Stop the Tirade against Euclid: Why Euclid should be more emphasized in Schools.

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This article purports to address the erroneous arguments in C.K. Raju's article: towards equity in Mathematics education 1. Goodbye Euclid!
Published in Bharathiya Samajik Chintan, VII (4), new series and points out the need to emphasize Euclid more than it is done in classroom today.

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Euclid had been my best friend since High School (since I was 13). I enjoyed him by studying his theorems, especially problem solving. Solving a beautiful, challenging problem in geometry gives me a pleasure akin to listening a beautiful piece of music and infuses power in me to meet greater challenges.

In whatever stage I am at, in my mathematical pursuits, a good piece of result which is new to me, based on the 'Elements' will propel me to finding more treasures hidden in it.

With such a strong liking for Euclid, when someone attacks Euclid I cannot remain silent.

This article is to address the issues in C.K.Raju's above named article that appeared in Bharathiya Samajik Chintan VII (4) new series.

C.K.Raju who is a well-known computer scientist and physicist, in India and a friend of mine, has been arguing vehemently, to teach religiously neutral mathematics by banishing Euclid from present day school curriculum and eliminating formal mathematics from University Mathematics Curriculum first in India and also world over!!!

This argument which has been around for a while, should be curbed right away looking at the ominous demands that Raju is making, (through his erroneous arguments, of course), before it corrupts young minds, further. I accidentally bumped into the article while browsing the net.

Now coming to the main issue, Raju's contention is the following: Theology influenced Greek Mathematics (especially the 'Elements'), by making it as he calls it 'Theologically Correct'. Since modern day formal mathematics is based on the Elements, Mathematics is essentially an instrument to perpetuate theological beliefs (more precisely, is only theological correct) and not universal and secular as is believed.

I call this total nonsense and absurd.

Making a mistake at a fundamental level, he builds a huge repertoire of arguments (in this paper and elsewhere) to arrive at totally false conclusions.

To bring back the poor Euclid, whom Raju has condemned to damnation (in his mind) let us start with Pythagoras.

It is believed by quite a few scholars starting from Voltaire, to present day US TV personalities like Albert Burke that Pythagoras in his extensive travels, also visited India.

Although Heroditus, (whom later historians call a 'Father of Lies') asserts that Pythagoras went to Egypt and learnt geometry from them, this view can be dismissed (although widely accepted), based on the fact that there is nothing in his symbolism that resembles to that in Egypt and some doubt if he learnt anything from Egyptians at all.

I consider his visit to India a fact. His leanings towards Vedantic philosophy and Vegetarianism, strongly lend credence to this belief. He learned the theorem (which now bears his name 'Pythagoras Theorem') from the sulaba sutras on his visit to India and also learnt astronomy and vedantic philosophy by his contact with Vedic scholars in India and took his knowledge to Greece.

In other words, there was transmission of knowledge from India to Greece from his time. In fact, there was transmission of knowledge from India to other civilizations, like Egyptian and Babylonian in ancient times, but we don't need this observation

for our arguments. It is also believed that a few other Greek scholars after him also visited India.

In other words, Vedantic wisdom and sciences like Mathematics and Astronomy found their way to Greece from India from the time of Pythagoras.

Greeks were great thinkers. They were the ones who initiated a scientific inquiry about the world around them through logical reasoning (or deductive proof) to satisfy the demands of logic and reasoning of the human mind.

These scholars like Pythagoras who brought the Vedantic wisdom of India to Greece believed on one hand they had [A] the knowledge of the Absolute truth (Vedic wisdom) and on the other hand [B] they were also interested in scientific pursuits, studying the world around them (Mathematics, Physics, Astronomy, etc.), in a way satisfying the demands of logic and reasoning. Hence, started the deductive process (or deductive proof in Mathematics, starting with the 'Elements')

Some of them like Pythagoras, and Plato justified [B] on the basis that it would help in the realization of [A]. For other scholars   [A] would not have mattered at all, like for e.g. in modern times like Newton, for whom theology mattered and Russell to whom theology was not of concern.

 [A] and [B]   are completely independent of each other. [A] had no influence on [B], although for some scholars [A] was a motivating factor for [B], after all the 'Elements' consists of the combined results obtained by several mathematicians at that time. In any case, [A] and [B] are completely independent of each other although for some [A] was a motivating factor for [B].

In other words the pursuit of rational scientific inquiry, satisfying the demands of logic and reasoning (i.e. the method of deductive proof, from which arose the 'Elements') is completely independent of the theological beliefs of some Greek scholars and the subsequent theological meaning that they attached to its pursuit.

