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The Theory About One Special Function For Quantum*

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Abstract

One especial function will be considered in this paper , and two hypotheses of quantum mechanics can be derived by the especial function ; which exists objectively and really in nature through analysis the theory which has been developed .

Introduction

The phenomenon of atoms is usually by solving Shrodinger equation in quantum mechanics (especially for hydrogen atom and it has been successful) [1] , but in any case the process is very difficult because the equation is complexity in mathematics (when subject investigated includes multiple elements) ; Also , the equation's wave function still faces many difficulties[2][3][4][5] . So I do some innovative thinking in here : if one special function is existence and quantal number appears in some way (the physical meaning of the special function will reveal in the following pages) , and the process of solving Shrodinger equation can be replaced ; The special function exists in the fact , and it can apply to atoms , Planck's hypothesis[6] and De Broglie's hypothesis[7] be derived by special function in this paper , it is importance for current theory , I firmly believe that the special function reveal the essence of quantum .

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* All opinions of this paper is according to my book and my last article .

1. One Special Function Which Has Quantum Properties

The special function first be given for quantum mechanics in my book [8] , and it has the form

$$U(x) = -D\left[Si^2(A\pi \cdot x) - (\pi/2)^2\right]^2, \ has: \ \ Si(x) = \int_0^x (\sin t/t)dt$$

where $Si(x)$ is trigonometric integral , D and A are pending constant of subject investigated ($D > 0$ in here) . In mathematics , the function $U(x)$ has minimum in $x = n/A$ $(n = 1,2,3,\cdots)$, and $U(x)$ is approximately equal to $-Dg_0^2/n^2$ (g_0 is pending constant ,the approximate method be used in this paper) , and a series of minimum is similar to energy levels E_0/n^2 of hydrogen atom (E_0 is ground-state energy energy) .

2. Physics Meaning of Special Function For Two Hypotheses

Physical characteristic strongly indicates that the special function may be important and deep theory for quantum mechanics . now I research one more comprehensive form of special function

$$U(r) = -D\left[Si^2\left(Ar^\eta\right) - (\pi/2)^2\right]^2 + B \tag{1}$$

or writing as

$$U(r) = -D\left[si^2\left(Ar^\eta\right) + \pi \cdot si\left(Ar^\eta\right)\right] + B, \ has: \ \ si(r) = -\int_x^\infty (\sin t/t)dt$$

where η and B are constant , D and A are still pending constant of subject investigated (and $D > 0$) , $r = \sqrt{x^2 + y^2 + z^2}$.

According to the opinions of my book and my last article[9], $U(r)$ is potential energy function for subject investigated (so we can set zero potential energy to $r \to \infty$, the usual convention , has $B = 0$ and negative potential energy) ; $U(r)$ forms any deep potential well in $(n\pi/A)^{1/\eta}$ (the nth potential well) , in order to present a clearer physics meaning ($U(r)$ only includes system's parameter) , writing $A_0 = (\pi/A)^{1/\eta}$, we obtain :

$$U(r) = -D\left[Si^2\left(\pi A_0^{-\eta} r^\eta\right) - (\pi/2)^2\right]^2 \tag{2}$$

the force function

$$f(r) = 4D\eta\left[Si^2\left(\pi A_0^{-\eta} r^\eta\right) - (\pi/2)^2\right]Si\left(\pi A_0^{-\eta} r^\eta\right)\sin\left(\pi A_0^{-\eta} r^\eta\right) \cdot r^{-1} \tag{3}$$

