P vs NP: Solutions of the Traveling Salesman Problem

Data Ordering and Route Construction Approach

The simplest solution is usually the best solution---Albert Einstein

Abstract

For one more time, yes, P is equal to NP. For the first time in history, the traveling salesman can determine by hand, with zero or negligible error, the shortest route from home base city to visit once, each of three cities, 10 cities, 20 cities, 100 cities, or 1000 cities, and return to the home base city. The formerly NP-hard problem is now NP-easy problem.

The general approach to solving the different types of NP problems are the same, except that sometimes, specific techniques may differ from each other according to the process involved in the problem. The first step is to arrange the data in the problem in increasing or decreasing order. In the salesman problem, the order will be increasing order, since one's interest is in the shortest distances. The main principle here is that the shortest route is the sum of the shortest distances such that the salesman visits each city once and returns to the starting city. The shortest route to visit nine cities and return to the starting city was found in this paper. It was also found out that even though the length of the shortest route is unique, the sequence of the cities involved is not unique.

Since an approach that solves one of these problems can also solve other NP problems. and the traveling salesman problem has been solved, all NP problems can be solved, provided one has an open mind and continues to think. If all NP problems can be solved, then all NP problems are P problems and therefore, P is equal to NP. The CMI Millennium Prize requirements have been satisfied.

Preliminaries

Given: The distances between each pair of cities.

Required : To find the shortest route to visit each of the cities once and return to the starting city. It is assumed that there is a direct route between each pair of cities.

Note

1. Number of distances required to travel to each city once and return equals the number of cities involved in the problem.

2 The symbol $C_{1,2}$ can mean the distance from City 1 to City 2.

The distance $C_{1,2}$ = the distance $C_{2,1}$.

Used as a sentence, $C_{1,2}$ can mean, from City 1, one visits City 2.

3. C_1 is the home base (starting city) of the traveling salesman.

4. $C_{1,2}(3)$ shows that the numerical value of $C_{1,2}$ is 3.

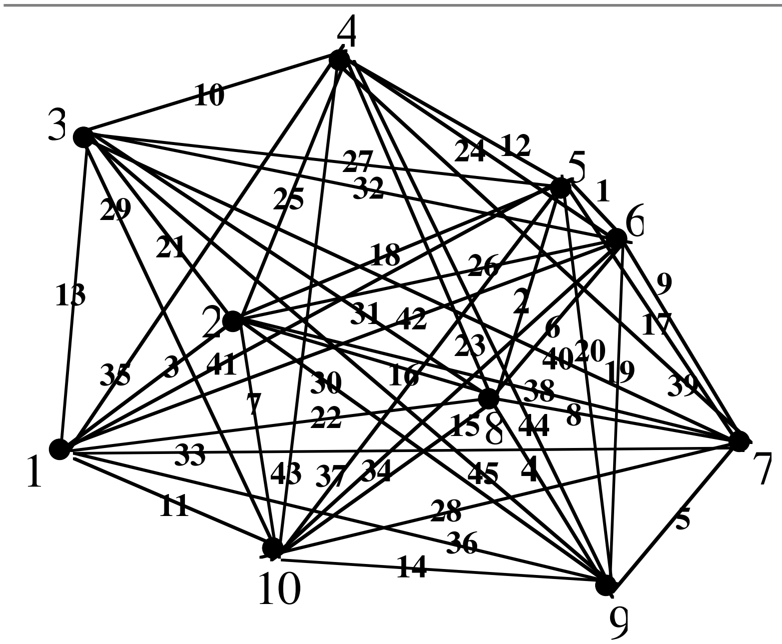
Determining the Shortest Route

Example From City 1, a traveling salesman would like to visit once each of **nine** other cities, namely, City 2, City 3, City 4, City 5, City 6, City 7, City 8, City 9, City 10; and return to City 1. Determine the shortest route**.**

As it was in the author's previous solutions of NP problems, the first step is to arrange the distances in this problem in increasing order. The main principle in this paper is that the shortest route is the minimum sum of the shortest distances such that the salesman visits each city once and returns to the starting city.

Since there are ten cities, ten distances are needed for the salesman to visit each of nine cities once and return to City 1.

For the departure from City 1, the first subscript of City 1 is 1, and for the return to City 1, the second subscript of the last city visited is 1.



Distances Between Each Pair of Cities

C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
$C_{1,2}$ **3**	$C_{2,3}$ **21**	$C_{3,4}$ **10**	$C_{4,5}$ **12**	$C_{5,6}$ **1**	$C_{6,7}$ **9**	$C_{7,8}$ **8**	$C_{8,9}$ **4**	$C_{9,10}$ **14**
$C_{1,3}$ **13**	$C_{2,4}$ **25**	$C_{3,5}$ **27**	$C_{4,6}$ **24**	$C_{5,7}$ **17**	$C_{6,8}$ **6**	$C_{7,9}$ **5**	$C_{8,10}$ **15**	
$C_{1,4}$ **35**	$C_{2,5}$ **18**	$C_{3,6}$ **32**	$C_{4,7}$ **39**	$C_{5,8}$ **2**	$C_{6,9}$ **19**	$C_{7,10}$ **28**		
$C_{1,5}$ **41**	$C_{2,6}$ **26**	$C_{3,7}$ **40**	$C_{4,8}$ **23**	$C_{5,9}$ **20**	$C_{6,10}$ **34**			
$C_{1,6}$ **42**	$C_{2,7}$ **38**	$C_{3,8}$ **31**	$C_{4,9}$ **44**	$C_{5,10}$ **37**				
$C_{1,7}$ **33**	$C_{2,8}$ **16**	$C_{3,9}$ **45**	$C_{4,10}$ **43**					
$C_{1,8}$ **22**	$C_{2,9}$ **30**	$C_{3,10}$ **29**						
$C_{1,9}$ **36**	$C_{2,10}$ **7**							
$C_{1,10}$ **11**								

