"Hidden" Parameters Describing Internal Motion Within Extended Particle Elements

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Recent attempts to consider isolated particles and real constitutive wave elements as localized, extended spacetime structures (i.e., moving within time-like hypertubes or M-Theoretic higher dimensional (HD) brane topologies) are developed within a causal extension of the Feynman-Gell-Mann electron model. These extended structures contain real internal motions, (i.e., internal hidden parameters) locally correlated with the "hidden parameters" describing the local collective motions of the corresponding pilot-waves. Recent experimental evidence is briefly discussed.

1. Introduction

Recent developments in the causal stochastic interpretation of Quantum Mechanics are presented to aid interpretation of new observations in Electromagnetic Theory (EM) associated with O(3) invariance and photon mass, $m_\gamma$ [1-5]. These "hidden" parameters describe internal motion within extended particle elements associated with a Feynman-Gell-Mann type causal electron model. They are related in this work to an extended version of the causal stochastic interpretation of electron theory based on the introduction of real internal spinning motions within the particles, and guiding pilot-wave constitutive elements. This procedure can be interpreted as a local correlation between these new internal motions and the "hidden parameters" describing the collective external pilot-wave motions already introduced to represent the Feynman-Gell-Mann pilot-wave motion [6].

This attempt to re-examine electron theory in the causal interpretation of Quantum Theory in terms of new internal and external motions is justified by the set of problems and questions left open after the astonishing success of QED predictions. Here we mention only:

- The problem of the electron’s size (i.e. the discrepancy between its Compton radius $R_C \approx 10^{-16}$ cm and the point like behavior $(R_e$ (charge)$<< R_C$) of its EM charge in high energy EM scattering (tied to the question of EM divergence);
- The problem of the nature of the electron's spin, of its EM self-interaction and the interpretation of its magnetic moment.
- The problem of the contribution of its charge to its mass;
- The interpretation of its anomalous magnetic moment and unknown origin or the Poincaré forces, which prevent the expansion or its charge distribution.

This introduction rests on an extension of Maxwell’s Theory of light to interpret recently observed phenomena [7-9]. It is based on Dirac’s suggestion [10] that the vacuum is a real physical medium built of a covariant polarized distribution of electromagnetic waves which carry excited linear Maxwellian and nonlinear soliton type photon waves ("piloted" by linear waves) [11]. If this is true one can introduce

- Nonzero electric field divergence and nonzero electron conductivity in vacuo tied to nonzero photon mass [1-5] corresponding to a non-expanding universe cosmology [4,12].
- New extended charged particle (electrons and photons) models built with point-like EM charges rotating around a center of mass [4,13-15] as discussed here (see Fig. 1).
Since in this model the pilot wave and the piloted particle are composed of extended elements (cores), we start with the assumption that each individual element moves within a time-like hypertube which recovers all internal and external interacting fields. As one knows, this implies the existence of a covariantly defined centre of mass, \( Y_\mu(\theta) \) where \( \theta \) is the proper time along \( Y_\mu \)'s path. Its internal mass distribution can be assumed to be contained within a relativistic spinning sphere (in \( Y_\mu \)'s rest frame, \( \Pi_0 \)) of radius \( R \) around an axis of rotation centered on \( Y_\mu \) with a moment of inertia \( I = \frac{1}{2}mR^2 \) in such a way that its equator rotates with a velocity \( \approx c \) in \( \Pi_0 \). This spherical mass distribution can be assumed to behave, for all practical purposes, like a rigid mass distribution that an external force applied to it can be separated into two components. i.e., a) a translational force on \( Y_\mu \) b) torques around \( Y_\mu \) and \( X_\mu \).

We present this model as follows. In the first part we analyze the internal motions of the free extended elements, which constitute the building blocks of the pilot wave and particle aspect of individual isolated electrons. This analysis implies the introduction of new internal variables (including their individual center of mass and charge) describing these (unobserved) internal motions: a procedure comparable to the introduction of the internal quantum potential and spin-orbit coupling. As we shall see it is possible to interpret within this model.

2. Internal Motions of Particle Individual Extended Elements in Terms of Causal Collective Behavior

If an individual electron is described as a real wave, \( \Psi \) comprising extended elements which can be analyzed in terms of collective motions propagating on a covariant stochastic subquantum Dirac type aether, and if these collective motions can be analyzed in terms of average drift motions within time-like hypertubes (2-branes) combined with stochastic random path perturbations (like molecules in a gas), then we can introduce at each point, \( Y_\mu \) a scalar density, \( \rho(Y_\mu) \) of these extended elements and the internal parameters, \( \lambda \) yield an average value \( \langle A \rangle \) at \( Y_\mu(\theta) \) where \( \theta \) defines the proper time along the average drift path followed by the condensed density, \( \rho(d\rho / d\theta = 0) \) within the collective motion. If the collective motions contain a non-dispersive soliton-like
particle like conserved density concentration, \( \rho(\theta) \) tied to nonlinear terms in the wave’s equation, the \( \rho \)'s will follow an average drift line (plus random fluctuations of course) so that the linear part of the \( \Psi \) field can be considered as a pilot wave. The model implies that the average individual extended element’s internal parameters are related to known electron properties, so that the following description of free extended wave (and particle) elements resemble a classical extended electron model proposed by MacGregor [13], Mckinnon [20] Ignatovich [21] and Vigier [6].

The starting point in this model is that each basic constitutive electron element contains a rotating point-like charge \( e \) within an extended structure (as initially suggested by Yukawa) and that this charge (centered at \( X_\mu \)) undergoes a helicoidal motion of constant radius \( R = \hbar / mc \) around \( Y_\mu \) (in \( \Pi_0 \)) so that we can write (in \( \Pi_0 \)) \( R_\mu = Y_\mu - X_\mu \) and \( R_\mu (dY_\mu / d\theta) = R_\mu (dX_\mu / d\tau) = 0 \), since there is a constant central force between \( Y_\mu \) and \( X_\mu \). We can also assume (following Faraday, et al. [6,22]) that its magnetic field contains two parts. The first external part is incorporated into the moving mass energy, \( \delta m_0 c^2 \) of the point-like charged part of the core. The second part, which does not rotate with it (according to Faraday’s experiments [6,22]) corresponds in Maxwell theory to a magnetic moment \( \mu = e\hbar / mc \cong eR / r \). The corresponding magnetic self-energy, \( W'' \) carried along by the point-like charge, can thus be treated as a self-inductance resulting from the current generated by our point-like electric charge so that we can write

\[
i = \left( \frac{e}{c} \right) \frac{\omega}{2\pi} = e \frac{\omega}{2\pi R}, \tag{1}\]

with the magnetic moment \( \mu = \pi R^2 \cdot i \). The corresponding magnetic self-energy, \( W'' \) then becomes

\[
W'' = \frac{1}{2} L \cdot i^2 = \frac{1}{2} L \left( e \frac{\omega}{2\pi R} \right)^2 \tag{2}
\]

which yields (since \( L = 4\pi R \) and \( i \cong e / 2\pi R \))

\[
W'' = \frac{e^2}{2\pi R} = \frac{\alpha}{2\pi} mc^2 \tag{3}
\]

where \( m \) denotes the total mass.

Expression (3) also results from the relation, \( w = v / R = c / R \), with

\[
i = \left( \frac{e}{c} \right) \left( \frac{v}{2\pi R} \right) \cong \frac{e}{2\pi R} \]

\[
\mu = \left( \frac{eR}{2} \right) \left( \frac{v}{c} \right) \cong \frac{eR}{2} = \frac{e\hbar}{2mc} \tag{4}\]

Indeed, \( W'' \) can also be considered the interaction of its non-rotating magnetic moment, \( \mu \) with the field (magnetic moment) corresponding to a magnetic radius, \( R_{\mu} \). As shown by Born and Schrödinger, we get [23]

\[
W'' \cong \frac{2\mu^2}{3R_{\mu}^3} \tag{5}
\]

The Einstein-de Broglie particle relation \( E = mc^2 = \hbar \nu \) follows immediately for single individual elements. Indeed, since we have \( \lambda \nu = c \), one rotation of \( X_\mu \) around \( Y_\mu \) so that \( \lambda = 2\pi R_\mu = \hbar / mc \) and \( c = \lambda \cdot \nu \), the corresponding angular momentum is thus \( 2\pi R \cdot mc = \hbar \), which yields
\( R_s = h / mc. \)

Since there is a central constant force between \( X_e \) and \( Y_e \) we can also define an internal spinning motion of the elements of the system within their time-like boundaries by their angular momentum tensors, \( S_{ij} \). Following MacGregor [13] these properties can be visualized by assuming that the cores and soliton electrons behave like rigid relativistic bodies in the sense:

- That all pairs of its internal extended elements are separated by constant space-like relativistic intervals during their motion;
- That if one characterizes each internal point-like internal element by a coordinate, \( z_{\mu} \) in the rest inertial frame \( \Sigma_0 \) of \( X_{\mu} (\dot{x} = 0) \) the particle (i.e. \( z_{\mu} = X_{\mu} \)) rotates twice around \( X_e \) when \( X_e \) undergoes one rotation around \( Y_e \) according to Dirac's analysis [24];
- That one can define two different radii related to different types of fields, i.e., 1) a radius \( R \) around \( Y_e \) which contains all material (charged and uncharged) elements, charged and neutral field sources within the hypertube, but is smaller than the EM self-field's extension; and 2) a radius \( R_E \ll R_e \) centered on \( X_{\mu} (\tau) \) which contains charged elements, i.e. sources of the self-electromagnetic fields.