In mathematics , the Taylor series for $f(r)$ at $n^{-1/\eta}A_0$ is

$$f(r) = \sum_0^\infty \frac{1}{k!}\left\{f^{(k)}\left(n^{1/\eta}A_0\right)\right\}\left(r - n^{1/\eta}A_0\right)^k, \ \ r \in U\left(n^{1/\eta}A_0\right) \tag{4}$$

because

$$f^{0}\left(n^{1/\eta}A_0\right)=0$$

$$f^{(1)}\left(n^{1/\eta}A_0\right)=4D\eta^2\pi A_0^{-2}\left[Si^2(n\pi)-(\pi/2)^2\right]Si(n\pi)\cos(n\pi)n^{(\eta-2)/\eta}$$

and

$$\left[Si^2(n\pi)-(\pi/2)^2\right]\approx(-1)^{n+1}g_0/n,\quad Si(n\pi)\approx\pi/2\ \left(n>>g_0(2/\pi)^2\right)$$

so we have in the first approximation

$$f(r)=-2Dg_0\pi^2A_0^{-2}\eta^2n^{-2/\eta}\left(r-n^{1/\eta}A_0\right)\ ;\ \left(n>>g_0(2/\pi)^2\right) \tag{5}$$

Now we consider that the particle which in the deep potential well can be be approximated by simple harmonic motion (don't consider relativistic effects in here) , particle which in nth potential well has total energy

$$E_{\text{total}}=Dg_0\pi^2A_0^{-2}\eta^2n^{-2/\eta}\mathrm{A}^2-Dg_0^2/n^2 \tag{6}$$

A is the amplitude (maximum displacement from the equilibrium position) in equation (6) , the dexter second part of equation (6) does not rest with particle's motion state , and only depends on the space where the particle exists , it means particle has inherent energy

$$U(n)=-Dg_0^2/n^2$$

according to the Properties of function $U(r)$, particle's escape energy

$$E=E(n)=Dg_0^2/n^2 \tag{7}$$

particle's vibration frequency

$$\nu=\nu(n)=\eta A_0^{-1}n^{-1/\eta}\sqrt{Dg_0/(2m)} \tag{8}$$

(since vibration frequency must be positive , thus $\eta>0$) .

By comparing equation (7) to (8) :

$$E/\nu=E(n)/\nu(n)=n^{(1/\eta)-2}\eta^{-1}A_0\sqrt{2mDg_0^3}$$

setting $\eta=1/2$:

$$E/\nu=A_0\sqrt{8mDg_0^3} \tag{9}$$

A, m and D are constant for subject investigated , so has :

$$E/\nu=b_0=constant \tag{10}$$

$$b_0=A_0\sqrt{8mDg_0^3} \tag{11}$$

The form of (10) is significant for quantum theory in here , because it appears with two basal hypotheses such as Planck's energy quantum hypothesis and De Broglie's hypothesis , if b_0 equals to Planck's constant h (it will be proven in the next), (10) is the two hypothesis (although we only discuss the energy and vibration frequency of particle in the early part of this paper , but vibration frequency is equal to frequency of the radiation if particle has electric charge, so (10) also is the form of Planck's hypothesis) .

3. Theoretical Verification And Special Function For Objects

Now we apply above theory to hydrogen-like atom . The energy levels of hydrogen-like atom are given by Bohr's theory and quantum mechanics (It is obviously the atom has different image in this paper) , has :

$$E_n = -\frac{Z^2 e^4}{8n^2 h^2 \varepsilon_0^2} \cdot \frac{m_1 m_2}{(m_1 + m_2)} \tag{12}$$

here Z is atomic number , e is the elementary charge , m_1 is the nucleus mass , m_2 is the electron mass . by considering (7)(11) and (12) :

$$Dg_0^2 = \frac{Z^2 e^4}{8h^2 \varepsilon_0^2} \cdot \frac{m_1 m_2}{(m_1 + m_2)} \quad ; \quad A_0 = \frac{b_0 h \varepsilon_0}{\sqrt{g_0} Z e^2} \cdot \frac{m_1 + m_2}{m_1 m_2} \tag{13}$$

the he inherent energy of electron :

$$U(r_n) = -\frac{Dg_0^2}{n^2} = -\frac{Dg_0^2 A_0}{n^2 A_0} = -\frac{Dg_0^2 A_0}{r_n} = -\frac{b_0}{8h\varepsilon_0 \sqrt{g_0}} \cdot \frac{Ze^2}{r_n} \tag{14}$$