Step A: Arrange the numerical values of the distances in increasing order

$C_{5,6}$ **1**	$C_{5,8}$ **2**	$C_{1,2}$ **3**	$C_{8,9}$ **4**	$C_{7,9}$ **5**	$C_{6,8}$ **6**	$C_{2,10}$ **7**	$C_{7,8}$ **8**	$C_{6,7}$ **9**
$C_{3,4}$ **10**	$C_{1,10}$ **11**	$C_{4,5}$ **12**	$C_{1,3}$ **13**	$C_{9,10}$ **14**	$C_{8,10}$ **15**	$C_{2,8}$ **16**	$C_{5,7}$ **17**	$C_{2,5}$ **18**
$C_{6,9}$ **19**	$C_{5,9}$ **20**	$C_{2,3}$ **21**	$C_{1,8}$ **22**	$C_{4,8}$ **23**	$C_{4,6}$ **24**	$C_{2,4}$ **25**	$C_{2,6}$ **26**	$C_{3,5}$ **27**
$C_{7,10}$ **28**	$C_{3,10}$ **29**	$C_{2,9}$ **30**	$C_{3,8}$ **31**	$C_{3,6}$ **32**	$C_{1,7}$ **33**	$C_{6,10}$ **34**	$C_{1,4}$ **35**	$C_{1,9}$ **36**
$C_{5,10}$ **37**	$C_{2,7}$ **38**	$C_{4,7}$ **39**	$C_{3,7}$ **40**	$C_{1,5}$ **41**	$C_{1,6}$ **42**	$C_{4,10}$ **43**	$C_{4,9}$ **44**	$C_{3,9}$ **45**

Step B: Interchange the first and second subscripts of each distance,
Note for example that the distance $C_{1,2}$ = the distance $C_{2,1}$.

$C_{5,6}$ or $C_{6,5}$ **1**	$C_{5,8}$ or $C_{8,5}$ **2**	$C_{1,2}$ or $C_{2,1}$ **3**	$C_{8,9}$ or $C_{9,8}$ **4**
$C_{7,9}$ or $C_{9,7}$ **5**	$C_{6,8}$ or $C_{8,6}$ **6**	$C_{2,10}$ or $C_{10,2}$ **7**	$C_{7,8}$ or $C_{8,7}$ **8**
$C_{6,7}$ o $C_{7,6}$ **9**	$C_{3,4}$ or $C_{4,3}$ **10**	$C_{1,10}$ or $C_{10,1}$ **11**	$C_{4,5}$ or $C_{5,4}$ **12**
$C_{1,3}$ or $C_{3,1}$ **13**	$C_{9,10}$ or $C_{10,9}$ **14**	$C_{8,10}$ or $C_{10,8}$ **15**	$C_{2,8}$ or $C_{8,2}$ **16**
$C_{5,7}$ or $C_{7,5}$ **17**	$C_{2,5}$ or $C_{5,2}$ **18**	$C_{6,9}$ or $C_{9,6}$ **19**	$C_{5,9}$ or $C_{9,5}$ **20**
$C_{2,3}$ or $C_{3,2}$ **21**	$C_{1,8}$ or $C_{8,1}$ **22**	$C_{4,8}$ or $C_{8,4}$ **23**	$C_{4,6}$ or $C_{6,4}$ **24**
$C_{2,4}$ or $C_{4,2}$ **25**	$C_{2,6}$ or $C_{6,2}$ **26**	$C_{3,5}$ or $C_{5,3}$ **27**	$C_{7,10}$ or $C_{10,7}$ **28**
$C_{3,10}$ or $C_{10,3}$ **29**	$C_{2,9}$ or $C_{9,2}$ **30**	$C_{3,8}$ or $C_{8,3}$ **31**	$C_{3,6}$ or $C_{6,3}$ **32**
$C_{1,7}$ or $C_{7,1}$ **33**	$C_{6,10}$ or $C_{10,6}$ **34**	$C_{1,4}$ or $C_{4,1}$ **35**	$C_{1,9}$ or $C_{9,1}$ **36**
$C_{5,10}$ or $C_{10,5}$ **37**	$C_{2,7}$ or $C_{7,2}$ **38**	$C_{4,7}$ or $C_{7,4}$ **39**	$C_{3,7}$ or $C_{7,3}$ **40**
$C_{1,5}$ or $C_{5,1}$ **41**	$C_{1,6}$ or $C_{6,1}$ **42**	$C_{4,10}$ or $C_{10,4}$ **43**	$C_{4,9}$ or $C_{9,4}$ **44**
$C_{3,9}$ or $C_{9,3}$ **45**			

Main Principle

The shortest route is the minimum sum of the shortest distances such that the salesman visits each city once, and returns to the starting city. Since there are ten cities, ten distances are needed to allow the salesman to visit once each of nine cities and return to the starting city. One will select ten distances, one at a time, to obtain ten well-connected distances to allow the salesman to visit each city once and return to City 1.

