An Elementary Proof of Gilbreath’s onjecture

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Abstract

Given the fact that the Gilbreath's Conjecture has been a major topic of research in aritmatic progression for well over a century,and as bellow:

2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61
1 2 2 4 2 4 2 4 6 2 6 4 2 4 6 6 2
1 0 2 2 2 2 2 2 4 4 2 2 2 2 0 4
1 2 0 0 0 0 0 2 0 2 0 0 0 2 4
1 2 0 0 0 0 2 2 2 2 0 0 2 2
1 2 0 0 0 2 0 0 0 2 0 2 0
1 2 0 0 2 2 0 0 2 2 2 2
1 2 0 2 0 2 0 2 0 0 0
1 2 2 2 2 2 2 2 0 0
1 0 0 0 0 0 0 2 0
1 0 0 0 0 0 2 2
1 0 0 0 0 2 0
1 0 0 0 2 2
1 0 0 2 0
1 0 2 2
1 2 0
1 2
1

The Gilbreath’s conjecture in a way as easy and comprehensive as possible.
He proposed that these differences, when calculated repetitively and left as bsolute values, would always result in a row of numbers beginning with 1,In this paper we bring elementary proof for this conjecture.

Introduction:

Given the fact that the Gilbreath's Conjecture has been a major topic of research in aritmatic progression for well over a century, the Gilbreath conjecture in a way as easy and comprehensive as possible.
Hopefully it will help the right person take this conjecture out of the unsolved list and into the list of accomplishments of mathematics.
To begin the story, the anecdote goes that an undergraduate student named norman Gilbreath was doodling on a napkin one day in a cafe and found a very interesting characteristic of the list of sequential prime numbers and the

diferences between them. He proposed that these diferences, when calculated repetitively and left as absolute values, would always result in a row of numbers beginning with 1 (after the first row). No one has been able to prove it.
In 1878, eighty years before Gilbreath's discovery, François Proth had, however, published the same observations along with an attempted proof, which was later shown to be false.
Andrew Odlyzko_verified that d_1^k is 1 for $k \leq n = 3.4 \times 10^{11}$ in 1993, but the conjecture remains an open problem. Instead of evaluating *n* rows, Odlyzko evaluated 635 rows and established that the 635th row started with a 1 and continued with only 0's and 2's for the next *n* numbers. This implies that the next *n* rows begin with a 1,see[1]

Notation

We define d_n^k is K th row, n th Number,and as $d_n^k = \left|d_{n+1}^{k-1} - d_n^{k-1}\right|$
We should prove that $d_1^k = 1$,for any k

Theorem: $d_1^k = 1$,for any k
Proof: assume that the Gilbreath's Conjecture is correct until p_m ,that is m th ,prime in first row by induction we prove that this Conjecture is correct for p_{m+1} ,so below table is correct by induction:

2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61……p_{m-2} p_{m-1} p_m

1 2 2 4 2 4 2 4 6 2 6 4 2 4 6 6 2

1 0 2 2 2 2 2 2 4 4 2 2 2 2 0 4

1 2 0 0 0 0 0 2 0 2 0 0 0 2 4

1 2 0 0 0 0 2 2 2 2 0 0 2 2

1 2 0 0 0 2 0 0 0 2 0 2 0

1 2 0 0 2 2 0 0 2 2 2 2

1 2 0 2 0 2 0 2 0 0 0

1 2 2 2 2 2 2 2 0 0

1 0 0 0 0 0 0 2 0

1 0 0 0 0 0 2 2

1 0 0 0 0 2 0

1 0 0 0 2 2

1 0 0 2 0

1 0 2 2

1 2 0

1 2

1

.

.

.
1
Notice that in above table for every sentence in any row ,we have $d_n^k = \left|d_{n+1}^{k-1} - d_n^{k-1}\right| < p_n \le p_m$,Now we prove that this table is correct for p_{m+1},

For simplifying this conjecture we state some Lemmas as below:

