

The Smarandache concatenated sequences and the definition of Smarandache-Coman constants

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Abstract. In two previous papers I presented the notion of "Coman constant" and showed how could highlight the periodicity of some infinite sequences of integers. In this paper I present the notion of "Smarandache-Coman constant", useful in Diophantine analysis of Smarandache concatenated sequences.

Definition:

We understand by "Smarandache-Coman constants" the numbers with n digits obtained by concatenation from the digital root of the first n terms of a Smarandache concatenated sequence, if the digital root of the terms of such a sequence form themselves a periodic sequence, with a periodicity equal to n . Note that not every Smarandache concatenated sequence is characterized by a Smarandache-Coman constant, just some of them; it is interesting to study what are the properties these sequences have in common; it is also interesting that sometimes more such sequences have the same value of Smarandache-Coman constant and also to study what these have in common.

Example:

The values of the digital root of the terms of the Smarandache consecutive sequence (12, 123, 1234, 12345, 123456, 1234567, 12345678, 123456789, 12345678910, 1234567891011, ...) are: 1, 3, 6, 1, 6, 3, 1, 9, 9, 1, 3, 6, 1, 6, 3, 1, 9, 9 (...) so these values form a sequence with a periodicity equal to nine, the terms 1, 3, 6, 1, 6, 3, 1, 9, 9 repeating infinitely. Concatenating these nine values is obtained a Smarandache-Coman constant, i.e. the number 136163199.

Let's take the following Smarandache concatenated sequences:

(1) The Smarandache consecutive numbers sequence

S_n is defined as the sequence obtained through the concatenation of the first n positive integers. The first ten terms of the sequence (A007908 in OEIS) are 12, 123, 1234, 12345, 123456, 1234567, 12345678, 123456789, 12345678910.

This sequence is characterized by a Smarandache-Coman constant with 9 digits, the number 136163199. Note that, obviously, the same constant will be obtained from the Smarandache reverse sequence (A000422), defined as the sequence obtained through the concatenation of the first n positive integers, in reverse order.

(2) The Smarandache concatenated odd sequence

S_n is defined as the sequence obtained through the concatenation of the first n odd numbers (the n -th term of the sequence is formed through the concatenation of the odd numbers from 1 to $2*n - 1$). The first ten terms of the sequence (A019519 in OEIS) are 1, 13, 135, 1357, 13579, 1357911, 135791113, 13579111315, 1357911131517, 135791113151719.

This sequence is characterized by a Smarandache-Coman constant with nine digits, the number 149779419.

(3) The Smarandache concatenated even sequence

S_n is defined as the sequence obtained through the concatenation of the first n even numbers (the n -th term of the sequence is formed through the concatenation of the even numbers from 1 to $2*n$). The first ten terms of the sequence (A019520 in OEIS) are 2, 24, 246, 2468, 246810, 24681012, 2468101214, 246810121416, 24681012141618, 2468101214161820.

This sequence is characterized by a Smarandache-Coman constant with nine digits, the number 263236299.

(4) The concatenated cubic sequence

S_n is defined as the sequence obtained through the concatenation of the first n cubes: $1(2^3)(3^3)\dots(n^3)$. The first ten terms of the sequence (A019522 in OEIS) are 1, 18, 1827, 182764, 182764125, 182764125216, 182764125216343, 182764125216343512, 182764125216343512729, 1827641252163435127291000.

This sequence is characterized by a Smarandache-Coman constant with three digits, the number 199.

(5) The antisymmetric numbers sequence

S_n is defined as the sequence obtained through the concatenation in the following way: $12\dots(n)12\dots(n)$. The first ten terms of the sequence (A019524 in OEIS) are 11, 1212, 123123, 12341234, 1234512345, 123456123456, 12345671234567, 1234567812345678, 123456789123456789.

This sequence is characterized by a Smarandache-Coman constant with nine digits, the number 26323629. Note that the same Smarandache-Coman constant characterizes the Smarandache concatenated even sequence.

(6) The "n concatenated n times" sequence

S_n is defined as the sequence of the numbers obtained concatenating n times the number n. The first ten terms of the sequence (A000461 in OEIS) are 1, 22, 333, 4444, 55555, 666666, 7777777, 88888888, 999999999, 101010101010101010.