Since one is looking for short distances, for the moment, one will work with the ten numbers (distances) up to the value, 14 units in the above table. See the box with thicker lines in the table, below. If necessary, one will move up the table to add some higher numbers and continue.

A $C_{5,6}$ or $C_{6,5}$ **1**	**G** $C_{2,10}$ or $C_{10,2}$ **7**	**N** $C_{1,3}$ or $C_{3,1}$ **13**	**U** $C_{6,9}$ or $C_{9,6}$ **19**
B $C_{5,8}$ or $C_{8,5}$ **2**	**H** $C_{7,8}$ or $C_{8,7}$ **8**	**P** $C_{9,10}$ or $C_{10,9}$ **14**	**V** $C_{5,9}$ or $C_{9,5}$ **20**
C $C_{1,2}$ or $C_{2,1}$ **3**	**J** $C_{6,7}$ or $C_{7,6}$ **9**	**Q** $C_{8,10}$ or $C_{10,8}$ **15**	**W** $C_{2,3}$ or $C_{3,2}$ **21**
D $C_{8,9}$ or $C_{9,8}$ **4**	**K** $C_{3,4}$ or $C_{4,3}$ **10**	**R** $C_{2,8}$ or $C_{8,2}$ **16**	**X** $C_{1,8}$ or $C_{8,1}$ **22**
E $C_{7,9}$ or $C_{9,7}$ **5**	**L** $C_{1,10}$ or $C_{10,1}$ **11**	**S** $C_{5,7}$ or $C_{7,5}$ **17**	**Y** $C_{4,8}$ or $C_{8,4}$ **23**
F $C_{6,8}$ or $C_{8,6}$ **6**	**M** $C_{4,5}$ or $C_{5,4}$ **12**	**T** $C_{2,5}$ or $C_{5,2}$ **18**	**Z** $C_{4,6}$ or $C_{6,4}$ **24**

Solution

Step C: One will now try to construct a ten-distance route using the entries from A to K. If successful, one would surely have constructed the shortest route, since only the least ten numerical distances would have been used. That is, one would have found the sum of the least ten distances.

Note for example that $C_{1,2}(3)$ shows that the numerical value of $C_{1,2}$ is 3. Such notation makes one become aware of a distance size during a route construction.

Below is an attempt to construct a ten-distance route.

$C_{1,2}(3)C_{2,10}(7)--C_{3,4}(10)--C_{5,8}(2)C_{8,6}(6)C_{6,7}(9)C_{7,9}(5)--(A)$

$C_{1,2}(3)\ C_{2,10}(7)--C_{3,4}(10)--C_{5,6}(1)C_{6,7}(9)C_{7,9}(5)C_{9,8}(4)--(B)$

In trying to construct routes in (A), or (B), above, one is unable to complete a ten-distance route, since all the distances needed are not available within entries in boxes A-K. For example, after $C_{2,10}$, the first subscript of the next distance should be 10, (the second subscript of $C_{2,10}$); and there is no distance with this subscript within A-K. Similarly, after $C_{3,4}$, the firs subscript of the next distance should be 4; but there is no distance with this subscript within boxes A-K. However, if boxes L, M, N and P are added, the needed distances would be available. One will therefore construct a ten-distance route using boxes A-P. Within boxes A-K, there are only two possible first distances, namely, $C_{1,2}$ and $C_{1,10}$. One of these distances with subscript 1 will be the starting (departure) distance, and the other distance would be the return distance. After the above expansion to boxes A-P, another possible additional departure or return distance would be $C_{1,3}$. Since there would now be three distances with the subscript 1, one of these distances would be redundant, since one of them is the departure distance, and another is the return distance. The additional availability of distances would still allow for the construction of the shortest route, since the addition of distances is very minimum.

Step D:: The dashes above indicate missing distances. After including the entries in boxes L, M, N and P to obtain the entries in the boxes A -P as shown , above, by the box with thick lines. After this minimum addition, one successfully constructed the shortest route to visit nine cities and return to City 1. The shortest route from City 1 to visit nine cities and return to City 1 is given by

$$\boxed{C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)C_{9,10}(14)C_{10,2}(7)C_{2,1}(3) = 81}$$

The details of how the above route was obtained is shown below in Steps 1-11.
One is interested in applying the entries in boxes A-P:

Begin from City 1 with $C_{1,2}$ or $C_{1,3}$ or $C_{1,10}$ and return to City 1 with $C_{2,1}$ or $C_{3,1}$ or $C_{10,1}$

Step 1: Begin with first city distance $C_{1,3}(13)$ (from box N, above)

Note: $C_{1,3}$ means distance from City 1 to City 3. (From City 1, salesman visits City 3.)