Lemma 1:Second row is correct ,i.e $d_m^2 = |p_{m+1} - p_m| < p_m$

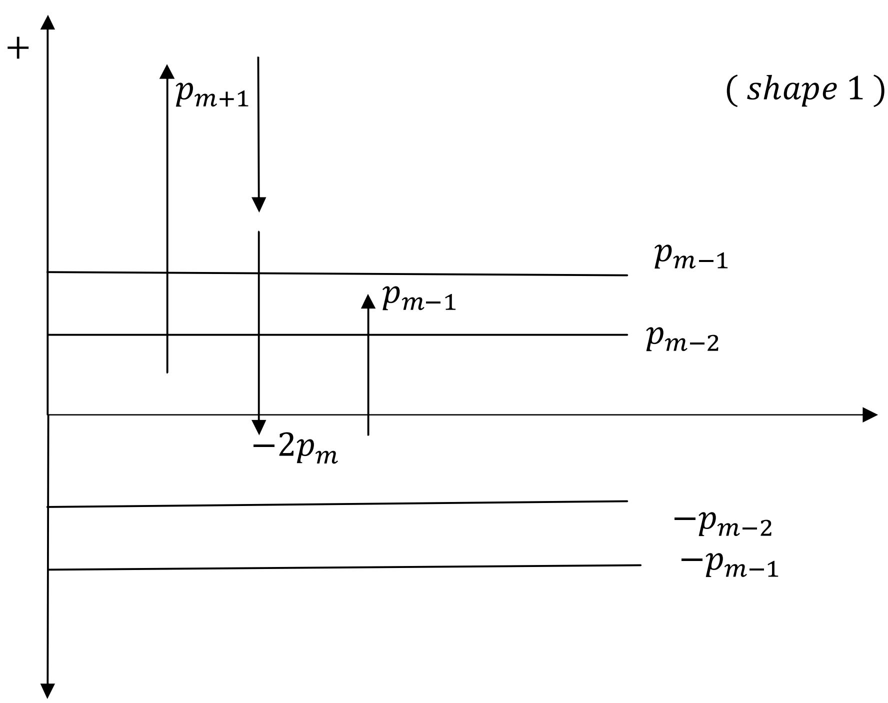
Proof, this is obviously correct by refer to [2]

Lemma 2:third row is correct ,i.e $d_{m-1}^3 = \big|\,|p_{m+1} - p_m| - |p_m - p_{m-1}|\big| < p_{m-1}$,

Proof :this is obviously correct by refer to [2]

Lemma 3:fourth row is correct ,i.e $d_{m-2}^4 = |d_{m-1}^3 - d_{m-2}^3| < p_{m-2}$

Proof :if $d_{m-2}^3 \ge d_{m-1}^3$,so $d_{m-2}^4 < p_{m-2}$, since $d_{m-2}^3 < p_{m-2}$,according to induction, otherwise we have:



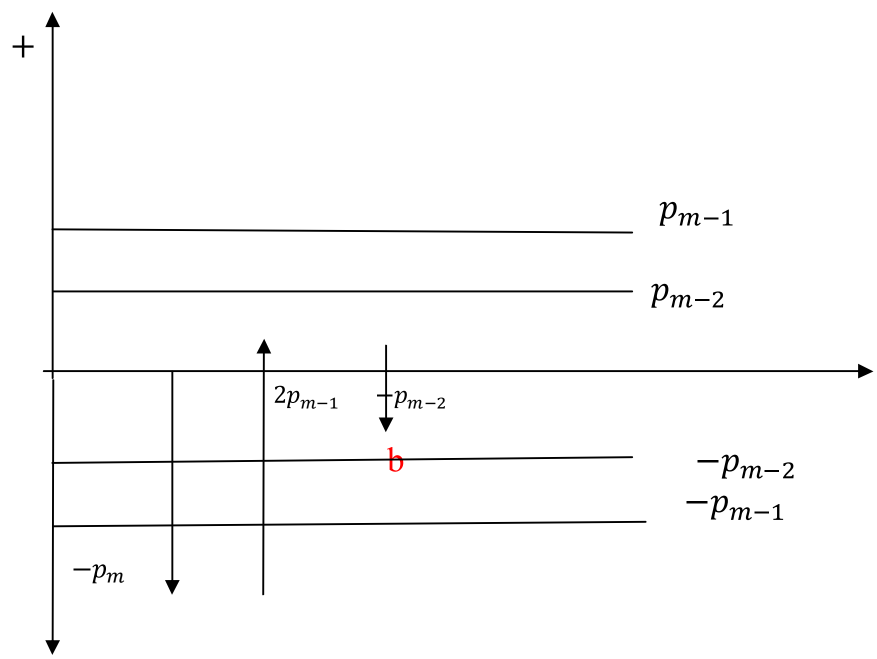
(*shape* 1)

If , $d_{m-1}^3 = \big| |p_{m+1} - p_m| - |p_m - p_{m-1}| \big| < p_{m-2}$, we have the result ,but if $p_{m-2} < d_{m-1}^3 < p_{m-1}$ like shape above we should prove:

$d_{m-2}^4 = |d_{m-1}^3 - d_{m-2}^3| < p_{m-2}$,but according to assume of induction

$d_{m-2}^3 = \big| |p_m - p_{m-1}| - |p_{m-1} - p_{m-2}| \big| < p_{m-2}$,the shape of $-d_{m-2}^3$ is as below:

