, the proton creates at the Bohr radius the electric field strength $E_B = \frac{e}{4\pi\varepsilon_0 a_B^2} = 5.1 \cdot 10^{11} \text{ V/m}$. It can be noted that

 E_0 is close enough to the value $\frac{a_B \alpha^2}{r} E_B$, where α is the fine structure constant. Besides the

multiplier $\frac{a_B \alpha^2}{r}$ appears in the expression for the electric field strength in the remote wave zone, that is the zone where the photon is generated: $E \approx \frac{\alpha^2 e}{16\pi\varepsilon_0 a_B r'} = \frac{\alpha^2 e}{4\pi\varepsilon_0 r r'}$, here r' is the distance from the electron to the observation point with regard to the delay of the electromagnetic wave propagation [5].

From the mechanical point of view we can consider in a simplified way the photon as a long thin cylinder, rotating with the angular frequency ω . If inside the cylinder there are N particles, each of which has a relativistic mass m, then in case of uniform distribution of particles the angular momentum of the cylinder must be equal to the Planck constant, as it is supposed for all photons:

$$\hbar = \frac{1}{2} Nmr^2 \omega \,. \tag{31}$$

From (31) it follows that the mechanical energy of the particles' rotation, calculated as half the product of the angular momentum \hbar and the angular velocity of rotation, is equal to the half of the photon energy: $W_r = \frac{1}{2}\hbar\omega = \frac{1}{2}W$. The other half of the photon energy must be the electromagnetic energy, which was taken into account in (30). Since the angular momentum of the electron in the atom is quantized and proportional to \hbar , from (31) it follows that the total relativistic mass Nm of the charged particles rotating inside the photon must be of the order of the electron mass, in order to ensure the angular momentum \hbar of the photon. However, the mass Nm is only a small part of the mass of the entire flux of charged particles of the vacuum field, that pass through the electron disk per time of the photon emission τ from (29). The total relativistic mass of particles of the entire flux per time τ is expressed by the product of the energy flux rate (25), the time τ and the area of the electron disk πr^2 , and then dividing by the square of the speed of light in order to pass from the energy to the mass: $\frac{\pi r^2 \tau P_{fq}}{c^2} = 3.8 \cdot 10^{-5}$ kg, which is much greater than the electron mass.

Let us consider the motion of some charged particle inside the photon, located on the radius r. This particle rotates at a certain velocity v around the axis of the photon, and besides it moves at the speed of light, as well as the photon, in the direction of its propagation. For the particle's period of rotation we can write:

$$T = \frac{2\pi}{\omega} = \frac{2\pi r}{v} = \frac{\lambda}{c}, \qquad \qquad \omega = \frac{v}{r}. \tag{32}$$

In this model of a photon, there is a relationship between the centripetal force, required for the particle's rotation, and the electric force, exerted on the particle with the charge q and the mass m. In view of (32) we have:

$$qE_0 = \frac{mv^2}{r} = m\omega^2 r \,. \tag{33}$$

Let us express from (30) E_0 and substitute it in (33) in order to determine the ratio $\frac{q}{m}$ for the charged particles inside the photon. In view of (29) for τ , as well as the assumed relation $r = 4a_B$ and the value of the photon angular frequency $\omega = 1.54946 \cdot 10^{16} \,\mathrm{s}^{-1}$, we find:

$$\frac{q}{m} = \frac{\omega^2 r}{E_0} = 16a_B^2 \sqrt{\frac{\pi \varepsilon_0 c \, \tau \omega^3}{\hbar}} = 2.7 \cdot 10^{16} \text{ C/kg.}$$
(34)