Step: 2: Since the second subscript of $C_{1,3}(13)$ is 3, the first subscript of the next distance will be 3. Inspect each of the above boxes to pick a distance whose first subscript is 3. Box K contains a distance with 3 as a first subscript. We choose the distance in box K, with the numerical value, 10. Connect the chosen distance with the distance in Step 1 to obtain the connected distance $C_{1,3}(13)C_{3,4}(10)$.

Step 3: Since the second subscript of the last distance is 4, the first subscript of the next distance should be 4. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 3), except that the first subscript of the next distance should be 4. Box M contains a distance with 4 as a first subscript. One chooses the distance $C_{4,5}(12)$ in box M, and attach to obtain the connected distances, $C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)$.

The excluded subscript numbers , except 1, represent the cities already visited.

Step 4: Since the second subscript of the last distance is 5, the first subscript of the next distance should be 5. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 3, 4), except that the first subscript of the next distance should be 5. One chooses the distance $C_{5,6}(1)$ in box A (with small numerical value, 1) to obtain the connected distances $C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)$.

Step 5: Since the second subscript of the last distance is 6, the first subscript of the next distance should be 6. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 3, 4, 5) except that the first subscript of the next distance should be 6, One chooses the distance $C_{6,7}(9)$ in box J to obtain the connected distances

$C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)$.

Step 6: Since the second subscript of the last distance is 7, the first subscript of the next distance should be 7. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 3, 4, 5, 6) except that the first subscript of the next distance should be 7. One chooses the distance $C_{7,8}(8)$ in box H to obtain the connected distances

$C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)$.

Step 7: Since the second subscript of the last distance is 8, the first subscript of the next distance should be 8. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 3, 4, 5, 6, 7) except that the first subscript of the next distance should be 8. One chooses the distance $C_{8,9}(4)$ in Box D to obtain the connected distances

$C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)$.

Step 8: Since the second subscript of the last distance 9, the first subscript of the next distance should be 9. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 3, 4, 5, 6, 7, 8), except that the first subscript of the next distance should be 9. One chooses the distance $C_{9,10}(14)$ in Box P, to obtain the connected distances $C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)C_{9,10}(14)$

Step 9: Since the second subscript of the last distance is 10, the first subscript of the next distance should be 10. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 3, 4, 5, 6, 7, 8, 9) except that the first subscript of the next distance should be 10. One chooses the distance $C_{10,2}(7)$ in box G to obtain the connected distances

$C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)C_{9,10}(14)C_{10,2}(7)$

Step 10: Since the second subscript of the last distance is 2, the first subscript of the next and last distance should be 2. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 3, 4, 5, 6, 7, 8, 9,10), except that the first subscript of the next distance should be 2 and the second subscript should be 1 (an exception) in order to return to City 1, the starting city. One chooses the distance $C_{2,1}(3)$ in box C to obtain the connected distances

$C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)C_{9,10}(14)C_{10,2}(7)C_{2,1}(3)$ (Ten distances)

Step 11: Add the distances in parentheses: 13 + 10 + 12 + 1 + 9 + 8 + 4 + 14 + 7 + 3 = 81 and obtain $\boxed{C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)C_{9,10}(14)C_{10,2}(7)C_{2,1}(3) = 81}$

The above in Step 11 is the shortest route of length 81 units.

EXTRA EXAMPLE (**not** the shortest route): Using $C_{1,2}$ as the first distance

Step 1 Begin with first city distance $C_{1,2}(3)$ (from box C, above)

Note: $C_{1,2}$ means distance from City 1 to City 2.

Step 2: Since the second subscript of $C_{1,2}(3)$ is 2, the first subscript of the next distance will be 2. Inspect each of the above boxes to pick a distance whose first subscript is 2. Box G contains, a distance with 2 as a first subscript. One chooses the distance in box G, with numerical value 7,

$\boxed{C_{1,2}(3)C_{2,10}(7)}$ Also Do: $C_{1,2}(3)C_{2,3}(21)$; $C_{1,2}(3)C_{2,5}(18)$ $C_{1,2}(3)C_{2,8}(16)$

However, since these connected distances contain values greater than 14, there is no need to continue their construction

Step 3: Since the second subscript of the last distance is 10, the first subscript of the next distance should be 10. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2), except that the first subscript of the next distance should be 10. Boxes G and L contain distances with excluded subscripts., One chooses the distance $C_{10,9}(14)$ in box P to obtain the connected distances, $\boxed{C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)}$

The excluded subscript numbers , except 1, represent the cities already visited.

Step 4: Since the second subscript of the last distance is 9, the first subscript of the next distance should be 9. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2, 10), except that the first subscript of the next distance should be 9. One chooses the distance $C_{9,8}(4)$ in box D (with numerical value, 4) to obtain the connected distances $\boxed{C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)}$ Also $C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,7}(5)$

$C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,5}(20)$ $C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,6}(19)$

Step 5 Since the second subscript of the last distance is 8, the first subscript of the next distance should be 8. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2, 10, 9), except that the first subscript of the next distance should be 8, One chooses the distance $C_{8,5}(2)$ in box B to obtain the connected distances

$\boxed{C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{8,5}(2)}$.

