Quantum Special Relativity – Part I

The theory presented in this paper is part of the quantum formulation of Einstein's theory of special relativity. The formulation is based on the assumption that both time and space are quantized. Thus, the Einstein's formula of length contraction, which is identical to the original Fitzgerald-Lorentz length contraction equation, is modified to account for the discrete nature of space. Although this formulation considers that both space and time are discrete physical entities, only the quantized nature of space will be considered in this paper.

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1. Introduction

In this section I shall briefly describe the phenomenon known as length contraction. Length contraction is the phenomenon whereby the length of an object with respect to an observer located in a moving reference frame will appear contracted in the direction of motion. The relation between the length of the object l_0 (proper length) with respect to a frame in which the object is at rest to the length of the object l ("contracted" length) measured by an observer moving with velocity v with respect to the stationary observer is:

$$l = l_0 \sqrt{1-\frac{v^2}{c^2}} \tag{1.1}$$

This is known as the Fitzgerald-Lorentz length contraction formula or Lorentz length contraction formula. Note that, according to this formula, the length of the object is maximum in the reference frame in which the object is at rest.

The purpose of this paper is to modify formula (1.1) to incorporate the discrete nature space. Thus, the theory presented here incorporates the Planck length:

$$L_P \equiv \sqrt{\frac{h\,G}{2\,\pi\,c^3}} \tag{1.2}$$

Because this formulation incorporates the Planck's constant (through the Planck length) it is natural to call this theory: *the quantum special theory of relativity* or simply: *Quantum Special Relativity* (QSP). The next section outlines the two postulates on which the formulation is based upon. This theory is very simple as it does not require any sophisticated mathematical tools. **Appendix 1** contains the nomenclature used in this paper.

2. Postulates

This formulation assumes that the nature of spacetime is discrete or quantized. This quantization is implemented through the following two postulates:

(1) **Time quantization postulate**

Time is discrete. This means that there is a time, T_{MIN}, which is the minimum time with physical meaning. In other words there is no time or time interval smaller than T_{MIN}. It is likely that T_{MIN} to be equal to the Planck time, T_P.

(2) **Length quantization postulate**

Length is discrete. This means that there is a length, L_{MIN}, which is the minimum length with physical meaning. In other words there is no length or distance smaller than L_{MIN}. It is likely that L_{MIN} to be equal to the Planck length, L_P.

3. The Problem of Fitzgerald-Lorentz's Length Contraction

Now let us consider the problem with the Fitzgerald-Lorentz length contraction formula. This formula is:

$$l = l_0\sqrt{1-\beta^2} \tag{3.1}$$

If we make l_0 equal to L_{MIN} then the value of l will be

$$l = L_{MIN}\sqrt{1-\beta^2} \tag{3.2}$$

Because $\sqrt{1-\beta^2}$ is less than 1 for all values of *v* in the range: $0 < v < c$, we deduce that the length, l, of the body in the direction of the body's movement will appear to be less than L_{MIN} (except for $v = 0$, in which case there is no movement). But this contradicts the second postulate (**Length quantization postulate**) of this formulation which says that there is no length smaller than L_{MIN}. Thus we draw the conclusion that if postulate 2 is correct then the Fitzgerald-Lorentz's length contraction formula is not. Now one can ask: What should be the value of l when $l_0 = L_{MIN}$? The value of l for L_{MIN} should be equal to L_{MIN} (not equal to $L_{MIN}\sqrt{1-\beta^2}$ as predicted by the Fitzgerald-Lorentz formula). This means that we need to modify the Fitzgerald-Lorentz length contraction formula to correct this problem. The corrected formula will be called: The *quantum length contraction* formula. This new formula is introduced in the following section.

4. Quantum Fitzgerald-Lorentz Contraction

The formula for the quantum Fitzgerald-Lorentz contraction (or quantum length contraction) is

$$l = l_0\sqrt{1-\beta^2} + \left(1-\sqrt{1-\beta^2}\right) L_{MIN} \tag{4.1}$$

This formula guaranties that the length, *l*, of an object will never appear to be smaller than the minimum length, L_{MIN}, for any observer in relative motion with respect to the object.

where L_{MIN} is the minimum length with physical meaning. In other words nature does not have any length or distance smaller than L_{MIN}. So far no experiment could determine the exact value of L_{MIN}. However it is likely that this length to be equal to the Planck length, L_P. The quantum length contraction formula presented in table 1 (Summary section) assumes, precisely, that $L_{MIN} = L_P$. Having said that we have to keep in mind that L_{MIN} could be smaller than the Planck length (including a nil value). We don't know.

Formula (4.1) guaranties that the length of a body will never appear to be smaller than the minimum length, L_{MIN}. This is reasonable since it doesn't make any sense than a body appears to be smaller than the minimum length imposed by nature (if there is one). **Appendix 2** illustrates the quantitative difference between the Fitzgerald-Lorentz contraction of Special Relativity and its counterpart of Quantum Special Relativity.

