The Black Hole as a Trans-Planckian Particle

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Motivated by T-duality, the 10D/4D correspondence is expanded to apply on trans-Planckian scales. On particle mass scales, the correspondence relates four-dimensional length scales with mass values derived from a ten-dimensional geometry. On trans-Planckian scales, the correspondence relates four-dimensional mass scales with length values derived from the ten-dimensional geometry. Trans-Planckian length scales corresponding to the masses of stellar, intermediate-mass and supermassive black holes are shown to derive from the higher-dimensional geometry. The trans-Planckian length scales calculated for supermassive black holes correlate with the masses of atomic nuclei across the Planck divide suggesting that the two types of object are analogous. Curiously, the galaxy cluster has a particle analogue.

A 10D/4D correspondence has been discovered [1], in which (with \( c = G = \hbar = 1 \))

\[
t_E^2 = 2m_Q^{-5}
\]

where \( t_E \) is the time-scale of an event occurring during the expansion of space following the Big Bang at \( t_E = 0 \) and \( m_Q \) is a mass scale characteristic of the event. The correspondence relates cosmological time, and length, scales to mass scales \( m_Q (1/a_0 \leq m_Q < m_p) \) derived from the geometry of a ten-dimensional spacetime; \( a_0 \) is the Bohr radius and \( m_p \) is the Planck Mass. In this paper we will investigate the 10D/4D correspondence on trans-Planckian scales.

Bolognesi has considered whether there is a dual description of gravity on trans-Planckian scales: a duality that exchanges a manifold of events with a manifold of momenta [2]. He has observed that the universe looks asymmetrical between space-time and energy-momentum and has conjectured that the asymmetry arises from our environmental conditions, i.e. we deal with space-time scales \( \gg l_p \), the Planck Length, and energy-momentum scales \( \ll m_p \). On the basis of such ideas, we will conjecture that a second 10D/4D correspondence, symmetrical with the first, applies at length scales \( l_2 < l_p \) and mass scales \( m_2 > m_p \). First, we will rewrite the correspondence described by (1) as
where $l_{1,4} > l_p$ and $m_{1,10} < m_p$. The subscripts 4 and 10 refer to the dimensions of the respective spacetimes. Motivated by the T-duality symmetry of string theory, we will replace $l_{1,4}$ in (2) with $m_{2,4}$ ($= 1/l_{1,4}$) and $m_{1,10}$ with $l_{2,10}$ ($= 1/m_{1,10}$), resulting in

\[ m_{2,4}^2 = 2l_{2,10}^{-5} \]  

(3)

where $m_{2,4} > m_p$ and $l_{2,10} < l_p$. The duality from which (2) and (3) result exchanges a 4D manifold of events with a 4D manifold of momenta, and exchanges a 10D manifold of momenta with a 10D manifold of events.

Evidence for the correspondence described by (3), and therefore for trans-Planckian duality, has been sought from black holes. Such objects are specified by their mass, spin and charge, which are also properties of elementary particles. In an analysis of a particular class of black hole solutions (extremal dilaton black holes), Holzhey and Wilczek have argued that some of the black holes behave like elementary particles; under different conditions the black holes can appear as extended spherical objects or as elementary point objects [3].

Elementary, and all other, particles, including atomic nuclei [4], lie upon coincident mass levels and sublevels within a network formed from three geometric sequences, the Planck sequences, each of which descends from the Planck Mass and derives from the geometry of a ten-dimensional spacetime [5]. Sequence 1 is of common ratio $1/\pi$, Sequence 2 is of common ratio $2/\pi$ and Sequence 3 is of common ratio $1/e$. Levels in Sequences 1, 2 and 3 are numbered $n_1$, $n_2$ and $n_3$ sequentially from Planck scale ($n=0$). The elementary particles are arranged in doublets or partnerships about mass levels [6]. The charged leptons and their pseudoscalar partners are arranged symmetrically about type 1 superlevels: levels characterised by numbers that are multiples of 3. The quark doublets are arranged symmetrically about type 2 superlevels: levels characterised by numbers that are multiples of 5. Additionally, each quark doublet lies at the close coincidence of type 2 superlevels in Sequences 2 and 3.

Across the Planck divide, the Planck sequences comprise levels of length scale. Levels in the three sequences are assigned the numbers $\nu_1$, $\nu_2$ and $\nu_3$. By conjecturing that $m_{2,4}$ in (3) represents the mass of a black hole, we have calculated $l_{2,10}$ for various black holes of known
mass and then plotted the resulting values on the levels of the Planck sequences, extended into the trans-Planckian realm.

The Tolman-Oppenheimer-Volkoff (TOV) limit (1.5-3.0 solar masses) sets an upper bound on the mass of a neutron star. If the mass of the collapsing part of a star is greater than the TOV limit a black hole will result. The trans-Planckian length scale \( l_{2,10} \) calculated for a 1.5 solar mass stellar black hole, the lightest conceivable such object, is \( 1.03 \times 10^{-50} \) m. This length scale is shown on the levels of Sequences 2 and 3 in Figure 1. It lies at the coincidence of Level 77.5, a half-superlevel of type 2, in Sequence 2 and Level 35, a superlevel of type 2, in Sequence 3. Furthermore, the TOV length scale lies at the close coincidence of type 2 superlevels in Sequences 1 and 3. We see that the trans-Planckian length scale corresponding to the TOV limit occupies a rare and distinct location within the ten-dimensional geometry.