This implies two evident physical consequences. One needs two radii for each extended element since one has two source distributions, i.e. one small radius, \( R_E \) for the charge distribution around \( X_{\mu} \), and one Compton-like radius, \( R_e \ll R_E \) for all the neutral electron elements since the extended electron contains point-like sources and fields.

Since \( X_{\mu} \) is surrounded by a moving electromagnetic field, the magnetic Faraday field's energy distribution moves with \( X_{\mu} (\tau) \) and carries self-energy. The charged sub-elements (which move with a velocity, \( \approx c \)) repel but are held together by the magnetic pinch forces resulting from their velocity (a Tokamak-like behavior) and the magnetic self-field does not rotate around \( X_{\mu} \), according to Maxwell's theory. The representation of the corresponding electromagnetic contribution to the charged part's total mass, \( \Delta m \) is a longitudinal vector potential, \( A_{\mu} \) and one must add to it the usual transverse potential contribution \( A_{\mu} \) emitted as a consequence of \( X_{\mu} \)'s acceleration in its orbital and spinning motion around \( Y_{\mu} \). The usual electromagnetic contributions to the core's energy \( W_E \) and \( W_H \) can be represented by \( W_E = 0. \) Since \( W_{H} \) is only 0.1% of the total energy \( mc^2 \), this total mass is essentially of gravitational origin associated with the internal orbital spinning motions of the electron.

If one thus assumes, as results from extended charge particle models, that a core (i.e. an electron's total mass with \( m = 0.511 \text{ MeV} \)) is the sum, in any given inertial frame, of the contributions of its various moving internal parts. For example, \( \tau \) in the rest frame, \( \Pi_0 \), of \( Y_{\mu} (0) \) one should add the contribution of the rigid rotating electrically charged core (spin) which contains the total charge \( e \) and radius \( R_E \) which rotates locally around \( X_{\mu} \) to the angular velocity of the orbital motion of \( X_{\mu} \) around \( Y_{\mu} \). The spin vector, \( S_{\mu} \) located at \( X_{\mu} (\tau) \) is in general not parallel to the axis of rotation (centered at \( Y_{\mu} \)) of the orbital circular motion of \( X_{\mu} \) around \( Y_{\mu} \). In other words, the charged core behaves like a spinning rotating plane around \( X_{\mu} (\tau). \) As we shall now show, their angle is determined by the relativistic conservation laws.

As one knows in the case of a spinning motion around an axis with an equatorial velocity, \( c(\omega = c / R) \), the relativistic spinning mass, \( M_{s} \), is related to the rest frame mass by the relation

\[
M_s = \frac{3}{2} m
\]

so that writing as usual \( I = (M_s R^2) / 2 \) we get \( I = (3 / 4)mR^2 = (1 / 2)M_s R^2 \) where \( M \) is the rotating part of the rest mass.

If we then define the spin angular momentum of the relativistic spinning sphere by
and introduce the "spinning mass Compton radius", \( R_c = \hbar / M \cdot c \) we obtain

\[ J = \frac{1}{2} \hbar, \]

so that this contribution yields \( m_s = (3/2)m \) (s, a = spinning, non-spinning)

\[ I = \frac{1}{2} m_s R^2, \quad \left( R_\mu = Y_\mu - X_\mu \right) \]

with \( \omega \equiv c / R \) and \( R = \hbar / M \cdot c \),

\[ J = I \omega = \frac{1}{2} m_s R \cdot c = \frac{\hbar}{2} \]

where \( \hbar / 2 \) is the projection of the spin on the z-axis centered on \( Y_\mu \). We can now calculate the mass-energy contribution of the moving charge and associated moving EM Maxwellian fields and the corresponding \( g \) factor. As one knows, if one denotes by \( v \) the velocity of \( Y_\mu \) (i.e. \( \lambda = 1 / (1 - v^2 / c^2) \)) one has in the associated inertial frame, \( \Sigma_{lab} \)

\[ m_{(lab)} = \gamma m_{(em)} \]
\[ J_{(lab)} = J_{(em)} \]
\[ M_{(lab)} = M_{(em)} \gamma \]
\[ \mathcal{g}_{(lab)} = \frac{\mathcal{g}_{(em)}}{\gamma} \]

where \( M \) represents the electron's and core's magnetic moment. This is not enough, however, since we know from our spin- \( \frac{1}{2} \) model, [13] that one has

\[ J = \frac{1}{2} \left( 1 + \frac{1}{2} \right)^{1/2} \cdot \hbar = \frac{\sqrt{3}}{e} \hbar \]

so that one should write, \( R = \sqrt{3} \cdot \hbar / mc \) since relativity theory yields, \( J = (1/2)mRc \), i.e. increases \( R \) and \( J \) by the factor \( \sqrt{3} \).

If we now recall that the spin axis \( z \) (centered on \( X_\mu \)) is not parallel to the axis (centered on \( Y_\mu \)) perpendicular to the equatorial plane of the motion of \( X_\mu \), this implies that the charge's motion generates a dipole with total magnetic moment \( \sqrt{3}eh / 2mc \) (along with a \( z \) component \( eh / 2mc \)) so that the magnetic moment which corresponds to this current loop is

\[ \mu = \frac{e}{2} R_\phi = \sqrt{3} \frac{eh}{2mc} \]

associated with the increased radius volume, \( R_\phi = \sqrt{3}R \).

The associated gyromagnetic ratio of the electron thus becomes
and the angle between the two axes of rotation (centered on $Y_{\mu}$ and $X_{\mu}$) corresponds to the value $	heta = \pm \arctan\left(\frac{1}{\sqrt{3}}\right) = \pm 54.70^\circ$. This is to be expected, since it has been shown that the corresponding quadrupole moment vanishes in that case, so that angular energy-momentum conservation, as confirmed by experiment, and the central force between $Y_{\mu}$ and $X_{\mu}$, are automatically preserved.

In the preceding calculations of $m_s$, we have left aside the contributions to the rotating mass (energy) of the electromagnetic fields generated by the dipole motion. Denoting by $W_E$ and $W_M$ their contributions, we see that one should take $W_E \cong 0 = \text{constant in this model.}$ Indeed, as a consequence of Maxwell's theory, Feynman's calculations and Faraday's experiments, we see that the Coulomb electric field around the charged core does not rotate, so that it does not contribute to $m_s$. The situation is different for $W_M$. Experiments have shown since Fermi's first experiments [24] the electron's magnetic field structures were much larger ($R_H \gg R_E$) than its electric charge distribution.

We also find that the value $g = 2$ was not quite exact; and that the value $M = \frac{e\hbar}{2mc}$, where $m$ is the observed electron mass, was a bit too small. Evidently this result can be interpreted in our model since, following Faraday [22], all the EM energy of the free electron does not rotate, and one should write

$$m_s = m_0 - \Delta m$$

(15)

where $\Delta m = m(\alpha / 2\pi)$ which according to QED [24] represents the non-rotating part of the internal electron energy. Which yields

$$\mu = \frac{e\hbar}{2mc} \left(1 + \frac{\alpha}{2\pi}\right)$$

(16)

and $g = \left(\frac{2mc}{e}\right)(\mu / J)$ so that

$$g - 2 = \frac{2\alpha}{e\pi}$$

(17)

and we have

$$\begin{cases}
W_H = 593\text{eV} \\
 m_s = m \left(1 - \frac{\alpha}{2\pi}\right) = 0.51041 \text{MeV} \\
 R = \sqrt{3} \cdot \frac{\hbar}{mc} \left(1 - \frac{\alpha}{2\pi}\right) = 6.6962 \times 10^{-11}\text{cm} \\
 g = 2 \times 1.001159652193
\end{cases}$$

(18)

In this model the electron has a very small charge radius $R_e \ll 10^{-16}$, an extended rotating charge and a mass and EM field distribution (around $Y_{\mu}$) with $\tilde{s} = \hbar$ where the centre of charge $X_{\mu}$ has a velocity $\approx c$.

### 3. Different Moving Mass and Electromagnetic Energy-Momentum Distributions in Individual Extended Cores

Within the classical and relativistic theory, the transition from point-like elements (associated arbitrarily with $\Psi$ waves endowed with mass, EM charge, spin, etc.) to extended elements into the hydrodynamical description of field behavior has evident qualitative consequences. The corresponding Lagrangian and Hamiltonian formalism now contains two types of variables associated
With the internal elements' motions located at any given point, and with the average collective motion of these elements around the said point, which correspond (i.e. react differently) to the local and external interaction around this point.

In other words, a description of a fluid recovers the description of its individual internal motions and the description of its waves' collective motions, described in terms of different internal parameters.

To clarify the consequences of this point, introduced some time ago in the literature [17] let us first briefly recall the extremely simple case of a relativistic fluid built with rotating rigid spheres of rest mass \( M_0 \), radius \( R \) and spinning around an axis with equators moving very close to the velocity of light, \( c \). As a consequence of this rotation, the relativistic spinning mass, \( M_s \) is related to \( M_0 \) by the relation \( M_s = (3/2) M_0 \) and the internal measured density of the mass remains constant. Also as a consequence, the relativistic moment of inertia, \( I \) becomes larger than the corresponding non-relativistic moment of inertia, \( I_c = (2/5) M_0 R^2 \) and becomes \( I = (1/2) M_s R^2 \) due to the increase of mass at a distance from the axis of rotation. The spin angular momentum of our relativistic spinning sphere then becomes [13]

\[
\hat{J} = I \hat{\omega}
\]  

where \( \hat{\omega} \) represents the angular velocity, which satisfies the relation in our model. We thus get

\[
J = \frac{1}{2} M_s \cdot R c = \frac{1}{2} \hbar.
\]

If we consider angular momentum seen from an external point just outside the element's equator - an expression which implies that all diameter points external or on the equator satisfy, with respect to its centre, \( O \), a relation similar to the usual Heisenberg equations for each value of \( r < R \). This description of an extended relativistic rotating massive sphere does not include electromagnetic charge. This model is thus insufficient since it does not apply to electron theory and corresponds to a massive neutrino if one assumes that it is held together by gravitational interactions [25].