5. Summary

The following table (table 1) shows the formulas for length contraction. For the quantum relativistic formula I have assumed that:

$$L_{MIN} = L_P \tag{5.1}$$

Name of the Formula	Original Relativistic Formula	Quantum Relativistic Formula
Length contraction	$l = l_0\sqrt{1-\beta^2}$	$l = l_0\sqrt{1-\beta^2} + \left(1-\sqrt{1-\beta^2}\right) L_P$

Table 1: The quantum formulas for the Lorentz length contraction. Note that L_{MIN} has been replaced by L_P (Planck length).

6. Conclusions

In summary, the formula presented in this paper is a straightforward generalization of the Fitzgerald-Lorentz formula of length contraction. Now one can as: When do we have to use the quantum formula? When the length involved in the event is of the same order of magnitude as L_{MIN} (close enough to this length) we should used the quantum formula for the length contraction instead of the original Fitzgerald-Lorentz formula. According to **Appendix 2** we draw the conclusion that the faster a body moves with respect to an observer located in an inertial reference frame, the greater the difference between the results given by the length contraction formulas of Special Relativity and Quantum Special Relativity. I would also like to mention that another way of deriving the formula presented here is to quantize the Lorentz transformations. But this would be the subject of another article. Another aspect to observe is time. It seems that it is not necessary to modify Einstein's time dilation formula:

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} \tag{6.1}$$

because if $t_0 = T_{MIN}$ then the "dilated" time t is

$$t = \frac{T_{MIN}}{\sqrt{1-\beta^2}} \tag{6.2}$$

and because $1/\sqrt{1-\beta^2}$ is greater than 1 for all values of v in the range: $0 < v < c$, the time t turns out to be greater than T_{MIN}. On the other hand for $v = 0$, the time t turns out to be identical to T_{MIN}. In other words the time t never gets smaller than T_{MIN}. This means that the time dilation formula does not contradicts postulate 1 (**Time quantization postulate**) provided that $t_0 \geq T_{MIN}$. This in turn implies that we have an asymmetry between space and time - we cannot treat time the same way we treated space!. Therefore one can ask: Why do we have this awkward asymmetry? To get the answer to this question we need to investigate this topic further. Finally, it is worth to remark that this quantum framework can be used in future experimental work to determine whether space is quantized or not.

Appendix 1
Nomenclature

The following are the symbols used in this paper

$h =$ Planck's constant
$c =$ speed of light in vacuum
$G =$ Newton's gravitational constant
$l_0 =$ proper length
$l =$ "contracted" length
$t_0 =$ proper time
$t =$ "dilated" time

$T_{MIN}=$ Minimum time with physical meaning
$L_{MIN}=$ Minimum length with physical meaning
$T_P=$ Planck time
$L_P=$ Planck length
$v=$ speed of a massive body with respect to certain observer
$\beta=$ ratio between the speed of a massive body, v, and the speed of light, c.

Appendix 2
Calculation of the Fitzgerald-Lorentz's Contraction for a Particle

Let us calculate the Fitzgerald-Lorentz contraction
(a) using Special Relativity, and
(b) using Quantum Special Relativity
for a particle whose proper length, l_0, is equal to 10 times the Planck length: $l_0 = 10\, L_P$. For the numeric calculations we shall assume that the velocity of the particle with respect to an inertial frame of reference is (i) $0.1c$; and then (ii) $0.99c$.

(a) Special relativity

The formula in this case is

$$l = l_0 \sqrt{1-\beta^2}$$

(Case a-i) $v = 0.1\mathrm{c}$

$$l(0.1\,c) = 10\, L_P \sqrt{1-0.1^2}$$
$$l(0.1\,c) \approx 1.608\ 098 \times 10^{-34}\, m$$

The ratio between this length and the Planck length is

$$\frac{l(0.1\,c)}{L_P} \approx 9.94988$$

(Case a-ii) $v = 0.99\,c$

$$l(0.99\,c) = 10\, L_P \sqrt{1-0.99^2}$$

$$l(0.99\,c) \approx 2.279\ 930 \times 10^{-34}\, m$$

The ratio between this length and the Planck length is

$$\frac{l(0.99\,c)}{L_P} \approx 14.1067$$

(b) Quantum special relativity

The formula in this case is

$$l = l_0\sqrt{1-\beta^2} + \left(1-\sqrt{1-\beta^2}\right)L_P$$

(Case b-i) $v = 0.1\text{c}$

$$l(0.1\,c) = 10\,L_P\sqrt{1-0.1^2} + \left(1-\sqrt{1-0.1^2}\right)L_P$$

$$l(0.1\,c) \approx 1.608\ 908\times 10^{-34}$$

The ratio between this length and the Planck length is

$$\frac{l(0.1\,c)}{L_P} \approx 9.9549$$

(Case b-ii) $v = 0.99\text{c}$

$$l(0.99\,c) = 10\,L_P\sqrt{1-0.99^2} + \left(1-\sqrt{1-0.99^2}\right)L_P$$

$$l(0.99\,c) \approx 3.668\ 136\times 10^{-34}\,m$$

The ratio between this length and the Planck length is

$$\frac{l(0.99\,c)}{L_P} \approx 22.6961$$

Comparing these results we draw the following conclusion: the faster a body moves with respect to an observer located in an inertial frame of reference, the greater the difference between the results given by the length contraction formulas of Special Relativity and Quantum Special Relativity.