Surely for people like Pythagoras and Plato, and later Greeks, their pursuit of scientific knowledge through deductive reasoning (method of deductive proof) aided or supplemented their theological beliefs. Other than that [A] and [B] are not connected anyway.

To give a more mundane argument, if someone says that he or she wants to be the Prime Minister of India or president of US, in order to serve the people of their country, does the motive, have any influence on the entire process (technically, 'the process') of getting elected? No. For some people it may be a motivating factor and for others it may just be a personal ambition.

Raju's fundamental mistake is this. Correct by reasoning (proof by deduction) and theological correctness or same (i.e. equivalent).

The fact is:
Correct by reasoning (proof by deduction) ------------------------>   theological correctness -- [1]
                                                        Leads to or implies

Implication the other way is not true.

Theological correctness   ------------/-------->   correct by reasoning -   [2]
                                        Does not necessarily imply

Speaking from the perspective of a lay person, as human beings, we clearly understand these facts. If something is reasonable or logical it doesn't conflict with theology (religious beliefs).

However, theological and religious beliefs are always not logical (do not always stand to reason). Thus, correct by reasoning and theological correctness are not equivalent.

Starting with Pythagoras and coming up to Einstein and later 20th Century physicists and mathematicians, Physics and Mathematics has lent a lot of credence to Vedic wisdom (theology). I quote an eminent American physicist Jon Archibald Wheeler, the first to involve in the theoretical development of the atomic bomb, in regards to 20th century physics he says "It is curious that people like Schroedinger, Niels Bohr and Oppenheimer are 'Upanishadic scholars".

This lends credence to [1] in the context of scientific inquiry in the sense that rigorous scientific inquiry starting with the Elements (correct by reasoning) has finally lead to the endorsement of Vedic wisdom (theological correctness).

The main thrust of Raju's arguments has been that logical correctness and theological correctness are equivalent (although this is not the way he states this,

this is what is essentially implied by his arguments). To counter his arguments, let us go back 2300 years, and start with Euclid. As observed earlier, mathematics having been put on a reasonably sound footing starting with the 'Elements' over centuries of development has aided physics of the 20th century in lending a lot of credibility to the truths expounded in the Vedas (Upanishads).

Assume you are a physicist, especially from India, (I am not a physicist, I am a Mathematician) and you are somehow uncomfortable with this outcome. One possible way out is accusing, the whole process of scientific enquiry having been orchestrated right at the beginning, so that the end results are not surprising, because at the very beginning the inquiry has been manipulated to produce the desired end result. This is precisely Raju's situation and his claims.

Unfortunately (for him) this is not the reality or situation. As I mentioned earlier, theology did nothing to influence the method of deductive proof (and hence the 'Elements') except for providing a motivation (and perhaps justification) for studying mathematics through deductive proof, for some of the early Greek mathematicians.

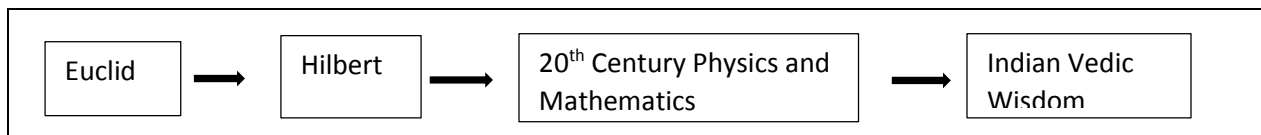Let us visualize the whole situation by drawing a few diagrams.

Figure 1

Let us call this a diagram describing the flow of scientific knowledge.
It illustrates how starting with Euclid (Euclid's Elements) Mathematical knowledge progressed till the time of Hilbert when Euclid's Elements were revised, to 20th Century physics and mathematics, finally lending credibility to Indian Vedic Wisdom. (I.e. logical correctness in science has finally led to theological correctness).

What Raju asserts is that, theological correctness, the result at the end, has been induced into Euclid (more precisely Euclid's 'Elements') right at the beginning. Let us draw a diagram for this.