If there is equilibrium between the centrifugal force and the attractive gravitational force along \( Y_\mu \) and \( X_\mu \) relation (19) yields a new realistic interpretation of the physical nature of Planck's constant [4] which is now related to the angular momentum of our model, which only depends on the \( R \cdot M_s \) product, a property which can be experimentally tested.

An extension of this chargeless model to an interpretation of electron motion has been proposed by Mac Gregor [13]. One adds to the model a very small localized distribution of charged matter on a core's equator, i.e., of total mass, \( \delta m \) carrying a charge \( e \) and radius \( R_E \ll A \), carried with a velocity \( c \), so that the whole model rotates as a block in the rest frame of \( O \). This pointlike charge distribution is the source of electric and magnetic self-fields (denoted \( E \) and \( H \)) influenced by external EM fields and held together by its own self-fields (since it behaves qualitatively like a Tokomak current pinched by its own magnetic field) with negligible electric self-energy \( W_E \) and small magnetic self-energy, \( W_H \). If \( \delta m \) is small this rigid model has the remarkable property that the total observed rotating spin, \( \hat{J} = \hbar / 2 \) (mass and charge) around \( Y_\mu \) and the electromagnetic spin (tied to \( X_\mu \)) are equal in the rest frame of the charged core (which practically coincides with the point \( O \)) as a consequence of the core's rigidity if \( \delta m \) is small enough.

Two physical consequences follow immediately from this model:

- The charge spherical distribution in its own rest frame is practically flattened into a very small disk in \( Y_\mu \)'s rest frame and the EM spin, \( \delta \hat{J} \) is tangent to the \( X_\mu \) velocity since the velocity is \( = c \) in the present model. In other words, the extended electron charged model recalls Bohr's original hydrogen model where the proton-electron Coulomb attraction is replaced by a \( Y_\mu - X_\mu \) \( m_e - \delta m \) gravitational interaction as the charge has to rotate twice on itself (following Dirac's argument) in order to recover its external EM distribution;
- If the mass and electric distributions belong to a single rigid material block, then there is a unique spin...
orientation in space-time. If we denote by \( \int_{X_{\mu}}^{(m)} \) the core's material mass angular momentum with radius in the rest frame of the mass center, \( Y_{\mu} \), a Lorentz transform will give its value and orientation at \( X_{\mu} \).

At this stage we consider the physical reasons for the real spin axis orientation from the observed orientation, \( J_{z} \) with \( \left( J = \hbar / 2 \right) \) in an external inertial frame. As one knows, an equatorial loop current produces in general observable multipolar electric effects, since its real rotation axis is not parallel in general to the axis observed in the experiments. Now one knows that in relativity theory the separate conservation in motion of angular momentum is only possible within a central field of forces. Since this is the case for our model, if we assume that the real interactions associated with measurement processes do not modify the magnitude of internal spin \( J \) which corresponds to values, \( J_{x}, J_{y}, J_{z} \) in the rest frame \( Y_{\mu} \) of the centre of charge, \( X_{\mu} \) with the Pauli matrices (so that \( J = (J_{x} + J_{y} + J_{z}) \cdot (\hbar / 2) \) we see 1) that \( J = (\sqrt{3} / 2) \hbar \) and 2) that the model has an effective vanishing electric quadrupole moment which is zero along \( Oz \) and vanishes along \( Ox \) and \( Oy \) (in \( Y_{\mu} \)) when averaged over a closed cycle (in \( Y_{\mu} \)) of precessional motion, which corresponds, in the rest frame of \( Y_{\mu} \) to two rotations of \( X_{\mu} \) around \( Y_{\mu} \) so that \( E = mc^{2} = \hbar \nu \). This implies of course that Dirac's analysis, corresponding during the motion to the non-crisscrossing Faraday lines of force centered on \( X_{\mu} \) now appear, in this model, as a consequence of central gravitational forces between \( Y_{\mu} \) and \( X_{\mu} \).

This also implies, as shown by MacGregor [13], that the forces associated with a \( J \) (or magnetic moment) of a charged particle, when combined with the internal central forces of the model, a reorientation of the core's real physical orientation in space - so that the angle between the real rotation axis \( \vec{J} \) in \( Y_{\mu} \) and the measured \( \vec{J}_{z} \) axis (with \( J_{z} = \hbar / 2 \) takes the value \( \Theta = \arccos(1 / \sqrt{3}) = 5.7 \). The model yields a direct interpretation of the gyromagnetic \( g \) factor with \( \alpha = e^{2} / h \). For the spin and the radius, one must distinguish for the same real physical reasons between the observed and real intrinsic qualities in that case.

The preceding physical interpretation (justification) by each individual core element of QED predictions implies some interesting consequences, i.e.:

a) The proposal that the internal charge core of the electron undergoes internal oscillations equivalent to the presence of an internal electron current, implies that Planck's constant \( \hbar \), initially discovered as a consequence of the collective behavior of black-body radiation, is in reality a constant related to the electron's internal charged core rotation (the original Stoney [4,26]). Its constancy can be shown to result from the self-electromagnetic fields [27].

b) The existence of stable internal oscillations is evident in this model. Following Maxwell, the charged core's oscillations imply accelerations. As the core accelerates, it must, by Ampere's law, build up a magnetic field. That build-up, by Faraday's law, will induce an electric field, whose direction, by Lenz's law, is opposed to the acceleration, so that its acceleration is the cause of its deceleration, which will reduce the magnetic field and induce a Faraday electric field since this process accelerates the core again.

This explains the core's internal oscillations. As discovered by Beckmann [28] if the frequency of the velocity of oscillation is \( \nu \) and the average velocity \( \bar{\nu} \) (about which the velocity fluctuates) and if the distance measured along the paths of \( Y_{\mu} \) between the points at which the electron attains successive maxima of its fluctuating velocity is \( \lambda \), one sees by elementary kinematics that we have the relation

\[
\nu = \nu \lambda
\]  

(21)

where \( \lambda \) is the length associated with one revolution of \( X_{\mu} \) around \( Y_{\mu} \).

The determination of the extend internal core's distribution of electric charge, \( \rho(x) \) and the possible forms of the corresponding self-induced electrostatic field in the frame of \( X_{\mu} (\tau) \) have been discussed recently [29]. Assuming that \( \varepsilon \) and \( \varepsilon_{0} \) represent the permittivity of the medium inside and outside the rigid (i.e. static) core in the rotating rigid frame \( \beta_{\mu} (r \leq R_{E} \text{ inside}) \) we have \( \phi(x) = Q / 4\pi\varepsilon_{0} \cdot |x| \) where \( Q \) is the core's total charge
and \( x \leq R_E \). Assuming \( \varepsilon_0 \Delta \phi(x) = 0 \) for \( |x| \geq R_E \) and \( \varepsilon \Delta \phi(x) = -\rho[\phi(x)] \) for \( |x| \leq R_E \) we get by writing \( \varepsilon = A / k^2 \) (\( k^2 \) being a real number) the total charge in the form

\[
Q = \int_{|x| \leq R_E} \Omega(x)\phi(x)d^3x
\]  

(22)

with \( Q_v = 4\pi r^2 D_\mu (r) = Q \) for \( r \geq R_E \). The corresponding electrostatic energy of the self-induced fields is

\[
W_E = \frac{1}{2} \int_{x \leq R_E} \Omega(x)\phi^2(x)d^3x
\]  

(23)

with an associated mass \( M \) given by

\[
\int_{x \leq R_E} \Omega(x)\phi^2(x)d^3x = 2Mc^2.
\]  

(24)

It has been shown that the continuity of the values of \( \phi \) for the value \( r = R_E \) implies that

\[
Q = 4c\varepsilon_0 \sqrt{\frac{\pi M}{AR_E}} \sin(kr)
\]  

(25)

which yields

\[
\phi(r) = \frac{e}{4\pi\varepsilon_0} \frac{\sin\left(\pi r / 2R_E\right)}{r}
\]  

(26)

so that if we take into account the oscillation of \( X_\mu \) around \( Y_\mu \) then \( R / R_E \geq 10^7 \).

This model implies that the extended electron's constitutive elements contain two different types of internal distributions:

- An extended charge distribution, i.e. a charged core centered on \( Y_\mu (r) \) with a small radius \( R_E \) moving with a velocity \( = c \) along an equator surrounded by an electromagnetic field which carries energy momentum and a mass, \( \sim 0.01 \text{ m} \); and
- An extended uncharged matter distribution with an energy-momentum distribution centered on \( Y_\mu (\theta) \) with a larger radius \( R \approx 0.1 \text{ cm} \) and a mass \( \approx 0.99 \text{ m} \) with \( m = 510.406e\text{V} = m_{\text{observer}} (1 - \alpha / 2\pi) \).

