Negating Four Color Theorem with Neutrosophy and Quad-stage Method

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Abstract: With the help of Neutrosophy and Quad-stage Method, the proof for negation of “the four color theorem” is given. In which the key issue is to consider the color of the boundary, thus “the two color theorem” and “the five color theorem” are derived to replace "the four color theorem".

Key words: The four color theorem, neutrosophy, quad-stage, boundary, proof for negation, the two color theorem, the five color theorem

1 Introduction

The four color theorem states that, given any separation of a plane into contiguous regions, producing a figure called a map, no more than four colors are required to color the regions of the map so that no two adjacent regions have the same color. Two regions are called adjacent if they share a common boundary that is not a corner, where corners are the points shared by three or more regions.

In 1976, Kenneth Appel and Wolfgang Haken published their proof of the four color theorem. It was the first major theorem to be proved using a computer.

Accordingly, this paper starts with the assumption that, in the case without considering the color of the boundary, "the four color theorem" is correct.

Figure 1 is an example of four-color map.

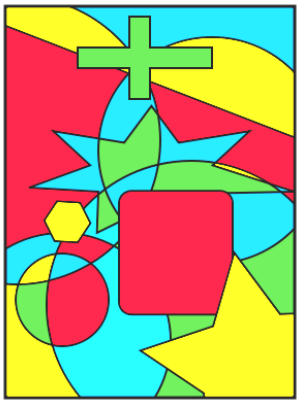

Figure 1 An example of four-color map

However, whether or not the color of the boundary should be considered? We believe that it should be taken into account.

In this paper, with the help of Neutrosophy and Quad-stage Method, the proof for negation of “the four color theorem” is given. In which the key issue is to consider the color of the boundary

2 Basic Contents of Neutrosophy

Neutrosophy is proposed by Prof. Florentin Smarandache in 1995.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <Anti-A> and the spectrum of "neutralities" <Neut-A> (i.e. notions or ideas located between the two extremes, supporting neither <A> nor <Anti-A>). The <Neut-A> and <Anti-A> ideas together are referred to as <Non-A>.

Neutrosophy is the base of neutrosophic logic, neutrosophic set, neutrosophic probability and statistics used in engineering applications (especially for software and information fusion), medicine, military, cybernetics, and physics.

Neutrosophic Logic is a general framework for unification of many existing logics, such as fuzzy logic (especially intuitionistic fuzzy logic), paraconsistent logic, intuitionistic logic, etc. The main idea of NL is to characterize each logical statement in a 3D Neutrosophic Space, where each dimension of the space represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of]-0, 1+[without necessarily connection between them.

More information about Neutrosophy can be found in references [1，2].

3 Basic Contents of Quad-stage

Quad-stage (Four stages) is presented in reference [3], it is the expansion of Hegel's triad-stage (triad thesis, antithesis, synthesis of development). The four stages are "general theses", "general antitheses", "the most important and the most complicated universal relations", and "general syntheses". They can be stated as follows.

The first stage, for the beginning of development (thesis), the thesis should be widely, deeply, carefully and repeatedly contacted, explored, analyzed, perfected and so on; this is the stage of general theses. It should be noted that, here the thesis will be evolved into two or three, even more theses step by step. In addition, if in other stage we find that the first stage's work is not yet completed, then we may come back to do some additional work for the first stage.

The second stage, for the appearance of opposite (antithesis), the antithesis should be also widely, deeply, carefully and repeatedly contacted, explored, analyzed, perfected and so on; this is the stage of general antitheses. It should be also noted that, here the antithesis will be evolved into two or three, even more antitheses step by step.

The third stage is the one that the most important and the most complicated universal relations, namely the seedtime inherited from the past and carried on for the future. Its purpose is to establish the universal relations in the widest scope. This widest scope contains all the regions related and non-related to the "general theses", "general antitheses", and the like. This stage's foundational works are to contact, grasp, discover, dig, and even create the opportunities, pieces of information, and so on as many as possible. The degree of the universal relations may be different, theoretically its upper limit is to connect all the existences, pieces of information and so on related to matters, spirits and so on in the universe; for the cases such as to create science fiction, even may connect all the existences, pieces of information and so on in the virtual world. Obviously, this stage provides all possibilities to fully use the complete achievements of nature and society, as well as all the humanity's wisdoms in the past, present and future. Therefore this stage is shortened as "universal relations" (for other stages, the universal relations are

also existed, but their importance and complexity cannot be compared with the ones in this stage).

The fourth stage, to carry on the unification and synthesis regarding various opposites and the suitable pieces of information, factors, and so on; and reach one or more results which are the best or agreed with some conditions; this is the stage of “general syntheses”. The results of this stage are called “synthesized second generation theses”, all or partial of them may become the beginning of the next quad-stage.