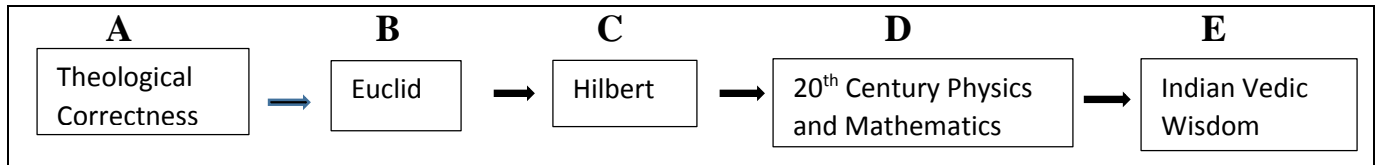| A	B	C	D	E
Theological Correctness	Euclid	Hilbert	20th Century Physics and Mathematics	Indian Vedic Wisdom

Figure 2

(Not a true state of affairs)

This diagram indicates that theological correctness has been induced into Euclid at the very beginning, but this is totally wrong.

The reason is this, Euclid is a guy who accepts only logical arguments (starting with a set of axioms, which is the whole philosophy of the 'Elements').

By saying that theological Correctness has been induced into Euclid, (more precisely into his 'Elements'), of course implicitly, (because the domain of theology is different from the domain of the 'Elements', which is the physical world), what one is saying is that everything that is theologically correct, is also logically correct, which is not true.

Therefore, the arrow emanating from A to B does not exist. However, one can always draw an arrow from B to A, since anything that stands to logic does not contradict theology (which is definitely true).

Let us draw a diagram for this

A	B	C	D	E
Theological Correctness	Euclid	Hilbert	20th Century Physics and Mathematics	Indian Vedic Wisdom
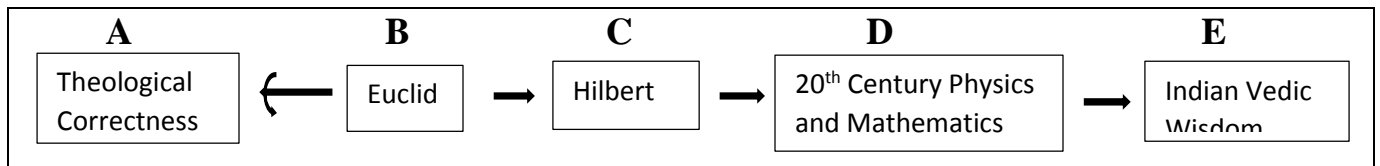
Now Raju is searching for a path from A to E to substantiate his fundamental (ulterior) emphasis, that the final outcome of centuries of scientific enquiry has been orchestrated right at the beginning. (I have spoken to him in 1980 when he was vehemently attacking vedic wisdom of India). However, no such path exists from A to E.

Now comes the crucial step. How about we get rid of this guy Euclid? Since, details of Euclid are obscure, he can make use of that to establish that Euclid did not exist (and is just a fictitious name) and in such a case one can draw an arrow from A to B and sure then there is a path from A to E. That would establish his (wrong) claim that right at the beginning the final outcome has been manipulated by inducing theological correctness, so that the whole inquiry at the end lead (seems to confirm) to vedic wisdom (theological correctness).

Whether he is doing this intentionally or unintentionally is beside the point.

He is totally wrong, because Euclid is a genuine historical character who existed and wrote the 'Elements'.

The proof is easy. For this, we seek the help of Apollonius of Perga, the 'Great Geometer'.

Apollonius wrote the 'conic sections' a very original book about conics. The first four books in Greek have survived (please refer to Apollonius of perga history, 'Mac Tutor' or T.L. Heath History of Greeks Mathematics),

In book 3, he clearly mentions Euclid directly and alludes to his 'Elements' This is 'incontrovertible' evidence about the existence of Euclid and his Elements and demolishes completely Raju's false claim that Euclid did not exist and is a fictitious name.

Therefore it is clear that Euclid is a genuine historical character who wrote the 'Elements'. The only doubt is his role in the authorship. He should have just been a compiler of the results in the 'Elements', occasionally providing missing details or proofs. Since it is a compilation of all available mathematical knowledge at that time, it could have been considered as an encyclopedia of Mathematics. Given this, it is very understandable why he is referred to as the 'Writer of the Elements' by Proclus (which Raju cites in his paper), since Euclid cannot claim any ownership of the results except having compiled them.

Since Euclid existed and wrote the 'Elements', therefore, in Fig 3, there is no arrow from A to B. This disproves Raju's claim that the 'Elements' has been made theologically correct and hence, further, disproves the claim that formal mathematics itself has been made theologically correct.

In fact it completely demolishes his entire thesis about the 'Cultural Foundations of Mathematics'.

There are a few more points (false claims) he makes elsewhere which I wish to address.