As discussed above both distributions are spinning, and as shown by Mac Gregor [13], at different angular velocities can be treated as "rigid" in the relativistic sense of the term. As one knows, an external force applied to this type of rigid body can be separated into two components, i.e.:

- A translational force that acts through the mass center;
- Torques that act through the charge and mass center;

from which one can predict the existence of a helical channeling window (Mott scattering) in electron-positron and electron-electron scattering, presently suggested by various experiments [20].
4. Charge and Self-Electromagnetic Field Motions With Free Extended Cores

Since the point-like charge $e$ within each extended element is actually surrounded in its rest frame $S_0$ by an irrotational Coulomb field $E_c$ (which is time varying for an observer moving through it, i.e. behaves like a moving charge carrying a flattened Coulomb field with it), and by an induced Faraday field, $\Psi$ which corresponds to inertial electro-magnetic reactions, we thus write, in $S_0$, the self-field in the form:

$$E = E_c + \Psi$$  \hspace{1cm} (27)

and have by definition

$$\nabla E_c = 0$$

$$\nabla \times \Psi = -\frac{\partial B}{\partial t}$$  \hspace{1cm} (28)

with the relations $B = \nabla \times A$, with $E_c = -\nabla \phi$ (i.e. $\Psi = \nabla \phi$) and where the current, $J = \rho v$ corresponds to the core's orbital motion $\nabla \times B - (1/c^2) (\partial E/c \partial t) = \mu J$, $\rho$ is the charge density and $v$ the current velocity. Using the Lorenz gauge ($\nabla \cdot A = (1/c^2) (\partial \phi/\partial \tau)$) and Maxwell's equations, we get for the self-field the relation

$$\Psi = -\frac{1}{c^2} \left( \phi \frac{dv}{dt} + v \frac{d\phi}{dt} \right)$$  \hspace{1cm} (29)

which implies that the force exerted on the charge by its own field is given by

$$e\Psi = \iiint \left[ \rho \phi \frac{dv}{dt} + \rho v \frac{d\phi}{dt} \right] dv$$  \hspace{1cm} (30)

accompanied by the Maxwellian equations

$$\phi = -\frac{D}{\varepsilon} \quad \text{and} \quad A = \frac{\mu}{4\pi} \iiint \frac{J}{r} \, dv = \frac{\phi v}{c^2}$$  \hspace{1cm} (31)

and we see by writing $d\phi/dt = (d\phi/dr) v \cdot \cos \theta$ and $dv = 2\pi r^2 \sin \theta dr d\theta$ and (where $r$ and $\theta$ denote the usual coordinates in $S_0$) that the second term vanishes by integration, so that

$$\Psi = -\frac{\phi}{c^2} \frac{dv}{dt}$$  \hspace{1cm} (32)

and the Faraday force is $e\Psi = -e\phi/c^2$. Moreover, if one works in $\Sigma_0$ one can replace $E$ by $\Psi$ and use $H = B/\mu$ so that the Poynting-Heaviside Theorem yields for the change of electromagnetic energy in a volume $V$ the relation

$$\iiint (\Psi \times H) \cdot dS + \iiint J \cdot \Psi \cdot dV = \iiint \left[ \frac{1}{2} e\Psi^2 + \mu H^2 \right] dV = 0$$  \hspace{1cm} (33)

if this energy is conserved.

Since $\Psi$ is proportional to $v$ and $H$ is proportional to $v$, then

$$\frac{\partial}{\partial t} \left( c_1 v^2 + c_2 v^2 \right) = 0$$  \hspace{1cm} (34)

which yields by differentiation and multiplication by $2 \dot{v}$ the constant orbital rotation of $X_\mu$ around $Y_\mu$, i.e.,
\[ \dot{v} + \omega^2 \cdot \dot{v} = 0 \]  

(35)

where \( \frac{c_2}{c_1} = \omega^2 \).

This shows that the helicoidal motion of the electromagnetic self-field of the rotating charge is associated with a total energy \( \delta mc^2 \) which should be subtracted from the total core energy \( mc^2 \) to obtain the rotating energy \( mc^2 \). Since we have

\[ m \cdot c^2 = mc^2 \left( 1 - \frac{\alpha}{2\alpha} \right) = (m - \delta m) c^2 \]  

(36)

with \( mc^2 = \hbar \nu \) and \( c \approx \lambda \nu \) we get the following table:

**TABLE 1**

<table>
<thead>
<tr>
<th>A. Nonrotating Rest Frame Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_0 = m(1 - \alpha / 2\pi)(2/3) )</td>
</tr>
<tr>
<td>( R = \sqrt[3]{(\hbar / mc)(1 + \alpha / 2\pi)} )</td>
</tr>
<tr>
<td>( W_E = 0 )</td>
</tr>
<tr>
<td>( e = \text{equatorial point charge} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Calculated Rotating Inertial Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_s = m(1 - \alpha / 2\pi) )</td>
</tr>
<tr>
<td>( W_H = mc^2 (\alpha / 2\pi) )</td>
</tr>
<tr>
<td>( I = \frac{1}{2} mR^2 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C. Calculated Spectroscopic Quantization</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J = \sqrt[3]{2} \cdot \hbar )</td>
</tr>
<tr>
<td>( \mu = \sqrt[3]{\hbar e} \cdot 2mc \cdot (1 + \alpha / 2\pi) )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D. Spectroscopic Quantities at Quantization Angle ( \Theta_{QM} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_z = 1/2 \cdot \hbar )</td>
</tr>
<tr>
<td>( \mu_z = e\hbar / (2mc \cdot (1 + \alpha / 2\pi) )</td>
</tr>
</tbody>
</table>

5. Spinors and Wave Equation Describing Internal Rotations of Extended Core Elements

The transition from point-like to extended core elements implies (in our model) the existence of internal rotations. These can be represented in various mathematical languages such as the tensor and spinor languages. For internal motions, the question is how they are related to \( Y_\mu \) and \( X_\mu \).

Of course, the description of such collective motions can be developed in different ways. The simplest is to split spacetime into small 4-volume elements into which we define average variables which correspond

- To the average values of the core’s internal motions within such domains; and
- To the average values, such as the density, \( \rho \) the drift current, etc. of the quantities which characterize locally these collective motions and to describe their evolution within drift hypertubes, recalling that the evolution of such quantities along paths tangent to a 4-vector, \( v_\tau \) associated with a proper-time, \( \tau \) is given by \( \partial^\mu (A \cdot v_\mu) \) since we are now dealing with conserved densities.

This amounts to a description of a collective wave in a fluid where we have introduced the variables which connect the local average internal motion of its constitutive extended elements (such as spin) with the external
variables associated with the collective motion of neighboring particles in contiguous hypertubes (like pressure), a process which enlarges the usual Maxwell-Boltzmann description to local average internal elements’ internal motions and implies that the wave equations of Quantum Mechanics describe simultaneously collective measurable (i.e., probabilistic) external and internal motions. The utilization of vectors or spinors in this description is thus only a matter of convenience.

If we start with a set of elements the transition to collective motions implies if one works within hypertubes containing all the $X_\mu$’s of the enclosed conserved set, that one can introduce within it an internal set of average quantities densities $A$ (representing their average position) whose proper-time derivative (w.r.t. the hypertube’s time-like axis parameter) is given by $\partial^\mu (A \cdot v_\mu)$ where $v_\mu$ is the 4-velocity of this axis.

To discuss individual motions of our extended cores we start from the description, notations and results of reference [30]. The motion of a single isolated core wave element is described by a centre of mass $Y_\mu(\theta)$ and the clock-like behavior of internal motions with a clock-needle $R_\mu = Y_\mu - X_\mu = S_{\mu\nu}$ with $R_\mu G^\mu = R_\mu \dot{X}_\mu = 0$ which rotates (⊥ to $u$ and $\dot{X}_\mu$) with the Einstein-de Broglie frequency $\Omega = (M / m)\omega / 2$, where $M_i = G_{\mu\nu}G^{\mu\nu}$ and $m = G_{\mu\nu}\dot{X}_\mu$. Following Dirac, the extended element’s charged core part thus rotates twice on itself while $X_\mu$ rotates once around $X_\mu$ in its rest frame $G_i = 0$ ($i = 1, 2, 3$).

In order to show that the associated real collective waves satisfy a Feynman-Gell-Mann type equation we shall, following Battey-Pratt and Racey [31]

1) Connect the tensor definitions of reference [32] which define each element's behaviour) with new internal variables defined in terms of two component spinors (i.e. rewrite the internal wave equations corresponding to equations for internal and collective core motions; and
2) Add new collective variables (such as a conserved element density) and introduce on each fluid droplet new collective interactions generating de Broglie’s and Bohm’s Quantum Potential Pilot Wave.

Point 1) immediately results from this well-known fact that any space-rotation of a wave element around $X_\mu(\tau)$ can be represented by a quaternion

$$\phi = \alpha + i\beta + j\gamma + k\delta$$

with $|\phi| = \phi\phi^* = \alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$ where $\phi^*$ is the quaternionic conjugate. Since one can write

$$\phi = \begin{vmatrix} \alpha + i\delta & -\gamma + i\beta \\ \gamma + i\beta & \alpha - i\delta \end{vmatrix}$$

any representation of a spherical rotation is now a special unitary matrix of order 2 (i.e. SU$_2$) whose operand form (introduced by Dirac) is the 2-component spinor

$$\begin{vmatrix} \alpha + i\delta \\ \gamma - i\beta \end{vmatrix}$$

The connection with the Darboux-Frenet frame [33-35] is evident. Denoting by $OZ = O_\mu^3$ the instantaneous spin
rotation axis in the rest frame of \( X_\mu(\psi) \) a spherical rotation starting from the spinor \( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) can be rotated into \( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) by the operator \( \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \) which rotates the core of our model by 180° about the z-axis, i.e. represented in the Lie group space by a quarter turn around a great circle in the 4D hypersphere and goes through the intermediate positions, \( \begin{pmatrix} e^{i\theta} \\ 0 \end{pmatrix} \) where \( \theta \) is the angular displacement along the great circle which now represents a core rotation of 2\( \theta \) about our z-axis\(^3\).