Also $C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{7,5}(17)$; $C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{7,8}(8)$; $C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{7,9}(5)$

Step 6: Since the second subscript of the last distance is 5, the first subscript of the next distance should be 5. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2, 10, 9, 8) except that the first subscript of the next distance should be 5. One chooses the distance $C_{5,6}(1)$ in box A to obtain the connected distances

$\boxed{C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{8,5}(2)C_{5,6}(1)}$; Also

$C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{7,6}(9)C_{6,4}(24)$

Note that $C_{5,6}(1)$ has the least numerical value, 1, among the eligible distances.

Step 7: Since the second subscript of the last distance is 6, the first subscript of the next distance should be 6. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2, 10, 9, 8, 5) except that the first subscript of the next distance should be 6. One chooses the distance $C_{6,7}(9)$ in box J to obtain the connected distances

$\boxed{C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{8,5}(2)C_{5,6}(1)C_{6,7}(9)}$

. Also, $C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{7,6}(9)C_{6,5}(1)C_{5,4}(12)$

The excluded subscript numbers , except 1, represent the cities already visited.

Step 8: Since the second subscript of the last distance 7, the first subscript of the next distance should be 7. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2, 10, 9, 8, 5, 6), except that the first subscript of the next distance should be 7. One chooses the distance $C_{7,4}(39)$ from the original data table to obtain the connected distances $\boxed{C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{8,5}(2)C_{5,6}(1)C_{6,7}(9)C_{7,4}(39)}$

Note that $C_{7,4}(39)$ has a relatively large numerical value, 39, among the eligible distances. One went up to a larger range of numbers to accommodate $C_{7,4}$. Because a value greater 14 has been used, upon completion of the route construction, the route found would not be the shortest route. The excluded subscript numbers , except 1, represent the cities already visited.

Step 9: Since the second subscript of the last distance is 4, the first subscript of the next distance should be 4. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2, 10, 9, 8, 5, 6, 7), except that the first subscript of the next distance should be 4. One chooses the distance $C_{4,3}(10)$ in box K to obtain the connected distances

$\boxed{C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{8,5}(2)C_{5,6}(1)C_{6,7}(9)C_{7,4}(39)C_{4,3}(10)}$

Alsso: $C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{7,6}(9)C_{6,5}(1)C_{5,8}(2)C_{8,4}(23)C_{4,3}(10)$

However, since these connected distances contain values greater than 14, there is no need to continue their construction

Note that $C_{4,3}(10)$ has the least numerical value, 2, among the eligible distances.

Step 10: Since the second subscript of the last distance is 3, the first subscript of the next and last distance should be 3. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2, 10, 9, 8, 5, 6, 7, 4) except that the first subscript of the next distance should be 3 and the second subscript should be 1 (an exception) in order to return to City 1, the starting city. One chooses the distance $C_{3,1}(13)$ in box N to obtain the connected distances

$$\boxed{C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{8,5}(2)C_{5,6}(1)C_{6,7}(9)C_{7,4}(39)C_{4,3}(10)C_{3,1}(13)=102}$$

Also $C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{7,6}(9)C_{6,5}(1)C_{5,8}(2)C_{8,4}(23)C_{4,3}(10)C_{3,1}(13)$

Step 11: Add the distances in parentheses: 3 + 7 + 1 4 + 4 + 2 + 1 + 9 + 39 + 10 + 13 and obtain 102.

For comparison purposes, before proceeding to the discussion and conclusion of the material covered already, one will next summarize the shortcomings of some previous methods for solving the traveling salesman problem,

Shortcomings of the Nearest Neighbor Approach and Grouping of Cities Approach

Shortcoming of the Nearest Neighbor Approach

Consider four cities at A, B, C, D. Let the home base of the salesman be at A.

Case 1: Applying the nearest neighbor approach, one would depart from City A along AD of length 6 units (Note: $6 < 9 < 10$). To visit each of the three cities once and return to A, one would either travel the distances AD + DB + BC+ CA (6 + 4 +12+10 = 32 units) or the distances AD + DC + CB + BA (6 + 9 +12 + 7 = 34 units.

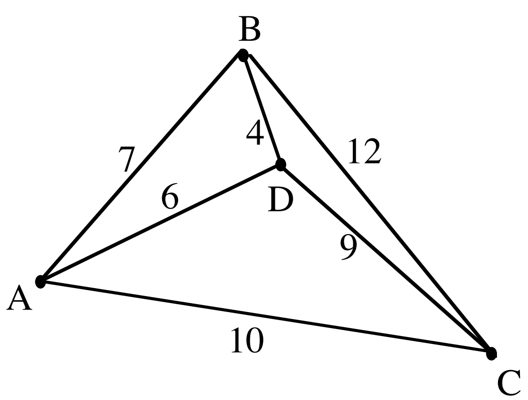
Case 2: If one departs along AB, one would either travel the distances
AB +BD + DC + CA (7 + 4 + 9 +10 = 30 units) or
AB + BC + CD + DA (7 + 12 + 9 + 6 = 34 units)

Case 3: If one departs along AC, one would travel either the distances
AC + CD + DB + BA (10 + 9 + 4 +7 = 30 units) or
AC + CB + BD + DA (10 + 12 + 4 + 6 = 32 units)

Observe above that the shortest route is **not** in Case 1, (of total distance 32 or 34 units) the nearest neighbor approach; but **is** in either **Case 2** or **Case 3 ,** of distance 30 units. Note that the totals in the first parts of Cases 2 and 3 are the same, the same individual distances, except for the order of the addition of the distances.