4 Negating the Four Color Theorem

The combination of Neutrosophy and Quad-stage is very useful for innovations in areas of science, technology and the like. For example, in reference [4], we expand Newton mechanics with Neutrosophy and Quad-stage method, and present New Newton Mechanics taking law of conservation of energy as unique source law.

The process of negating "the four color theorem" with Neutrosophy and Quad-stage method, and deriving “the two color theorem” and “the five color theorem” to replace "the four color theorem", can be divided into four stages.

The first stage (stage of “general theses”), for the beginning of development, the thesis (namely "the four color theorem") should be widely, deeply, carefully and repeatedly contacted, explored, analyzed, perfected and so on.

About these aspects, especially the brilliant accomplishments of proving "the four color theorem" with computer, many discussions could be found in related literatures, therefore we will not repeat them here, while the only topic we should discuss is finding the shortcomings in the existing proofs of "the four color theorem". In fact, many scholars believe that the existing proofs of "the four color theorem" with computer are not satisfactory. Therefore, many new proofs are still appeared unceasingly. For example, a very simple proof of this theorem was given in reference [5] recently.

For the different proofs of "the four color theorem", we can name the results as "the Appel-Haken’s four color theorem", "the Chen Jianguo’s four color theorem", and the like.

In addition, some experts still ask the question that whether or not "the four color theorem" is correct.

On other viewpoints about "the four color theorem", we will discuss them in detail below, in order to avoid duplication.

The second stage (the stage of “general antitheses”), the opposites (antitheses) should be discussed carefully.

For "the four color theorem", there are many opposites (antitheses). For example: "the two color theorem", "the three color theorem", "the five color theorem", "the six color theorem", and so on. As "the four color theorem" is denied, we will select the suitable one from these theorems.

The third stage is the one of the most important and the most complicated universal relations. The purpose of this provision stage is to establish the universal relations in the widest scope.

To link and combine with Neutrosophy, we will take into account the intermediate part; on a map, what is the "intermediate part"? After careful analyses, we can identify two main types of "intermediate part". The first one is already considered in the existing proofs of

"the four color theorem", such as the third region between two regions. The second one is not considered in the existing proofs of "the four color theorem", such as the boundary between two regions. Obviously, so far the color of boundary is not considered in any existing proof also. However, whether or not the boundary and its color should be considered" We believe that they should be taken into account, because the boundary is the objective reality on a map.

The fourth stage (the stage of "general syntheses"), our purpose is to negate "the four color theorem", and reach the results that are the best or agreed with some conditions.

Firstly, we suppose that, in the case without considering the color of the boundary, "the four color theorem" is correct. In other words, in this case, "the Appel-Haken's four color theorem", "the Chen Jianguo's four color theorem", and the like, are all correct.

Secondly, we introduce the concept of "boundary part" (or "totality of boundary"). Because the boundary on a map has a certain width, the "boundary part" (or "totality of boundary") can be defined as: a special connected region constituted by all the boundaries.

After considering the boundary and its color, there are two situations should be considered. The first one is that, the original color distribution (namely the color distribution on a map that is agreed with the principle of "the four color theorem"; or the color distribution before considering the boundary) can be changed. The second one is that, the original color distribution cannot be changed.

For the first situation, only two colors will be sufficient: one color could be used for all the boundaries; another color for all the regions. For example, on a country's black and white map, the color of all the boundaries of states or provinces could be black; while the color of all the states or provinces could be white. Obviously, this is also the general drawing method for the black and white map. At this time, "the four color theorem" is replaced by "the two color theorem".

Figure 2 is a black and white world map.

Figure 2 A black and white world map

For the second situation, because the original color distribution cannot be changed (four colors are required), the fifth color is required. Otherwise, supposing that the color of the boundaries is one of the four colors for regions, for the reason that the boundary has a

certain width and can be considered as the special region, therefore the color of this special region will be the same color as at least one ordinary region, thereby the principle of color distribution will be violated (the colors of the adjacent two regions will be the same). Obviously, in this case five colors are required. For example, in Figure 1, the colors for different regions are red, yellow, green and blue respectively, while the color for boundaries is black.

In Figure 1, the number of the required colors is as follows

$$4+1=5$$

At this time, "the four color theorem" is replaced by "the five color theorem"..

Thus, we already prove that, as considering the boundary and its color, "the four color theorem" is incorrect.

5 Conclusions

As considering the boundary and its color, if the original color distribution (namely the color distribution on a map that is agreed with the principle of "the four color theorem"； or the color distribution before considering the boundary) can be changed, "the four color theorem" is replaced by "the two color theorem"; if the original color distribution cannot be changed, "the four color theorem" is replaced by "the five color theorem".

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