This sequence is characterized by a Smarandache-Coman constant with nine digits, the number 149779419. Note that the same Smarandache-Coman constant characterizes the Smarandache concatenated odd sequence.

(7) The permutation sequence

S_n is defined as the sequence of numbers obtained through concatenation and permutation in the following way: 13...(2*n - 3)(2*n - 1)(2*n)(2*n - 2)(2*n - 4)...42. The first seven terms of the sequence (A007943 in OEIS) are 12, 1342, 135642, 13578642, 13579108642, 135791112108642, 1357911131412108642, 13579111315161412108642.

This sequence is characterized by a Smarandache-Coman constant with nine digits, the number 313916619.

(8) The Smarandache $n2*n$ sequence

S_n is defined as the sequence for which the n-th term $a(n)$ is obtained concatenating the numbers n and $2*n$. The first twelve terms of the sequence (A019550 in OEIS) are 12, 24, 36, 48, 510, 612, 714, 816, 918, 1020, 1122, 1224.

This sequence is characterized by a Smarandache-Coman constant with three digits, the number 369.

(9) The Smarandache nn^2 sequence

S_n is defined as the sequence for which the n-th term $a(n)$ is obtained concatenating the numbers n and n^2 . The first fifteen terms of the sequence (A053061 in OEIS) are 11, 24, 39, 416, 525, 636, 749, 864, 981, 10100, 11121, 12144, 13169, 14196, 15225.

This sequence is characterized by a Smarandache-Coman constant with nine digits, the number 26323629. Note that

the same Smarandache-Coman constant characterizes the Smarandache concatenated even sequence and the Smarandache antysimmetric numbers sequence.

(10) The Smarandache power stack sequence for $k = 2$

$S_n(k)$ is the sequence for which the n -th term is defined as the positive integer obtained by concatenating all the powers of k from k^0 to k^n . The first ten terms of the sequence are 1, 12, 124, 1248, 12416, 1241632, 124163264, 124163264128, 124163264128256, 124163264128256512.

This sequence is characterized by a Smarandache-Coman constant with six digits, the number 137649.

Comments:

(1) I conjecture that any sequence of the type nk^n is characterized by a Smarandache-Coman constant:

: for $k = 3$ the sequence 13, 26, 39, 412, 515, 618, 721, 824, 927, 1030, 1133, 1236 is characterized by the Smarandache-Coman constant 483726159;

: for $k = 4$ the sequence 14, 28, 312, 416, 520, 624, 728, 832, 936, 1040, 1144, 1248 is characterized by the Smarandache-Coman constant 516273849 etc.

(2) I conjecture that any sequence of the type nn^k is characterized by a Smarandache-Coman constant:

: for $k = 3$ the sequence 11, 28, 327, 464, 5125, 6216, 7343, 8512, 9729, 101000, 111331 is characterized by the Smarandache-Coman constant 213546879 etc.

(3) Not any power stack sequence is characterized by a Smarandache-Coman constant:

: for $k = 3$ the Smarandache sequence is 1, 13, 139, 13927, 1392781, 1392781243, 1392781243729, 13927812437292187 and the values of Coman function for the terms of the sequence are 1, 4, 4, 4 (...), the digit 4 repeating infinitely so is not a sequence characterized by a Smarandache-Coman constant.

(4) I conjecture that not any sequence with the general term of the form $1(2^k)(3^k)...(n^k)$ is characterized by a Smarandache-Coman constant:

: the values of digital root for the terms of the concatenated square sequence 1, 14, 149, 14916, 1491625, 149162536, 14916253649, 1491625364964,

149162536496481, 149162536496481100, ... (A019521 in OEIS) are 1, 5, 5, 3, 1, 1, 5, 6, 6, 7, 2, 2 (...) and so far has not been shown any periodicity.

Conclusion:

We found so far 10 Smarandache-Coman constants, 7 with nine digits, *i.e.* the numbers 136163199, 149779419, 26323629, 313916619, 483726159, 516273849, 213546879, two with three digits, *i.e.* the numbers 199 and 369, and one with six digits, the number 137649.