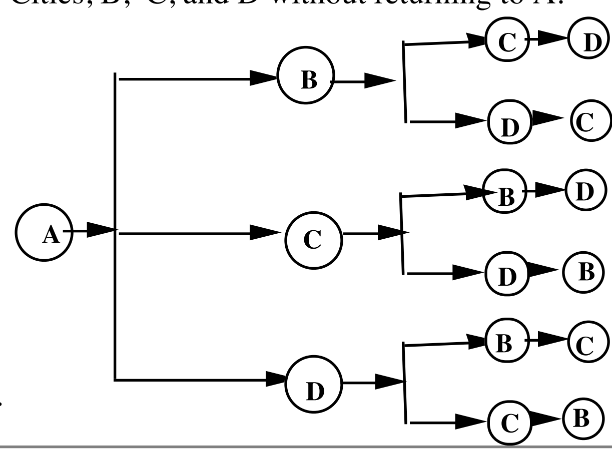
It is to be observed that departing to the nearest city at D, 6 units away, did not produce the shortest total distance. However, departing to either the city at B, or the city at C produced the shortest route of length 30 units, even though B or C is not the nearest neighbor.

The "culprit" is BC or CB of distance 12 units. If one departs to city at D, one is compelled to travel the longest distance of 12 units, since the options to visit the cities at B and D cannot avoid the 12 units distance. The error for Case 1 is about either 6% or 13%, respectively. As the number of cities increases, the errors will multiply.



$C_{A,B}$ = distance between City A and City B.

Possible routes for a salesman to visit each of the Cities, B, C, and D without returning to A.



$C_{A,B}$ **7**	$C_{A,B}$ **7**	$C_{A,C}$ **10**	$C_{A,C}$ **10**	$C_{A,D}$ **6**	$C_{A,D}$ **6**
$C_{B,C}$ **12**	$C_{B,D}$ **4**	$C_{C,B}$ **12**	$C_{C,D}$ **9**	$C_{D,B}$ **4**	$C_{D,C}$ **9**
$C_{C,D}$ **9**	$C_{D,C}$ **9**	$C_{B,D}$ **4**	$C_{D,B}$ **4**	$C_{B,C}$ **12**	$C_{C,B}$ **12**
28	**20**	**26**	**23**	**22**	**27**

Shortcoming of the Grouping of Cities Approach

Mini-Brute Force plus "Divide and Conquer" Approach

Example : Using a three-distance template to determine the shortest route

From City 1, a traveling salesman would like to visit once each of **nine** other cities, namely, City 2, City 3, City 4, City 5, City 6, City 7, City 8, City 9, City 10; and return to City 1.

Guidelines

Step 1: From City 1 (home base of salesman), consider the possible sub-routes for visiting three other cities (say, Cities 2, 3, and 4) without returning to City 1, and determine the shortest route from City 1 to visit once each of these cities.

Step 2: I From the last city visited by the shortest route, one will next determine the shortest sub-route for visiting three other cities, say, Cities 5, 6 and 7.

Step 3: From the last city visited according to the shortest route for visiting Cities 5, 6 and 7, one will determine the shortest route for visiting Cities 8, 9, and 10. The sums of the distances of the above shortest routes will added, and the distance from City 10 to City 1 will also be added the shortest routes sum. (Review example on previous page, and imitate)

Step 4: The results of Steps 1-3 can be combined into a single table as below (18 columns)

$C_{1,2}$ 3	$C_{1,2}$ 3	$C_{1,3}$ 13	$C_{1,3}$ 13	$C_{1,4}$ 35	$C_{1,4}$ 35	$C_{4,5}$ 12	$C_{4,5}$ 12	$C_{4,6}$ 24	$C_{4,6}$ 24
$C_{2,3}$ 21	$C_{2,4}$ 25	$C_{3,4}$ 10	$C_{3,2}$ 21	$C_{4,3}$ 10	$C_{4,2}$ 25	$C_{5,6}$ 1	$C_{5,7}$ 17	$C_{6,5}$ 1	$C_{6,7}$ 9
$C_{3,4}$ 10	$C_{4,3}$ 10	$C_{4,2}$ 25	$C_{2,4}$ 25	$C_{3,2}$ 21	$C_{2,3}$ 21	$C_{6,7}$ 9	$C_{7,6}$ 9	$C_{5,7}$ 17	$C_{7,5}$ 17
34	38	48	59	66	81	22	38	42	50

$C_{4,7}$ 39	$C_{4,7}$ 39	$C_{7,8}$ 8	$C_{7,8}$ 8	$C_{7,9}$ 5	$C_{7,9}$ 5	$C_{7,10}$ 28	$C_{7,10}$ 28
$C_{7,5}$ 17	$C_{7,6}$ 9	$C_{8,9}$ 4	$C_{8,10}$ 15	$C_{9,8}$ 4	$C_{9,10}$ 14	$C_{10,9}$ 14	$C_{10,8}$ 15
$C_{5,6}$ 1	$C_{6,5}$ 1	$C_{9,10}$ 14	$C_{10,9}$ 14	$C_{8,10}$ 15	$C_{10,8}$ 15	$C_{9,8}$ 4	$C_{8,9}$ 4
57	49	26	37	24	34	46	47

Step 5: Combine the boxed columns (shortest sub-routes) above, and add the distance $C_{10,1}$ ($C_{10,1}$ = 11, is the distance from the last city, City 10, to the home base city of the salesman.