1) Raju says Mathematics is not universal and secular. Wrong. Mathematics is indeed universal and secular.
   To support his claim. When someone asked him if $2 + 2 = 4$, is not universal he counters it by saying that if you take 2 stones and 2 more stones and if you break 1 stone into 2 pieces then the above equation does not hold. This is one of the most foolish arguments that I have ever heard. If you break one of the stones into 2, the left hand side of the equation is not $2 + 2$ anymore, it is $2 + 3$ which is 5. Further, in putting forth such an absurd argument he himself is using a 2-valued logic which he is opposing from the very beginning.

2) Now coming to his arguments, about 2-valued logic, our logic depends on our commonsense perception of the world. To give a crude example even a child knows that if he puts his hand in fire, he will get burnt, so he should not put his hand in fire.
   There is no such thing as he will, and at the same time he will not get burnt, the 3 or 4-valued logic he advocates.
   Coming to Mathematics, logic is of course 2-valued. Given the definition of an even and odd number, and given a number, it is either even or odd. Given a point and a line, either a point lies on it or it does not… etc., isn't this obvious?

   His argument is as follows. Either logic is decided 1) Culturally or 2) Empirically.
   If it is decided empirically, it depends on Physics. My question is why? I say this is false.
   It just depends on our commonsense perception of the world. Why should it depend on physics? Of course, no doubt, physics is the study of physical world.

If commonsense perception of the world does not agree with quantum physics, for which reason I think, he says quantum logic is not 2- valued, that is fine.

This anomaly in 2-valued logic exists in Vedantic philosophy as well. If you approach a seer and tell him, based on Vedanta, since everything in this world is, Brahman, therefore 'I am God', the seer would probably tell you this is both true and false. One who has studied and understood Vedantic philosophy can easily see the truth of this statement. Clearly, there is an anomaly with 2- valued logic here. Such anomalies exist at the philosophical level or in the case of science at the quantum level. If 2- valued logic decided empirically (based on common sense perception of the world) does not fit into quantum theory, that is fine. It doesn't have to. That does not in any way prove his conclusion that logic is decided culturally. Further in putting forth such an argument, he is again using a 2-valued logic to substantiate an argument against 2-valued logic. This is 'totally nutty'.

3. Raju mentions elsewhere that notions of infinity are tied to notions of eternity. Absurd again. The notion of infinity has nothing to do with time, and hence, eternity. One can convince a student of 8th grade about the notion of infinity by asking him to give the largest integer. If he comes up with one, you could ask him to add one more and get a larger integer. Where is time coming into the picture here? In fact, Euclid in his 'Elements', gives the proof of the infinitude of primes by assuming the existence of a largest prime and coming up with another prime bigger than that, proof, which an intelligent high school student can understand. Euclid existed before Christ and Christianity was born, to which Raju attributes the notion of eternity!

4) Elsewhere Raju also mentions the associative law of addition of numbers, in Mathematics, criticizing its acceptance since it involves an infinite process (adding infinite decimals). The foundations of real number system is made by the correspondence between real numbers and points on the line. Thus adding 3 real numbers is equivalent to adding 3 lengths which can be added in any order. This is the basis of the associative law. It does not involve any infinite process. In fact the completeness of the real number system is established by such correspondence (For more on this please refer Hardy's 'Pure Mathematics' Chapter1).

Also, Calculus does not depend on Physics (and hence on the notions of time and eternity). As is well known, It can be developed based on geometry. In fact, very rudimentary traces of calculus lie in the 'Method of Exhaustion' developed by Archimedes, dating back to 3 centuries before Christ.

Having established the existence of Euclid and his authorship of the 'Elements' it is not necessary to dwell any further into Raju's erroneous arguments, But there is one more point I would like to make about the mystery geometry of Egypt to which he attributes the 'Elements'.

Egyptian Mathematics (including the mystery Mathematics) is computational in nature, sometimes even involving brute-force computations.There is not a trace of the type of deductive-proof Mathematics found in Egyptian Mathematics(including the mystery mathematics) as is carried out in the 'Elements'.To attribute the 'Elements' to Egyptians is indicative of a lack of proper understanding of ancient Egyptian Mathematics and in fact Mathematics as a whole.

Now coming to the class room, both the deductive and empirical should be emphasized in the class room. I do not know the situation in India since I am in a faraway land. I feel the empirical and the computational aspects of Mathematics should be combined with deductive proof to give a more comprehensive view of Mathematics. Euclid should be more emphasized in schools than is currently done, since the goal is not just computation, but efficient computation. For this, more stress on deductive proof based mathematics (theorem – proving) will provide students with the type of training needed for doing efficient computation and devising new methods of efficient computation.

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