For the level of stars, the charge to mass ratio should be the highest for the charged magnetar, as a neutron star with the mass $M_s = 2.7 \cdot 10^{30}$ kg that, according to our assumption, bears the electric charge $Q_s = 5.5 \cdot 10^{18}$ C. This gives: $\frac{Q_s}{M_s} = 2 \cdot 10^{-12}$ C/kg. At the level of atoms, the same is true for the proton, for which $\frac{e}{M_p} = 9.6 \cdot 10^7$ C/kg. What does the relation (34) give to us? From this relation it follows that we must refer to a lower level of matter, that is, the praon level of matter. For the charged praon at rest, the mass to charge ratio, in view of the results of Section 6, is: $\frac{q_{pr}}{m_{pr}} = 4.6 \cdot 10^{27}$ C/kg. Now we will take into account that in (34) the mass of the charged particle is the relativistic mass, i.e. the ratio of the particle's energy to the square of the speed of light. This mass can be written as: $m = \gamma m_{pr}$, where γ is the Lorentz factor for the particle, moving almost at the speed of light. Substituting the mass $m = \gamma m_{pr}$ in (34) and using the value $\frac{q_{pr}}{m_{pr}}$ for the praon, we can determine the Lorentz factor: $\gamma = 1.7 \cdot 10^{11}$.

Earlier in Section 6, we referred to the fact that the protons in cosmic rays reach the energy $E_r = 6 \cdot 10^{19} \,\text{eV}$, while the rest energy of the proton is $E_p = 9.38 \cdot 10^8 \,\text{eV}$. Consequently, for the

most energetic cosmic-ray protons the Lorentz factor is as follows: $\gamma_p = \frac{E_r}{E_p} = 6.4 \cdot 10^{10}$. We

see that the Lorentz factors for praons and protons are close enough to each other. All this means that the photon is a tightly bound flux of praons, the energy of which is maximum and corresponds to the energy of cosmic rays at the nucleon level of matter. Besides, praons are related to protons, just as protons are related to a charged neutron star – a magnetar. From photon's neutrality it follows that it must consist both of positively and negatively charged praons.

9. Conclusion

Based on the assumption that the electric force appears due to the action of the fluxes of charged particles that exist in the vacuum field, we derived an expression for the electric field strengths inside the ball (14) and outside it (7). These expressions are in good agreement with the formulas for the field strengths in electrostatics. From the field strengths we can easily proceed to the scalar potentials of the electric field, since the strength is up to a sign determined as the potential gradient.

Once we find the electric scalar potential, then with the help of a special procedure [14] we can find the 4-potential, the stress-energy tensor of the electromagnetic field, the electromagnetic field equations, the electromagnetic force, as well as the contribution of the electromagnetic field into the equation for the metric. This means that the electromagnetic field theory both in the flat Minkowski space and in the curved spacetime is fully proved at the substantial level through the charged particles fluxes of vacuum field. And the dependence of metric on the electromagnetic field potential allows us to take into account the influence of the inhomogeneous charged particles fluxes on the results of space-time experiments, based as a rule on the use of electromagnetic waves and devices.

In (19) and (22) we made an estimate of the energy density of the charged component of the vacuum field, in (23) we presented the cross section of charged particles' interaction with the matter, in (25) we estimated the rate of the energy flux of the charged particles in one direction. Based on the principles of the theory of infinite nesting of matter, the densest objects at each level of matter are assumed as the sources of the charged particles of vacuum field – neutron stars and magnetars, nucleons and atoms, praons as the components that make up nucleons, etc. These objects emit neutrinos, photons and high-energy cosmic rays that can make contribution to the vacuum field at all levels of matter.

In the formula (27) we expressed the body charge in terms of the emission rate of those fluxes of charged particles of the vacuum field, which interacted with the body's matter and transferred their momentum to it. Due to this interaction, the contribution was made by the charged component of the vacuum field into the mass as the measure of body's inertia. The inertia of the body is manifested in its acceleration, when the balance changes between the falling on the body and outgoing energy fluxes of the vacuum field. We can distinguish in the vacuum field three components, one of which with the energy density ε_c is associated with the strong gravitation and the rest energy of particles, determines the integrity of nucleons and atomic nuclei, and is mainly responsible for the inertia of bodies. Another component with the energy density ε_s is responsible for the ordinary gravitation, and the third component in the form of charged particles with the energy density ε_{cq} leads to electromagnetism. The last two components make their own contribution to the mass of bodies.