$C_{1,2}$ 3	$C_{4,5}$ 12	$C_{7,9}$ 5
$C_{2,3}$ 21	$C_{5,6}$ 1	$C_{9,8}$ 4
$C_{3,4}$ 10	$C_{6,7}$ 9	$C_{8,10}$ 15
34	22	24

Total = 34 + 22 + 24 + 11 = 91

Shortest route to visit each of the nine cities once and return =

$C_{1,2} + C_{2,3} + C_{3,4} + C_{4,5} + C_{5,6} + C_{6,7} + C_{7,9} + C_{9,8} + C_{8,10} + C_{10,1} = 91$ units.

Observe above that Cities, 2, 3, 4, 5, 6, 7, 8, 9, and 10 have been visited; and by $C_{8,10}$, the salesman is at City 10; and to return to City 1, one adds $C_{10,1}$.

Grouping of cities approach

$C_{1,2}(3)C_{2,3}(21)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,9}(5)C_{9,8}(4)C_{8,10}(15)C_{10,1}(11) = \mathbf{91}$

Comparison of Approaches for Finding Shortest Routes

Case 1: For the **Nearest Neighbor approach**, the error lies in being compelled to travel an avoidable longer distance as illustrated on page 10.

Case 2: For the **Grouping of Cities approach**, the error emanates from ignoring some of the shortest distances in determining the shortest route. The length of the shortest route by the grouping of cities approach was found to be 91 units, (for sample problem in this paper)

Case 3: For the **Data Ordering and Route Construction approach,** the length of the shortest route determined was 81 units. The error in Case 2 relative to Case 3 is about 13%, In observing the numerical values of the distances for the shortest routes in Cases 2 and 3 as well as the entries in the table used in the construction of the shortest route for Case 3, below, note that Case 3 used numerical values from the table in boxes A-P. (minimum boxes). Even though Case 2 was obtained by a different approach, one can observe that values 15 and 21 in Case 2 are from boxes beyond boxes A-P.

Case 3 **Shortest route**

Data ordering and route construction approach

$C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)C_{9,10}(14)C_{10,2}(7)C_{2,1}(3) = \mathbf{81}$ **<--- R1**

Numerical distances: 1, 3, 4, 7, 8, 9, 10, 12, 13, 14

Case 2

Grouping of cities approach

$C_{1,2}(3)C_{2,3}(21)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,9}(5)C_{9,8}(4)C_{8,10}(15)C_{10,1}(11) = \mathbf{91}$

Numerical distances: 1, 3, 4, 5, 9, 10, 11, 12, **15, 21**

A	$C_{5,6}$ or $C_{6,5}$	**1**	**G**	$C_{2,10}$ or $C_{10,2}$	**7**	**N**	$C_{1,3}$ or $C_{3,1}$	**13**	**U**	$C_{6,9}$ or $C_{9,6}$	**19**
B	$C_{5,8}$ or $C_{8,5}$	**2**	**H**	$C_{7,8}$ or $C_{8,7}$	**8**	**P**	$C_{9,10}$ or $C_{10,9}$	**14**	**V**	$C_{5,9}$ or $C_{9,5}$	**20**
C	$C_{1,2}$ or $C_{2,1}$	**3**	**J**	$C_{6,7}$ or $C_{7,6}$	**9**	**Q**	$C_{8,10}$ or $C_{10,8}$	**15**	**W**	$C_{2,3}$ or $C_{3,2}$	**21**
D	$C_{8,9}$ or $C_{9,8}$	**4**	**K**	$C_{3,4}$ or $C_{4,3}$	**10**	**R**	$C_{2,8}$ or $C_{8,2}$	**16**	**X**	$C_{1,8}$ or $C_{8,1}$	**22**
E	$C_{7,9}$ or $C_{9,7}$	**5**	**L**	$C_{1,10}$ or $C_{10,1}$	**11**	**S**	$C_{5,7}$ or $C_{7,5}$	**17**	**Y**	$C_{4,8}$ or $C_{8,4}$	**23**
F	$C_{6,8}$ or $C_{8,6}$	**6**	**M**	$C_{4,5}$ or $C_{5,4}$	**12**	**T**	$C_{2,5}$ or $C_{5,2}$	**18**	**Z**	$C_{4,6}$ or $C_{6,4}$	**24**

Note the following:

$C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)C_{9,10}(14)C_{10,2}(7)C_{2,1}(3) = 81$is equivalent to

$C_{1,3}(13) + C_{3,4}(10) + C_{4,5}(12) + C_{5,6}(1) + C_{6,7}(9) + C_{7,8}(8) + C_{8,9}(4) + C_{9,10}(14) + C_{10,2}(7) + C_{2,1}(3) = 81$

(From City 1 to City 3; from City 3 to City 4; from City 4 to City 5; from City 5 to City 6; from City 6 to City 7; from City 7 to City 8; from City 8 to City 9; from City 9 to City 10; .from City 10 to City 2; and finally, from City 2 to City 1.)

