The Smarandache-Coman function and nine conjectures on it

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Abstract. The Smarandache-Coman function is the function defined on the set of non-null positive integers with values in the set of non-null positive integers in the following way: SC(n) is the least number such that SC(n)! is divisible by n + r, where r is the digital root of the number n. In other words, SC(n) = S(n + r), where S is the Smarandache function. I also state, in this paper, nine conjectures on this function which seems to be particularly interesting: beside other characteristics, it seems to have as values all the prime numbers and, more than that, they seem to appear, leaving aside the non-prime values, in natural order.

Definition:

The Smarandache-Coman function is the function defined on the set of non-null positive integers with values in the set of non-null positive integers in the following way: SC(n) is the least number such that SC(n)! is divisible by n + r, where r is the digital root of the number n. In other words, SC(n) = S(n + r), where S is the Smarandache function.

Note: The digital root of a number is obtained through the iterative operation of summation of the digits of a number until is obtained a single digit; examples: the digital root of the number 28 is 1 because 2 + 8 = 10 and 1 + 0 = 1; the digital root of the number 1729 is 1 because 1 + 7 + 2 + 9 = 19 and 1 + 9 = 10 and 1 + 0 = 1; the digital root of the number 561 is 3 because 5 + 6 + 1 = 12 and 1 + 2 = 3; so, the digital root of a number can only have one from the following nine values: 1, 2, 3, 4, 5, 6, 7, 8 or 9.

The values of SC function are:

: 2, 4, 3, 4, 5, 4, 7, 8, 6, 11, 13, 5, 17, 19, 7, 23, 10, 9, 5, 11, 6, 13, 7, 5, 8, 17, 6, 29, 31, 11, 7, 37, 13, 41, 43, 6, 19, 5, 7, 11, 23, 6, 10, 13, 9, 47, 7, 17, 53, 11, 19, 59, 61, 7, 7, 29, 5, 31, 8, 11, 17, 7, 6, 13, 67, 23, 71, 73, 10, 11, 79, 9, 37, 19, 13, 6, 41, 7, 43, 11, 6, 83, 17, 29, 89, 13, 31, 19, 97 (...)

Observation 1:

Within the first 89 values of SC(n) are found all the first 25 primes from 2 to 97. More than that, they all appear for the first time in order: there is not a prime \( p_3 > p_2 \) between \( p_1 \) and \( p_2 \), where \( p_1 < p_2 \) and both \( p_1 \) and \( p_2 \) appear for the first time in Smarandache-Coman sequence.

Observation 2:

Note that, from the first 89 values of SC(n):
- 69 are primes (25 of them distinct);
- 3 are odd non-primes (all of them equal to 9);
- 17 are even non-primes (4 of them distinct: 4, 6, 8, 10).

Observation 3:

Up to \( n = 89 \), the longest chain of consecutive prime values of SC(n) is obtained for \( n \) from 46 to 58: 47, 7, 17, 53, 11, 19, 59, 61, 7, 7, 29, 5, 31.

Conjecture 1:

All the prime numbers appear as values in the SC sequence (the sequence of the values of SC function).

Conjecture 2:

All the prime numbers appear for the first time in natural order in SC sequence: there is not a prime \( p_3 > p_2 \) between \( p_1 \) and \( p_2 \), where \( p_1 < p_2 \) and both \( p_1 \) and \( p_2 \) appear for the first time in SC sequence.

Conjecture 3:

All the even numbers appear as values in the SC sequence.

Conjecture 4:

There exist an infinity of primes \( p \) for which \( SC(p) = q \), where \( q \) is prime.

The sequence of the primes \( (p, q) \) is:

- \((1, 2), (3, 3), (5, 5), (7, 7), (11, 13), (13, 17), (23, 7), (29, 31), (31, 7), (37, 19), (41, 23), (47, 7), (53, 61), (61, 17), (67, 71), (71, 79), (73, 37), (79, 43), (83, 17), (89, 97)\)...
Conjecture 5:
For all the pairs of twin primes \((p, q)\), where \(p \geq 11\), is true that, if \(p\) appears for the first time in SC sequence as \(SC(n)\), then \(SC(n + 1) = q\).

Conjecture 6:
There exist an infinity of numbers \(n\) such that \(SC(n) = m\) and \(SC(n + 1) = m + 1\), where \(m + 1\) is prime. Such pairs of \((m, m + 1)\) are: \((10, 11)\), \((28, 29)\), \((46, 47)\), \((82, 83)\)...

Conjecture 7:
There exist an infinity of numbers \(n\) such that \(SC(n) = m\) and \(SC(n + 1) = m - 9\), where \(m - 9\) is prime. Such pairs of \((m, m - 9)\) are: \((20, 11)\), \((22, 13)\), \((26, 17)\)...

Conjecture 8:
There exist an infinity of values primes \(p\) of \(SC(n)\) for which the sum \(s\) of all the values of \(SC(n)\) up to and including \(SC(p)\) is prime. Such pairs of \((p, s)\) are: \((7, 29)\), \((13, 67)\), \((17, 89)\), \((11, 173)\), \((7, 199)\), \((17, 229)\), \((7, 313)\), \((13, 547)\), \((11, 691)\), \((59, 769)\), \((13, 971)\), \((23, 1061)\), \((17, 1597)\), \((97, 1877)\)...

Conjecture 9:
There exist an infinity of pairs \((p = S(n), r = S(n + 2))\), both \(p\) and \(r\) primes which appear for the first time in SC sequence, with the property that \(r = p + 4\), such that \(q = S(n + 1)\) is prime. Such triplets \((p, q, r)\) are: \((13, 5, 17)\), \((19, 7, 23)\), \((37, 13, 41)\), \((67, 23, 71)\)...

3