In such an extended rotating core model a rotation that is a linear function of time is referred to as spin \([36]\). With our notations the corresponding rotation is thus represented by the operator

\[
\begin{pmatrix} e^{int} & 0 \\ 0 & e^{-int} \end{pmatrix}
\]

in the core's rest frame centered on \( X_\mu(\tau) \). When the core is moving with a velocity \( v \) w.r.t. an external observer, \( \Sigma \) the initial condition

\[
\begin{bmatrix} \gamma + i\delta \\ \gamma + i\beta \end{bmatrix} = \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix}
\]

in its rest frame \( \mathcal{L}_0 \) appears in the form

\[
\Phi = \begin{bmatrix} \Phi_1 e^{-t \omega / (1-v^2/c^2) / \beta} \\ 0 \\ \Phi_2 e^{t \omega / (1-v^2/c^2) / \beta} \end{bmatrix}
\]

\[\text{(43)}\]

to the static observer as a consequence of the Lorentz transformation \( t' = (t - \mathbf{v} \cdot \mathbf{\tau} / c^2) / \beta \) with \( \beta = (1-v^2/c^2)^{1/2} \) and \( \mathbf{\tau} \cdot \mathbf{\tau} = v_\chi x + v_\gamma y + v_\zeta z \). The observer sees the centre \( x \) of a contracted core moving past him with a velocity, \( v \) and observes a variation of the rotation's phase with time, but also from position to position. This is a straight forward consequence of the chosen \( D(\mathcal{O}) \) representation of the Lorentz Group. As a consequence, each particular phase of the motion moves with a velocity \( |V| = c^2 / |v| \) in the direction of \( v \) (See Figure xxx), as in de Broglie's initial assumption, and regions of constant phase are perpendicular to the motion of the model.

For our external observer, the core rotates around \( X_\mu \) with an angular velocity \( \omega (1-v^2/c^2)^{1/2} \) as a consequence of time dilation, and this rotation combined with \( V \) produces a decreasing pitch (w.r.t. increasing velocity) since he sees an angular velocity of \( \omega / (1-v^2/c^2)^{1/2} \) the helical configuration: a well known result

\[\text{\(^3\) Since our model, as in references \([4,14,15]\), is continuously connected with surrounding space, one must distinguish between inversion by parity \( P \) and reversal (by time inversion \( T \)) of spin.} \]
of the distinction between the contravariant (i.e. \( \omega(1 - v^2 / c^2)^{1/2} \)) and the covariant form (i.e. and the covariant form of the rotation energy of the core.

Now as noticed by Battey-Pratt and Racey, [31] the introduction of the preceding new internal spinor variables implies that they are related (for an observer) to the variables \( X_\mu \) and \( Y_\mu \) describing locally the core's external motion by a wave equation with \( \nabla^2 - \frac{1}{c^2} \left( \frac{\omega^2}{\beta^2} \right) \Phi \)

\[
\Phi = \frac{\omega^2}{c^2} \tag{44}
\]

since an immediate calculation yields

\[
\frac{\partial^2 \Phi}{\partial t^2} = -\frac{\omega^2}{\beta^2} \Phi \text{ and } \nabla^2 \Phi = -\frac{\omega^2 v^2}{c^4 \beta^2} \Phi. \tag{45}
\]

If we recall that in the single element case we have shown that \( E = Mc^2 = hv \) so that

\[
\frac{M^2 c^2}{\hbar^2} = \frac{\omega^2}{c^2} \tag{46}
\]

we see that the relation (44) (which represents with new parameters the core's internal rotation) takes the classical form of a Klein-Gordon equation (16)

\[
\Phi = \frac{m^2 c^2}{\hbar^2} \Phi \tag{47}
\]

an astonishing fact indeed, since we now connect spin with mass in a discreet extended clock-like wave element. This is not all, however. The similarity to the Feynman-Gell-Mann equation appears immediately.

6. Collective Core Motions

The Lagrangian description of a set of core collective motions (waves) evidently implies physical (i.e. mathematical) relations between the collective variables and the local average variables describing (locally) individual constitutive set elements in a small 4-volume centered on a point \( X_\mu (\tau) \). This can be done in two steps:

1. The local relation (interpretation) of collective spinor parameters describing a real small collective linear pilot wave equation with the local internal variables of their constitutive extended elements.
2. Their relation with the non-dispersive non-linear internal soliton-like solutions representing observed particles in this model.

We start from the assumption that both states' collective motions are described locally by 4-component spinors \( \Psi_\alpha \) satisfying the connection and identities (discovered by Pauli [17]) connecting them with the representations \( \Phi(\gamma_3/\gamma_2) \) of the Lorentz group, and therefore satisfying automatically the Pauli identities with the 4x4 matrices \( \gamma_\mu \). With the usual Bjorken-Drell relations and notations \( (\hbar = c = 1) \) we first assume that the pilot-wave Lagrangian without constraints can be written:

\[
L = m^2 \bar{\Psi} \Psi - \left( i \gamma_5 - e A^+ \right) \Psi \cdot \left( i \gamma_5 - e A \right) \Psi + \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \mu^2 A^+ A \tag{48}
\]

which yields for the \( \Psi \) field Feynman-Gell-Mann type field equations
\[
\left[ (i\hat{\nabla} - e\hat{A})^2 - m^2 \right] \Psi = 0
\]  
(49)

with two conserved currents

\[
J_{1\mu} = \frac{1}{m} \text{Re} \left[ \bar{\Psi} (i\hat{\nabla}_\mu - eA_\mu) \Psi \right] \quad \text{and} \quad J_{2\mu} = \frac{1}{2m} \partial^\nu \left[ \bar{\Psi} \sigma_{\mu\nu} \Psi \right]
\]  
(50)

along with

\[
\sigma_{\mu\nu} = \tfrac{1}{2} (\lambda_\mu, \gamma_\nu)
\]  
(51)

and spin vector density

\[
S_\mu = \frac{1}{m} \text{Re} \left[ \bar{\Psi} \gamma_\mu (i\hat{\nabla} - e\hat{A}) \Psi \right].
\]  
(52)

We complete this description with two physical constrains (assuming \( \hbar = c = 1 \)) which reduce the Dirac equation to the Feynman-Gell-Mann equation:

- That \( \Psi \) also satisfies the Dirac equation

\[
(i\hat{\nabla} - e\hat{\nabla}) \Psi = m\Psi
\]  
(53)

- That the invariant \( i\bar{\Psi} \gamma_5 \Psi \) vanishes, i.e.,

\[
i\bar{\Psi} \gamma_5 \Psi = 0
\]  
(54)

which imply that \( \Psi \) can be built with a two-component spinor \( W \)

\[
\Psi = \begin{pmatrix} W \\ W \end{pmatrix} \quad \text{with} \quad W = \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}
\]  
(55)

and that \( \Psi \) now satisfies the usual Feynman-Gell-Mann equation

\[
\left[ \left( i\hat{\nabla} - e\hat{\nabla} \right)^2 + \sigma \left( \hat{H} + i\hat{E}^2 \right) \right] \omega = m^2 c^2 \omega
\]  
(56)

which we can now analyze in terms of internal and external (collective) variables.

As well known, by analyzing the purely mathematical connection between 4-component spinors, \( \Psi \) and the finite dimensional representations of the Lorentz group \( D(\frac{2}{2}, \frac{3}{2}) \), Pauli showed long ago that one has the following local mathematical identities, \( i.e. \) (with \( \gamma_\mu = (1/\sqrt{2}) \epsilon_{\mu\nu\alpha\beta} \gamma_\nu \gamma_\alpha \gamma_\beta \)):

- Two invariant \( g = \bar{\Psi} \Psi \) and \( i\bar{\Psi} \gamma_5 \Psi = 0 \) in this model;
- A current and spin density \( J_\mu = i\bar{\Psi} \gamma_\mu \Psi \) and \( S_\mu = -\bar{\Psi} \gamma_\mu \Psi \) with \( J_\mu J_\mu = \rho^2 \), \( S_\mu S_\mu = \rho^2 \) and \( J_\mu S_\mu = 0 \); an angular momentum density \( M_\mu \), i.e. \( M_{\mu\nu} = \frac{1}{2} \bar{\Psi} (\gamma_\mu \gamma_\nu - \gamma_\mu \gamma_\nu) \Psi \) with \( \rho \cdot M_{\mu\nu} = -\epsilon_{\mu\nu\alpha\beta} S_\alpha J_\beta \); a momentum density \( K_\mu = i\bar{\Psi} \left[ \partial_\mu \right] \Psi \); an energy momentum density \( T_\mu \), with

\[
\rho \cdot T_{\mu\nu} = \rho \cdot \left( -\bar{\Psi} \left[ \partial_\mu \right] \gamma_\nu \Psi \right) = K_\mu \cdot J_\nu \partial_\nu J_\lambda \cdot M_{\mu\lambda}
\]  
(57)

which yields a simple physical interpretation of relation (44) with the constraints (53) and (54). Indeed, if we write
\[ W = \rho^{1/2} \cdot e^{iS/\hbar} U \]  

(58)

and if we now utilize the hydrodynamical interpretation of relation (58) with the help of the quaternion formalism introduced by Battey-Pratt (38), we can physically interpret the terms appearing in relations (57).