We will also note the difference in how the origin of the electrical force is understood. In our approach, the fluxes of charged particles of the vacuum field are the source of electrical force, they exist as a necessary complement to the matter in the form of elementary particles and the bodies composed of them, are involved in the processes of gravitational clustering of the scattered matter, and are generated by the emission from the densest objects, such as praons, nucleons and neutron stars.

In electrostatics, the electric force is not explained. In quantum electrodynamics by means of selecting the Lagrangian of the field's interaction with the matter the formula is derived that resembles the formula for the electric energy of the interaction between two charges in electrostatics [15]. As interpretation the pattern is suggested, in which the charged bodies exchange virtual photons with each other, which leads to the electrical interaction. Besides, here the uncertainty principle is used, limiting the lifetime of virtual photons. Due to virtuality, the photons are attributed very exotic properties, including the possibility of energy negativity or the presence of the momentum without energy. The photons' energy is considered to be proportional to the Planck constant, and therefore the possibility of existence of photons and particles, belonging to the lower levels of matter and with another Planck constant, is not considered. The obvious disadvantage of this approach is the difficulty to explain the origin of virtual particles as such and their unique properties.

If we consider the fluxes of charged particles in the vacuum field as the source of the electric forces, it becomes possible to consider their scattering in the process of quantum transitions in atoms. In [5] the substantial model of electron in the form of a disk is considered, in which the charged matter rotates differentially, and ensures the magnetic moment of the electron. In addition, the electron spin is explained as the result of the shift of the disk's center relative to the nucleus and rotation of this center in addition to the matter rotation in the electron cloud. If the electron transits into the quantum state with lower energy, it emits a photon, which carries with it the angular momentum that is proportional to the Dirac constant. In this process, the scattering of charged particles of the vacuum field on the electron disk, taking into account the action of the magnetic and electric fields in the wave zone, leads to the formation of a photon as an object preserving its structure for a long time.

In Section 8, we studied the model of the photon, emitted in atomic transition in the hydrogen atom. Associating the photon parameters and its structure with the parameters of the emitter – the charged electron disk, we managed to determine the charge to mass ratio for the particles that make up the photon. As a result, it turned out that photons consist of praons of very high energies, comparable to the energies that cosmic rays would have if these rays emerged at the nucleon level of matter near the protons. These relativistic praons must form the basis of the charged particles of the vacuum field. Indeed, in the interaction of praons of the vacuum field with the electron in atomic transition, the twisting of praons takes place under action of the fields along the axis of the electron disk, and the appearing photon carries away the excess angular momentum of the electron from the atom. Meanwhile, part of praons of the vacuum field is part of the photon, so that the speed of the photon actually is the speed of praons in the fluxes of particles of the vacuum field.

In contrast to the chaotic motion of the praons in the vacuum field, the praons in the photon are rigidly bound to each other by both electromagnetic and gravitational forces. The situation here is similar to the situation with the nucleons, which only in special circumstances can form extremely stable formations – the atomic nuclei. According to the gravitational model of strong interaction [5], the nucleons in atomic nuclei are attracted to each other by strong gravitation and repel each other by means of the torsion field, arising from the rapid rotation of the nucleons. In order to form the nucleus, the nucleons must interact with each other only in a strictly defined orientation of the spins and magnetic moments and must have sufficient initial energy that allows rotating the nucleons up to the desired rotation speed by means of gravitational induction. The praons in the photon can interact with each other in a similar way. We can even calculate the gravitational constant for the praon level of matter with the help of the coefficients of similarity from Section 5 and the

strong gravitational constant $\Gamma = \frac{e^2}{4\pi\varepsilon_0 M_p M_e} = 1.514 \cdot 10^{29} \,\mathrm{m}^3 \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^{-2}$ in the following way,

using the theory of dimensions and SP Φ symmetry, according to [1]: $G_{pr} = \frac{\Phi}{PS^2} \Gamma = 3.3 \cdot 10^{68} \,\mathrm{m}^3 \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^{-2}$. In the gravitational field with this large gravitational

constant, the praons of the photon can form sufficiently rigid structure, so that the photon could fly large cosmic distances without decaying.