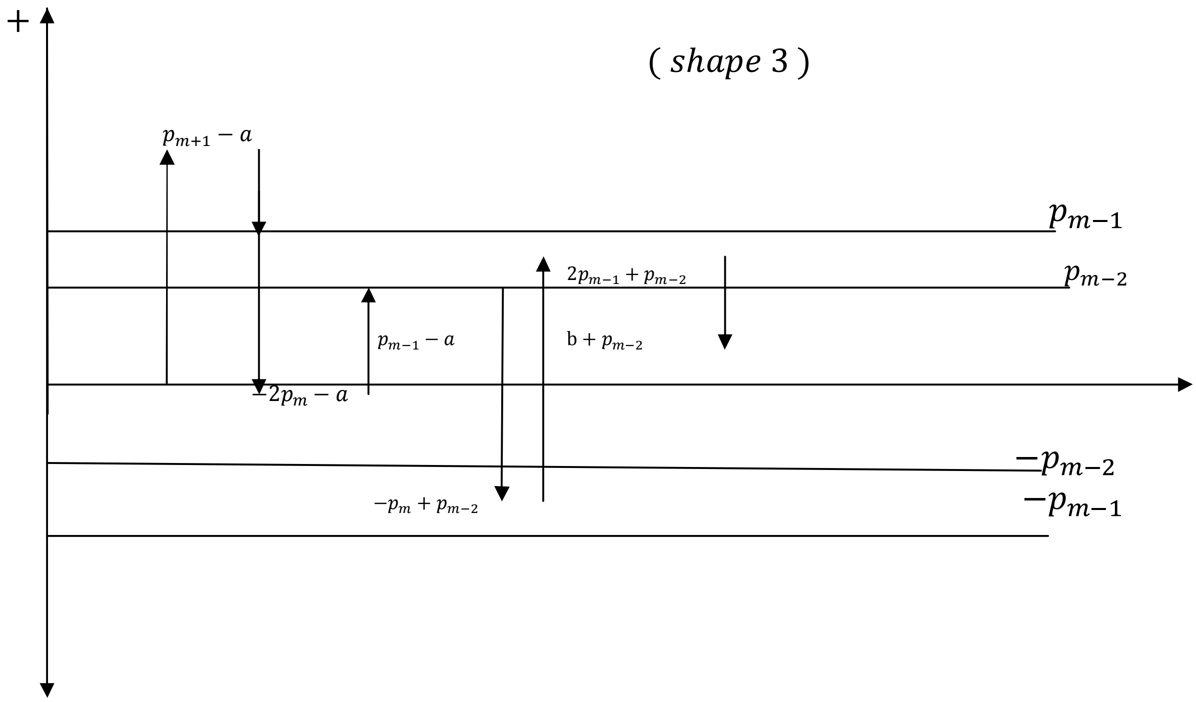
(*shape* 2)



Since $p_{m-2} < d_{m-1}^3 < p_{m-1}$,so $d_{m-1}^3 = p_{m-2} + a$ that

$a < p_{m-1} - p_{m-2} < p_{m-2}$,Now suppose that we shift (shape 1) a downward or $-a$,and other hand we shift (shape 2) p_{m-2} upward ,and connected two shape to each other such that (shape 2) is started in the end of (shape1)

(*shape* 3)



According to (shape 3) $< p_{m-2}$,notice that the number of the point transferred in (shape 1) are the same of (shape 2) so the number of the end of the arrows in the(shape 1) and(shape 2) are the same ,the number of the d^4_{m-2} are 6,

$$\big((p_{m+1} - p_m) - (p_m - p_{m-1})\big) - 3p_{m-2} + \big((p_m - p_{m-1}) - (p_{m-1} - p_{m-2})\big) + 3p_{m-2} < \big((p_{m+1} - p_m) - (p_m - p_{m-1})\big) - 3a - \big((p_m - p_{m-1}) - (p_{m-1} - p_{m-2})\big) + 3p_{m-2} = b + p_{m-2} < p_{m-2}$$

So we proved the lemma. Notice : if we have any change in absolute values in both shapes the result would be without change.

Lemma 4: k th row is correct ,i.e $d^k_{m-(k-2)} = \left|d^{k-1}_{m-(k-3)} - d^{k-1}_{m-(k-2)}\right| < p_{m-(k-2)}$

Proof: the prove of this lemma is similar to lemma 3,notice that for $d^4_{m-2} = |d^3_{m-1} - d^3_{m-2}| < p_{m-2}$ we have 6 points, for d^5_{m-3} ,we have 12 points, so for $k \geq 3$,we have $3 \times 2^{k-3}$ points, then for $k = m+1$, $d^{m+1}_1 < 2$,so $d^{m+1}_1 = 1$

So we proof the Theorem.

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