One first remarks that, as already published and discussed in the literature \[1,6,17\] and without the constraints (53) and (54), the relation (48) associated with waves \( \Psi = Q \cdot \omega \) (with \( \omega \omega = \pm 1 \)) has been shown to correspond to a quantum potential

\[ U = \frac{\Box Q}{Q} - \omega \omega \partial_\mu \omega \partial_\mu - \partial_\mu \omega \partial_\mu \omega \]  

(59)

and related to the usual quantum calculations.

Now from \( L = (\hbar c / 2) \left\{ (\bar{\Psi} \gamma_\mu \partial_\mu \Psi - \bar{D}_\mu \Psi \partial_\mu \Psi) + 2 \chi \bar{\Psi} \Psi \right\} = 0 = (\hbar c / 2) t_{\mu \nu} + \rho m_0 c^2 = 0 \) a Lagrangian, the Dirac constraint (53,54) can be derived (reintroducing \( \hbar \) and \( c \)) (with \( \xi = mc^2 / \hbar \) and \( D_\mu = (\partial / \partial x^\mu) - (ie / c) A_\mu \)) because as shown by Takabayasi \[37\], Halbwachs \[17\], etc., if analyzed in hydrodynamical terms with \( \bar{\Psi} \gamma_\mu \Psi = 0 \) this yields the Dirac equations \( \lambda_\mu D_\mu \Psi = - \chi \Psi \) and \( \bar{D}_\mu \Psi \lambda_\mu = \chi \Psi \) so that \( L = 0 \). They also yield the conserved current

\[ g_\mu = - \left( 1 / c^2 \right) t_{\mu \nu} v_\nu = (i/2\rho) \bar{\Psi} \left[ \partial_\mu \right] \Psi \]  

(60)

where \( \sigma_\alpha \) is the spin density.

Introducing then the dual of the vector density, \( \sigma^\alpha \) by the definition

\[ \hat{\sigma}_{[\alpha \beta \gamma]} = \hat{\sigma}_{[\alpha \beta \gamma]} = \hat{\sigma}_\mu = it_{\alpha \beta \mu} \sigma_\mu \] with \( \mu \neq \alpha, \beta, \gamma \)  

(61)

and utilizing the Takabayasi projection operator on a plane orthogonal to vector \( U_\mu \) i.e.

\[ \eta_{\mu \nu} = \delta_{\mu \nu} + \frac{U_\mu U_\nu}{c^2}. \]  

(62)
With the definition $W_{\mu}^{(\nu)} = \eta_{\mu\nu} W_{\nu}$ for all vectors we get the expression
\[ f_{\mu\nu\lambda} = \frac{1}{2} \delta_{\mu\nu} \cdot U_{\lambda} + \left\{ i c t \mu_{\nu\alpha} \sigma_{\alpha} \left( \frac{U_{\alpha} U_{\lambda}}{c^2} + \delta_{\alpha\lambda} \right) \right\} \]
\[ = \delta_{\mu\nu} \cdot U_{\lambda} + \Theta_{\mu\nu} \]
so that starting as usual from the identity expressing total angular momentum conservation. i.e.
\[ t_{\mu\nu} - t_{\nu\mu} = 2 \delta_{\lambda\mu} f_{\mu\nu\lambda} \]
\[ = \dot{\Theta}_{\mu\nu} + \Theta_{\mu\nu} \]
we get the relation
\[ g_{\mu} U_{\nu} - g_{\nu\mu} + \left[ \delta_{\mu\nu} U_{\lambda} \right] = \dot{\Theta}_{\mu\nu} + \Theta_{\mu\nu} + \tau_{\mu\nu} . \]

Any attempt to describe the average internal behaviour of a localized particle-like wave packet "piloted" by an external linear wave raises the problem of the physical stability of the particle aspect of matter. If observed extended elements of particles and pilot-waves are extended wave packets, which thus recover internal motions, can one describe them within the frame of the usual linear wave equations or should one add non-linear terms to those equations to endow them with non-dispersive (non-spreading) properties at least during their lifetimes? This problem has already been discussed in the literature by de Broglie et al. [38] and we shall only briefly summarize here some established results related to the present model. In order to satisfy observed physical properties of quantum particles, the first property is that if we consider each wave element (in the pilot wave and in the particle-like soliton) as bilocal structures with an internal centre of mass around which spirals a point-like centre of charge, the average motion of individual particle elements (i.e. constitutive pilot-waves and piloted solitons) should be considered as an approximate continuous distribution (defined by the density $\rho$) of a parameter $Y_{\mu}(\Theta)$ (or $X_{\mu}$) of their mass centers associated (carrying) spin Vectors $S_{\mu}(\Theta)$ defining local orbital spin corresponding to local average rotation of their associated centers of charge around $Y_{\mu}(\Theta)$.

The second property is that if the components of the (average) wave function, satisfy the same wave equation a non-linear term can be introduced into it, which would only be effective (big enough) inside the extended soliton part. This would also explain the piloting mechanism.

As a possible solution we shall only present here an extension of the solution proposed by Mackinnon. [39] If we assume:
1) That in the rest frame of its centre of mass the extended average element centers at a point $Y_{\mu}(\Theta)$ at the centre of a volume $\Delta V$ is associated with the charged point $X_{\mu}(\tau)$ at a constant distance $R = X_{\mu} - Y_{\mu}$;
2) That $Y_{\mu}(\tau)$ in its rest frame is the origin of an orthogonal set of three axes (where $R_{\mu}$ lies in the $X,Y$ plane) represented by a pair of spinor components $\varphi(\varphi_{1}, \varphi_{2})$, where the $X$-$Z$ axis is the rotation axis;
3) That we can leave aside the space-like distance $R_{\mu} = Y_{\mu} - X_{\mu}$ (i.e. neglect the corresponding internal oscillations) and work directly in the rest frame of $I_{0}$ of $X_{\mu}$, since $X_{\mu}$ and $Y_{\mu}$ in a free core remain within the same time-like hypertube;
4) That, following Mackinnon [39] we start from the assumption that if we construct in $\Pi_{0}$ at $X_{\mu}$ a system of three orthogonal axes rotating around a vector $\sigma_{\mu}$ and $\hat{X} \equiv 0$ then the corresponding phase vibration (of $Y_{\mu}$ w.r.t. $X_{\mu}$) must be the same for all external observers. It is represented [40] by a two-component spinor $\Psi\left(\Psi_{1} (X_{\mu}), \Psi_{2} (X_{\mu})\right)$ associated with the representation $D\left(\frac{1}{2}, 0\right)$ and $D\left(0, \frac{1}{2}\right)$ of the Lorentz group [20]. This implies that if $U$ denotes the velocity of $X_{\mu}$ w.r.t. a direction $z$ in an inertial
frame, the wave packet representing all possible inertial plane waves (on $X_\mu$) with all velocities $\pm c$ in the interval is given by, the non-dispersive wave expression

$$F_{12}(x,t) = K \{ \exp \left[ i \omega(k_0) t - k_0 x \right] \} \left\{ \sin \frac{\Theta}{\Theta} \right\} \cdot \Phi_{12}$$

(66)

with

$$\Delta k = \frac{m_e c}{\hbar} \left( 1 - \beta^2 \right)^{-1/2}$$

$$k_0 = \frac{m_e \nu}{\left( 1 - \beta^2 \right)^{-1/2}}$$

$$\beta = \frac{U}{c}$$

$$\Theta = \Delta k \left( z - vt \right)$$

$$k = \text{constant}$$

A two component spinor $\Phi (\Phi_1, \Phi_2)$ then satisfies the wave equation

$$F_{1,2} - \frac{m_0 c^4}{\hbar^4} F_{1,2} = \left( c^2 - \nu^2 \right) \exp \left( i \left[ \omega(k_0) t - k_0 x \right] \cdot \nabla \right) \cdot \frac{\sin \Theta}{\Theta}$$

(68)

where $\lambda$ is a constant. A simple extension of preceding calculation suggests that, adding a solution $\Theta_N$ of (xx) to a solution $\Theta_L$ of its linear left-hand side with the same phase $S(x,t)$ implies that the soliton (particle) wave $\Phi_N$ is piloted by $\Phi_L$ which satisfies the Feynman-Gell-Mann equation.