Discussion and Conclusion

The length of the shortest route was found to be 81 units; but the sequence of cities of the shortest route is not unique. One sequence of the cities of the shortest route is given by $C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)C_{9,10}(14)C_{10,2}(7)C_{2,1}(3)$, say R1 If the direction of travel of this route is reversed, one obtains the route given by $C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,8}(4)C_{8,7}(8)C_{7,6}(9)C_{6,5}(1)C_{5,4}(12)C_{4,3}(10)C_{3,1}(13)$. Another route of length 81 units is $C_{1,3}(13)C_{3,4}(10)$ $C_{4,5}(12)$ $C_{5,8}(2)C_{8,6}(6)C_{6,7}(9)C_{7,9}(5)C_{9,10}(14)C_{10,2}(7)C_{2,1}(3)$
Therefore, the sequence of cities of the shortest route is not unique, but the length of the route is unique.

Justification of the shortest route.

From City 1, ten distances are needed to visit nine cities and return to City 1.
If each of the distances, $C_{m,n}$, in the ten-distance route were from the least ten distances (i.e., box A-K) in the table, one could immediately conclude that such a ten-distance route is the shortest route. In observing the possible shortest route, $C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)C_{9,10}(14)C_{10,2}(7)C_{2,1}(3)$,R1, not all the distances are from the least ten distances in the table, and one cannot immediately conclude that R1 is the shortest route. However, the next three distances (except 11 which is not applicable here), 12, 13, and 14 are included in R1.
These additions are minimum additions, and therefore, the shortest route of length 81 units is given by $C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)C_{9,10}(14)C_{10,2}(7)C_{2,1}(3)$. Perhaps, one should say a shortest route, since the sequence of cities is not unique.
Observe below that any ten-distance route which contains a distance greater than 14 (largest distance in R1) is at least 6 units greater than that of R1.

$C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,8}(8)C_{8,9}(4)C_{9,10}(14)C_{10,2}(7)C_{2,1}(3)=$ **81**	**R1**
Numerical distances: 1, 3, 4, 7, 8, 9, 10, 12, 13, 14	

$C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,7}(5)C_{7,6}(9)C_{6,5}(1)C_{5,8}(2)C_{8,4}(23)C_{4,3}(10)C_{3,1}(13)=$ **87**	**R2**
Numerical distances: 1, 2, 3, 5, 7, 9, 10, 13, 14, **23**	

1

$C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,7}(9)C_{7,9}(5)C_{9,8}(4)C_{8,2}(16)C_{2,10}(7)C_{10,1}(11)=$ **88**	**R4**
Numerical distances 1, 4, 5, 7, 9, 10, 11, 12, 13, **16**	

A $C_{5,6}$ or $C_{6,5}$ **1**	**G** $C_{2,10}$ or $C_{10,2}$ **7**	**N** $C_{1,3}$ or $C_{3,1}$ **13**	**U** $C_{6,9}$ or $C_{9,6}$ **19**
B $C_{5,8}$ or $C_{8,5}$ **2**	**H** $C_{7,8}$ or $C_{8,7}$ **8**	**P** $C_{9,10}$ or $C_{10,9}$ **14**	**V** $C_{5,9}$ or $C_{9,5}$ **20**
C $C_{1,2}$ or $C_{2,1}$ **3**	**J** $C_{6,7}$ or $C_{7,6}$ **9**	**Q** $C_{8,10}$ or $C_{10,8}$ **15**	**W** $C_{2,3}$ or $C_{3,2}$ **21**
D $C_{8,9}$ or $C_{9,8}$ **4**	**K** $C_{3,4}$ or $C_{4,3}$ **10**	**R** $C_{2,8}$ or $C_{8,2}$ **16**	**X** $C_{1,8}$ or $C_{8,1}$ **22**
E $C_{7,9}$ or $C_{9,7}$ **5**	**L** $C_{1,10}$ or $C_{10,1}$ **11**	**S** $C_{5,7}$ or $C_{7,5}$ **17**	**Y** $C_{4,8}$ or $C_{8,4}$ **23**
F $C_{6,8}$ or $C_{8,6}$ **6**	**M** $C_{4,5}$ or $C_{5,4}$ **12**	**T** $C_{2,5}$ or $C_{5,2}$ **18**	**Z** $C_{4,6}$ or $C_{6,4}$ **24**

The future in the approach for solving the traveling salesman problem lies in the approach (data ordering and route construction)) whereby one concentrates on the smallest distances, and by judicious selection, construct the shortest route. Such an approach reduces the redundant use of brute-force. For the nine cities visit, using brute-force, one would have to consider about 362,880 possibilities. Each possibility would be a column of nine distances. One of these 362,880 columns would be the shortest route to visit the nine cities without returning to City1.

Bye-bye: nearest neighbor approach. You compelled the salesman to travel a longer distance. Bye-bye: grouping of cities approach. You ignored some of the shortest distances. Welcome: Data Ordering and Route Construction. Continue to refine and you would always be welcome

The error in the shortest route of length 81 units determined is zero or negligible.

The above paper has been added to the author's previous solutions of NP problems in the paper, P vs NP: Solutions of NP problems (Example 7) at viXra:, 1408.0204

Adontem