In Section 5, for the ratio of the absolute value of energy in the field of strong gravitation

to the energy of electric field of the proton we found: $\frac{|E_g|}{E_e} = \frac{4\pi\varepsilon_0 \Gamma M_p^2}{e^2} = \frac{M_p}{M_e}$. Аналогичное равенство следует и для праона, для чего необходимо постоянную сильной гравитации Γ заменить на постоянную гравитации для праонного уровня материи G_{pr} и

подставить массу и заряд праона из раздела 6: $\frac{4\pi \varepsilon_0 G_{pr} m_{pr}^2}{q_{pr}^2} = \frac{M_p}{M_e}$.

Concurrent consideration of the evolution of objects at different levels of matter, such as the level of praons, nucleons and neutron stars, allows us to draw conclusions not only as to the origin of gravitational and electromagnetic forces. For example, if for a neutron star with the mass $M_s = 1.35$ Solar mass and the stellar radius $R_s = 12$ km we calculate the average binding energy per nucleon, we will obtain $E_b = \frac{kGM_nM_s}{2R_s} = 7.5 \cdot 10^{-12}$ J or 47 MeV per nucleon, which is greater than the binding energy of atomic nuclei. Taking into account that neutron stars are born in supernova explosions, when the explosion energy is carried away by neutrinos and emission, and is converted into the kinetic energy of the discharged shell, a significant part of the binding energy is emitted from the star and transferred into the environment. In [13], we estimated that 61% of all praons are part of nucleons, and the rest 39% form new particles (which are structurally the analogues of white dwarfs at the level of elementary particles) or exist separately. The same proportion remains at the level of stars: 61% of all nucleons over time will be part of neutron stars, and the rest of nucleons remain either as a gas or as the matter of white dwarfs. New particles as the analogues of white dwarfs, due to their significant presence in cosmic space, can ensure the red shift effect in the spectra of distant galaxies, explain the background radiation and the dark matter, etc.

Consequently, the concentration of free protons in the visible Universe must be of the same order as the averaged over the entire space concentration of nucleons in stars, that is of the order of concentration of baryons $n = 0.13 \text{ m}^{-3}$, according to the findings of the Lambda-Cold Dark Model [16]. With this in mind, the product of the concentration of baryons and the binding energy of a neutron star in the calculation per nucleon will give us the estimate of the maximum energy density of emission in cosmic space: $nE_b = 1 \cdot 10^{-12} \text{ J/m}^3$. Indeed, the energy density in the relic radiation equals $4 \cdot 10^{-14} \text{ J/m}^3$, and the energy density in the stellar radiation, magnetic fields and cosmic rays is of the same order of magnitude, as well as the kinetic energy of the motion of gas particles. The sum of these energy densities does not exceed the maximum energy density nE_b .

In conclusion, we will estimate the length of free path of the charged particles of the vacuum field in the cosmic space, taking as the charge concentration in a first approximation the value $\eta = 0.13$ of the elementary charge per cubic meter, which is equal to the average concentration of baryons in the Universe. This approach gives only the minimum value of the free path length, since on the average the matter in the Universe is neutral, and η must reflects the average concentration of the total charge of the Universe. From the ratio $\vartheta\eta x \approx 1$ at a given concentration of charges and the value ϑ according to (23), we find the free path length of charged particles: $x = 2.9 \cdot 10^{30}$ m. This value is 3 orders of magnitude greater than the visible size of the Universe, which is estimated by the value of 14 billion parsecs or $4 \cdot 10^{27}$ m. Consequently, the charged particles can easily reach our Universe from a distance, where they can be produced in a concentration sufficient to meet the required energy density. We do not support the model of the Big Bang, which limits the lifetime of the Universe to the

value of 13.8 billion years, explaining in a different way the phenomena associated with this model [13]. Then the charged particles of the vacuum field can have enough time to get into our Universe from the outside and reach here the equilibrium concentration with the value $n_{\rm rer} = 4 \cdot 10^{87} \,\mathrm{m}^{-3}$.

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