The assumption of extended particle cores (with internal, $R_\mu$ motions) implies, of course, the introduction of different Lorentz frames. Indeed, to describe them one should add to external observer frames $\Sigma$ (one passes from one frame to another by a Poincaré transformation):

- An instantaneous, comoving inertial frame whose origin, $Y_\mu$, is at rest and its Lorentz frame has $a_\mu^{\pm} = dY_\mu / d\Theta$, $a_\mu^{\mp} \cong R_\mu$, so that the orbital rotation of $X_\mu$ vanishes;
- An instantaneous comoving inertial frame, $I_0$ whose origin, $X_\mu$ is at rest and its Lorentz frame has $b_\mu^{\pm} \cong X_\mu = dX_\mu / d\tau$ and no spin but which rotates with an angular momentum tied to the rotation of $R_\mu = Y_\mu - X_\mu$;
- A non-inertial frame, $N^a$ centered on $X_\mu$ in which the accelerating electron charge $X_\mu$ is at rest and its instantaneous spin is zero;
- A non-inertial reference frame $M^a$ which $Y_\mu$ is at rest ($dY / d\Theta = 0$) and the instantaneous orbital motion of $X_\mu$ is zero;
- A non-inertial reference frame $N^a$ supported in a gravitational field. The principle of equivalence implies that $N^a = N^a$ when $a = -g$. The necessity of introducing the preceding frames has been discussed recently (without electron spin) by Petkov. [41]

This introduction implies (as will be developed in subsequent work) that at
1) The velocity of light is anisotropic in $N^a$ and $N^g$;
2) The electric fields in $I_0$ and $N^a$ are identical;
3) The charged volumes in $N^a$ and $N^g$ are anisotropic;
4) The $X^\mu$'s follow local geodetic paths in $N^a$ and $N^g$ in the distorted internal geometry within $R$;
5) Another important point [42] is that if we recall that the point-like charge centered on $X^\mu$ rotates twice on itself [6,24] while $X^\mu$ undergoes one rotation around $Y^\mu$ then one sees that if one assumes that this internal Zitterbewegung resonates with the corresponding external zero-point field to ensure the continuity of Faraday’s lines of force on the electrons charged sphere, then one expects that $E = \hbar \nu = mc^2$ and the deBroglie relation $(\omega, \gamma \nu) c^{-2} = m_c \gamma \nu / \hbar = p / \hbar$ defines this gearing pilot mechanism.

In this model internal/core (17) and particle oscillations beat in phase with the external zero-point frequency of the extended pilot wave elements.

### 7. Divergence of the Electromagnetic Field

In this chapter our discussion has centered primarily on properties relating to extended electron dynamics; however as discussed in detail elsewhere [14,15] the model applies equally well (as summarized in this and the next section) to internal photon motion and integration of the EM and G fields. A non-vanishing divergence of the electric field given below can be added to Maxwell’s equations which results in space-charge distribution. A current density arises in vacuo and longitudinal electric non-transverse electromagnetic terms (i.e. magnetic field components) appears (like $B^{(1)}$) in the direction of propagation.

Both sets of assumptions were anticipated by de Broglie and Dirac. They imply that the real zero-point (vacuum) electromagnetic distribution

- Is not completely defined by $F_{\mu\nu}$ but by a four-vector field distribution given by a four-vector density, $A_\mu$ associated with a de Broglie-Proca equation i.e.

  $$\square A_\mu(x_\alpha) = -\frac{m^2 c^2}{\hbar^2} A_\mu(x_\alpha)$$  \hspace{1cm} (69)

  and its complex conjugated equation.

- The $A_\mu$ field potential equation also contains a gradient term so one has in vacuum:

  $$A_\mu = A_\mu^T + A_\mu^L + \lambda \partial_\mu S$$  \hspace{1cm} (70)

  with $A_\mu A^\mu \to 0$ and a small electrical conductivity in vacuo.

### 8. Possible New Consequences of the Model

Since such models evidently imply new testable properties of electromagnetic and gravitational phenomena we shall conclude this work with a brief discussion of the points where it differs from the usual interpretations and implies new possible experimental tests.

If one considers gravitational and electromagnetic phenomena as reflecting different behaviors of the same real physical field i.e. as different collective behavior, propagating within a real medium (the aether) one must start with a description of some of its properties.

We thus assume that this “aether” is built (i.e. describable) by a chaotic distribution $\rho(x_\mu)$ of small extended structures represented by four-vectors $A_\mu(x_\alpha)$ round each absolute point in $I_0$. This implies
• the existence of a basic local high density of extended sub-elements in vacuum
• the existence of small density variations $\delta \rho(x, \lambda) A_\lambda(x, \mu)$ above $\delta \rho > 0$ for light and below $(\delta \rho < 0)$ for gravity density at $x_\mu$.
• the possibility to propagate such field variations within the vacuum as first suggested by Dirac [43].

One can have internal variations: i.e. motions within these sub-elements characterized by internal motions associated with the internal behavior of average points (i.e. internal center of mass, centers of charge, internal rotations) and external motions associated with the stochastic behavior, within the aether, of individual sub-elements. As well known the latter can be analyzed at each point in terms of average drift and osmotic motions and $A_\mu$ distribution. It implies the introduction of non-linear terms.

To describe individual non-dispersive sub-elements within $I_0$, where the scalar density is locally constant and the average $A_\mu$ equal to zero, one introduces at its central point $Y_\mu(\theta)$ a space-like radial four-vector $A_\mu = r_\mu \exp(iS / \hbar)$ (with $r_\mu r^{\mu} = a^2 =$ constant) which rotates around $Y_\mu$ with a frequency $\nu = m_\gamma c^2 / \hbar$. At both extremities of a diameter we shall locate two opposite electric charges $e^+$ and $e^-$ (so that the sub-element behaves like a dipole). The opposite charges attract and rotate around $Y_\mu$ with a velocity $\equiv c$. The $+e$ and $-e$ electromagnetic pointlike charges correspond to opposite rotations (i.e $\pm \omega/2$) and $A_\mu$ rotates around an axis perpendicular to $A_\mu$ located at $Y_\mu$, and parallel to the individual sub-element’s four momentum $\partial_\mu S$.

![Diagram conceptualizing two oppositely charged sub-elements rotating at $\nu \equiv c$ around a central point 0 behaving like a dipole “bump” and “hole” on the topological surface of the covariant polarized Dirac vacuum.](image)

Assuming electric charge distributions correspond to $\delta m > 0$ and gravitation to $\delta m < 0$ one can describe such sub-elements as holes ($\delta m < 0$) around a point 0 around which rotate two point-like charges rotating in opposite directions as shown in Fig. 1 below.

These charges themselves rotate with a velocity $c$ at a distance $r_\mu = A_\mu$ (with $r_\mu r_\mu = $ Const.). From 0 one can describe this by the equation

$$\square A_\mu - \frac{m_\gamma^2 c^2}{\hbar^2} A_\mu = -\left[\Box(A^\star A)\right]^{1/2}, A_\mu$$

with $A_\mu = r_\mu \exp[iS(x_\mu) / \hbar]$ along with the orbit equations for $e^+$ and $e^-$ we get the force equation

$$m \cdot \omega^2 \cdot r = e^2 / 4\pi^2$$

(72)
and the angular momentum equation:

\[ m_r \cdot r^2 \cdot \omega = \hbar / 2 \]  

(73)

Eliminating the mass term between (31) and (33) this yields

\[ \hbar \omega = e^2 / 2r \]  

(74)

where \( e^2/2r \) is the electrostatic energy of the rotating pair. We then introduce a soliton-type solution

\[ A^0_\mu = \sin \left( \frac{K \cdot r}{K \cdot r} \right) \exp \left[ i(\cot - K_0 x) \right] \]  

(75)

where

\[ K = mc / h, \quad \omega = mc^2 / h \quad \text{and} \quad K_0 = mv / h \]  

(76)

satisfies the relation (31) with \( r = ((x-vt)^2 \cdot (1-v^2 / c^2)^{-1} + y^2 + z^2)^{1/2} \) i.e.

\[ \Box A^0_\mu = 0 \]  

(77)

so that one can add to \( A^0_\mu \) a linear wave, \( A_\mu \) (satisfying \( \Box A_\mu = (m^2 c^2 / \hbar^2) A_\mu \)) which describes the new average paths of the extended wave elements and piloted solitons. Within this model the question of the interactions of a moving body (considered as excess or defect of field density, above or below the aether’s neighboring average density) with a real aether appears immediately.

As well known, as time went by, observations established the existence of unexplained behavior of light and some new astronomical phenomena which led to discovery of the Theory of Relativity.

In this work we shall follow a different line of interpretation and assume that if one considers particles, and fields, as perturbations within a real medium filling flat space-time, then the observed deviations of Newton’s law reflect the interactions of the associated perturbations (i.e. observed particles and fields) with the perturbed average background medium in flat space-time. In other terms we shall present the argument (already presented by Ghosh et al. [44]) that the small deviations of Newton’s laws reflect all known consequences of General Relativity.

9. New Background Conditions of the Dirac Vacuum

If one assumes in conjunction with the de Broglie-Bohm-Vigier Causal Stochastic Interpretation (CSI) of quantum theory [18,45-47] that de Broglie matter-waves describe a wave-particle duality built up with real extended space structures with internal oscillations of particle-like spin, it is possible to justify Bohr’s physical assumptions and predict new properties of a real Dirac covariant polarized vacuum [10,45].

Bohr’s major contribution to modern physics was the model of photon emission-absorption in Hydrogen in terms of random energy jumps between stable quantum states and atomic nuclei. This discovery was one of the starting points for the Copenhagen Interpretation of quantum theory. We suggest this structural-phenomenology by general covariance applies equally as well to the symmetry conditions of the Dirac vacuum backdrop also; but as one knows the purely random description of quantum jumps suggested by Bohr is obviated by the CSI of quantum mechanics [18,45,46,48] suggesting this interaction is piloted. We feel the CSI interpretation is required for our exciplex model to work because it is the internal motion of a massive photon that enables coupling to the Dirac vacuum.