At mass scales \(< m_p \) such locations, upon coincident superlevels, are associated with elementary particles. It seems that stellar black holes and elementary particles, or particles in general, are related objects. Determining whether stellar black holes in general occupy sublevels within the Planck sequences, as do all particles [4], will require very precise measurement of their masses, which mostly lie in the narrow range 5-30 solar masses.

We will now move on to intermediate-mass black holes, which have masses in the range \( \sim 10^2 \) to \( \sim 10^5 \) solar masses. Newly-discovered stable quasi-periodic oscillations of the highly luminous X-ray source M82 X-1, thought to be an intermediate-mass black hole, have enabled a less ambiguous (compared with earlier methods) measurement to be made of the mass of such an object [7]. A value of 428±105 solar masses was found. The trans-Planckian length scale \( l_{2,10} \) corresponding to 428 solar masses is plotted on the levels of Sequences 1, 2 and 3 in Figure 2. It lies at the coincidence of Level 32.5, a half-superlevel of type 2, with Level 82.5, a half-superlevel of type 2, in Sequence 2. In fact, the length scale lies at the close coincidence of type 2 half-superlevels in all three sequences. Like the TOV trans-Planckian length scale, that of M82 X-1 occupies a rare and distinct location within the ten-dimensional geometry. It will be interesting to know where in the higher-dimensional geometry other intermediate-mass black holes are located.
Figure 1: The length scale corresponding to a TOV limit of 1.5 solar masses on the trans-Planckian levels of Sequences 1, 2 and 3. Scales are constrained to lie on a straight line (shown here in blue) since level numbers in the three Planck sequences are in constant ratio.
Figure 2: The length scale corresponding to the mass (428 solar masses) of the intermediate-mass black hole in M82 X-1, on the trans-Planckian levels of Sequences 1, 2 and 3.
We will now consider supermassive black holes (SMBH). These range in mass from $10^5$ or $10^6$ solar masses to $\sim 10^{10}$ solar masses. The trans-Planckian length scales corresponding to these mass values, as calculated from (3), are shown plotted on the levels of Sequences 2 and 3 in Figure 3(a). The length scales are centred on the exceptionally precise type 1 superlevel coincidence (93, 42). Figure 3(a) may be compared with Fig 3(b), in which the proton and its apparent partner, the tightly bound atomic nucleus $^{56}$Fe, a scalar boson, are shown to be arranged symmetrically about (93, 42) on the mass levels of Sequences 2 and 3 [8]. The similarity of the two patterns suggests that supermassive black holes and stable atomic nuclei are analogous objects.

In the Planck Model, the dark matter candidates of mass 7.1 keV [9, 10] and 35 GeV [11] are arranged symmetrically about the type 1 superlevel coincidence (42, 48) in Sequences 1 and 3, which is of mass 16 MeV [12]. Since the electron and its proposed superpartner, the $K^-$ meson, are also arranged symmetrically about (42, 48) in Sequences 1 and 3, we have suggested that the two dark matter particles are superpartners; the scale characteristic of dark matter is then 16 MeV. The trans-Planckian length scale $l_{2,10}$ that lies on (42, 48) in Sequences 1 and 3 has the value $2.13 \times 10^{-56}$ m. Applying the 10D/4D correspondence on trans-Planckian scales results in a characteristic mass of $3 \times 10^{14}$ solar masses. This is the mass of a galaxy cluster, the largest gravitationally bound object in the universe. Galaxy clusters comprise galaxies, hot gas and, mostly, dark matter. They have grown over time and now mostly have masses between $10^{14}$ and $10^{15}$ solar masses [13]. The trans-Planckian length scale corresponding to the mass of a $3 \times 10^{14}$ ($=10^{14.5}$) solar mass galaxy cluster is shown plotted on the levels of Sequences 1 and 3 in Figure 4(a). The dark matter scale 16 MeV is shown on the mass levels of Sequences 1 and 3 in Figure 4(b). The similarity of the two patterns suggests that galaxy clusters and dark matter particles are analogous objects.

We have seen that the supermassive black hole is the trans-Planckian counterpart of a particle, the atomic nucleus. Now it seems that the galaxy cluster, a gravitationally bound assemblage of (mostly) dark matter particles, is the trans-Planckian counterpart of the dark matter particles themselves.
References

4. B. F. Riley, *Particle mass levels*, viXra:0909.0006
Figure 3: (a) Supermassive black holes on levels of trans-Planckian length scale in Sequences 2 and 3. The distribution of SMBH length scales is centred on (93, 42). (b) The proton and the $^{56}$Fe nucleus arranged symmetrically about (93, 42) on the mass levels of Sequences 2 and 3.
Figure 4: (a) The length scale corresponding to the mass, $3 \times 10^{14}$ solar masses, of a typical galaxy cluster, on the trans-Planckian levels of Sequences 1 and 3. (b) The mass scale, 16 MeV, associated with the proposed dark matter particles of mass 7.1 keV and 35 GeV, on the levels of Sequences 1 and 3.