r_n approaches continuous change if A_0 is small enough , equation (14) is the function of electrostatic potential energy , the theory accomplishes quantum-classical transition (I don't think that it is just a coincidence , It strongly indicates the function of Coulomb's electrostatic force is only a approximate function) . Has

$$b_0 = h \approx 6.62606896 \times 10^{-34}\ \mathrm{J \cdot s} \tag{15}$$

$$g_0 = \pi^2 / 4 \approx 2.46740110 \tag{16}$$

$Z = 1$ (hydrogen atom) , radius of steady state :

$$r_n = n^2 \cdot A_0 = \frac{2n^2 h^2 \varepsilon_0}{e^2 \pi} \cdot \frac{(m_1 + m_2)}{m_1 m_2} \tag{17}$$

the first radius :

$$A_0 = \frac{2h^2 \varepsilon_0}{e^2 \pi} \cdot \left(\frac{m_1 + m_2}{m_1 m_2} \right) \approx 1.05835441 \times 10^{-10}\ \mathrm{m} \tag{18}$$

(it is twice the Born's radius) , by substitution A_0 into equation (14) , the ground-state energy of hydrogen atom is :

$$E_1 \approx -13.6eV$$

so the special function be proven indirectly .

The coulomb force between two objects should be written as :

$$f_{21}(r) = -D\frac{\partial}{\partial r}\left\{ \left[Si^2\left(\pi\sqrt{r/A_0}\right) - (\pi/2)^2 \right]^2 + B \right\},\ has: (q_1 q_2 < 0)$$

$$f_{21}(r)=+D\frac{\partial}{\partial r}\left\{\left[Si^2\left(\pi\sqrt{r/A_0}\right)-(\pi/2)^2\right]^2+B\right\},\ has:(q_1q_2>0)$$

$$D=\frac{32K^2}{\pi^2h^2}\cdot\frac{q_1^2q_2^2m_1m_2}{m_1+m_2}\quad,\quad A_0=\frac{2h^2\varepsilon_0}{\pi}\cdot\frac{m_1+m_2}{q_1q_2m_1m_2}$$

where h is Planck's constant , ε_0 is the vacuum permittivity; r is the distance between object 1 and 2 ; q_1 is the electric charge of object 1 , q_2 is the electric charge of object 2 ; m_1 is the mass of object 1 , m_2 is the mass of object 2 .

Or written as :

$$f_{21}(r)=-D\frac{\partial}{\partial r}\left\{\left[Si^2\left(\pi\sqrt{r/A_0}\right)-(\pi/2)^2\right]^2+B\right\},\ has:(q_1q_2<0)$$

$$f_{21}(r)=+D\frac{\partial}{\partial r}\left\{\left[Si^2\left(\pi\sqrt{r/A_0}\right)-(\pi/2)^2\right]^2+B\right\},\ has:(q_1q_2>0)$$

$$D=\frac{32K^2}{\pi^2h^2}\cdot\frac{q_1^2q_2^2m_1m_2}{m_1+m_2}\quad,\quad A_0=\frac{h^2}{2K\pi^2}\cdot\frac{m_1+m_2}{q_1q_2m_1m_2}$$

where K is Coulomb's constant .

4. Supposition

I suppose that the gravitation also has more accurate function form :

$$f_{21}(r)=-D\frac{\partial}{\partial r}\left\{\left[Si^2\left(\pi\sqrt{r/A_0}\right)-(\pi/2)^2\right]^2+B\right\}$$

$$D=\frac{32G^2}{\pi^2h^2}\cdot\frac{m_1^3m_2^3}{m_1+m_2}\quad,\quad A_0=\frac{h^2}{2G\pi^2}\cdot\frac{m_1+m_2}{m_1^2m_2^2}$$

where G is gravitational constant; r is the distance between object 1 and 2 ; m_1 is the mass of object 1 , m_2 is the mass of object 2 .

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