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4 According to Newton massive bodies move in the vacuum, with constant directional velocities, i.e. no directional acceleration, without any apparent relative “friction” or “drag” term. This is not true for accelerated forces (the equality of inertial and gravitational masses are a mystery) and apparent absolute motions proposed by Newton were later contested by Mach.
Some experimental evidence has been found to support this view [48,49] showing the possibility that the interaction of these extended structures in space involve real physical vacuum couplings by resonance with the subquantum Dirac ether. Because of photon mass the CSI model, any causal description implies that for photons carrying energy and momentum one must add to the restoring force of the harmonic oscillator an additional radiation (decelerating) resistance derived from the EM (force) field of the emitted photon by the action-equal-reaction law. Kowalski has shown that emission and absorption between atomic states take place within a time interval equal to one period of the emitted or absorbed photon wave. The corresponding transition time corresponds to the time required to travel one full orbit around the nucleus. Individual photons are extended spacetime structures containing two opposite point-like charges rotating at a velocity near \( c \), at the opposite sides of a rotating diameter with a mass, \( m_e \approx 10^{-45} \) g and with an internal oscillation \( E = mc^2 = \hbar \nu \). Thus a new causal description implies the addition of a new component to the Coulomb force acting randomly and may be related to quantum fluctuations. We believe this new relationship has some significance for our model of vacuum C-QED blackbody absorption/emission equilibrium [4].

The result from real causal interactions between the perturbed local background “ether” and its apparently independent moving collective perturbations imply absolute total local momentum and angular momentum conservation resulting from the preceding description of vacuum elements as extended rigid structures.

10. Conclusion

We conclude this model with three remarks.

1. If one assumes elementary particles are extended in space, then one enters a new field of research, since one should describe (in such a frame) their internal motions and connect them with observable properties of their external motions.

2. Such attempts evidently violate the limits imposed on physical models by the Copenhagen interpretation (believed to be incomplete), since one thus assumes the existence of some still unobservable properties only justified by their indirect physical consequences and their internal motions occur in distorted space-time geometry like the Einstein energy dependent spacetime metric, \( \tilde{M}_e \) [50] which in terms of new thinking should be extended to an HD string theoretic vacuum that takes into account the parameters of a covariant Dirac polarized vacuum.

3. The model proposed in this work (albeit too simple) suggests a similarity between the proposed internal periodic motions of electrons and the periodic motions, at much larger scale, of atoms and molecules, i.e. extends to internal particle motions some of the concepts suggested by the causal stochastic interpretation of Quantum Mechanics. Whether this is true or not will be settled by the future development of microscopic physics [49].
APPENDIX I. General Comments by Louis H. Kauffman

Comments on "Hidden Parameters Describing Internal Motion Within Extended Particle Elements" by J-P Vigier by Louis H. Kauffman

1. This paper addresses electron theory in the context of questions about the size of the electron in relation to its pointlike behavior, the problem of the nature of electron spin and its electromagnetic self-interaction, the problem of the contribution of the electron charge to the electron mass, and the problem of the anomalous magnetic moment.

2. The paper bases its modeling on the assumption that the vacuum is a physical medium built of a covariant polarized distribution of electromagnetic waves. In this model, each individual element moves within a time-like hypertube.

3. Certainly if one assumes that elementary particles are extended in space, then one should describe their internal motions and connect these with observable properties from the outside. This makes this sort of modeling a challenge to the pure symmetry approach to elementary particles where the particle is identified with its external quantum symmetry group. In this context it is legitimate to assume that there is associated with a a particle a distinction of geometric type in the ambient three-dimensional space. Of course, it is also possible to articulate this distinction in terms of internal spaces of higher dimensions as occurs in string theories and earlier in Kaluza-Klein theory. At this point we reach an interface between the topology of embedded manifold structures in three-dimensional space and corresponding structures in higher dimensions. If the hypertubes appear from the outside as tangled knotted and woven structures, this will have to be compared with their interior view that will contain the geometry and topology of the interior spaces.

4. The remarks in Comment 3, lead mathematically to new notions about knots and links in three dimensional space that can generalize the role of knots and links in both string theory and in Chern-Simons theory. In both of these cases the embeddings of one-dimensional manifolds (or tubes about one-dimensional manifolds) are augmented by extra structure that is called out as internal structure or the structure of a gauge field on a bundle over the three-dimensional space. In all these cases it is the relationship among these structures that is of consequence for particle properties and particle interactions. What we need to think about on the mathematical side is how to hold the context of a knot with extra structure when this structure has global complexity as does a gauge field or a string quantization. The notion of external/internal that is challenged here will potentially lead to new mathematics and new physics.

5. The point of view taken here also challenges the strict notion of measurement in the Copenhagen/von Neumann school of quantum mechanics. Particles being field structures should not have their measurements treated as an idealized projection to an eigenstate, but rather, the entire context of the measurement should be taken into account as a quantum field theoretic phenomenon. This is more complex, that a simple projection and it is more realistic, more geometrical and more topological. It will be worth the effort to find this richer formulation of measurement in relation to geometric/topological particle structure.

APPENDIX II. General Comments by Peter Rowlands

The paper describes a model of particle structures extended in space-time, with structural features incorporating ‘hidden’ parameters which describe ‘the local collective motions of the corresponding pilot-waves’. This is part of a long-term project by Vigier in applying the pilot wave model of de Broglie to overcoming some of the problems inherent in the Copenhagen interpretation of quantum mechanics. The conclusion to the paper says that the semi-classical model proposed is ‘too simple’, but that it suggests a way of linking particles with proposed internal structures with atoms and molecules which are known to have such structures.

Present experimental evidence is consistent with a point-like structure for fundamental particles; data from Penning traps suggests that the radius of the electron, if it exists, must be less than $10^{-22}$ m. String / membrane theory, however, has proposed that fundamental particles can be represented in some sense as extended objects, which would help to overcome the problem of infinite self-energy needing to be removed by renormalization, and the finite size is linked to the HD brane concept in the abstract of this paper.

A point-like structure for particles does not mean that the particles will behave as point-like in a classical way. There are aspects of particle behavior which generate properties akin to extension, even in a point-like particle – for example, vacuum polarization, Zitterbewegung and the related Lamb shift. There is also the classical radius,
relating mass and charge, and the Compton radius, a measure of the particle's mass. And, of course, Heisenberg uncertainty means that a point-like particle cannot be located at a classical point. In addition, aspects of quantum systems can often be usefully modeled by semiclassical approaches, e.g. the Bohr theory. So, even assuming a Copenhagen interpretation of quantum mechanics and a strictly point-like structure for a fundamental particle, it is relevant to ask how far a model of an extended structure can encompass such intrinsically quantum properties as Zitterbewegung. The value of such a model, therefore, does not necessarily require us to prove it to be true, but depends on the extent to which we can use it to generate results, especially numerical ones, for experimental investigation and extension of theory into new areas.

Equation (5) p. 3, \( W_H \equiv 2 \mu^2 / 3R_M \) is of interest because the factor 2/3 makes it close to the expression that Heaviside obtained for the mass \( m \) produced by a sphere of radius \( r \), with a charge \( e \) uniformly distributed through it:

\[
\frac{e^2}{4\pi\varepsilon_0 r} = \frac{2}{3} mc^2
\]

I have always thought that, for an electron, the radius

\[
r = \frac{3}{2} \frac{e^2}{4\pi\varepsilon_0 m_e c^2}
\]

was more important than the 'classical radius' [54] (Zero to Infinity, p. 612), and certainly with respect to the polarized vacuum, Zitterbewegung, etc. (Something like this or the classical radius connects directly with the electron mass, which derives from Zitterbewegung. This is also true of the Compton wavelength.) Of course, an electron is not a diffused sphere of charge, but this is not a totally inaccurate expression of vacuum, which, from the electron's point of view is a series of 'virtual' positron-electron pairs. Vacuum is nonlocal and so fits in with a picture of a diffused concept of charge rather than a localized one.

This also fits in with the formula (6) on p. 4, where the relativistic spinning mass is 3/2 times the rest frame mass derived from the classical radius.

I also have a long-standing calculation of the vacuum energy of the universe (the so-called 'dark energy') being 2/3 of the total energy (Zero to Infinity, p. 605)[54], and I have also related this indirectly to the 2/3 in the electron mass calculation. This is a prediction by about 20 years of the experimental result (early versions date from 1979, 1982, 1992, 1994), not a retrodiction.

Descartes is quoted as the originator of vortex-like atoms. I certainly accept that vortices are fundamental to Descartes, but am not convinced that this extended to vortex atoms.

'unique spin orientation in space-time'. The nilpotent formulation of quantum mechanics defines the uniqueness of fermions solely through the instantaneous direction of the spin axis, which contains all the information that is known about a fermionic state (Zero to Infinity, p. 144)[54]. The nilpotent formulation derives from a double vector space, one space being defined as ordinary, observable, space, the other as unobservable, vacuum, space (see the accompanying paper, P. Rowlands, Dual Vector Spaces and Physical Singularities). The uniqueness of axis is in both spaces.

The diagram immediately calls to mind the work of two contributors to PIRT 2010 (and earlier meetings). A. Giese has a particle model with + and – charges rotating round each other at the speed of light. G. Grantham has a vacuum that is made up of a lattice of electron-positron pairs. I think of both of these as being a kind of model of Zitterbewegung, which I have as occurring at a particle 'singularity', on the boundary between real and vacuum spaces. (See P. Rowlands, Dual Vector Spaces and Physical Singularities) So I think of an extension in structure in real space as being like a 'physical' semiclassical model of the more abstract and quantum mathematical structure of a dual vector space (the 'vacuum' space being a mathematical combination of all the unobservable parameters in physics – mass, time, charge). This also fits with the string / membrane concept, in principle, though it rules out any of the individual string models as being ultimately or fundamentally true. (P. Rowlands, Dual Vector Spaces and Physical Singularities)

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