Dynamics of Motion in Gravitational Fields of First Type

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Abstract

In this paper let us consider the general law of motion with regard to two particular fields of force with central symmetry: the static gravitational field for massive systems and the electrostatic field for charged systems. The general law of motion in both cases is defined by a linear differential equation of the first order with respect to the speed and by a linear differential equation of the second order with respect to position. The solution of the motion law is researched here whether when prospective external resistences to motion are neglected or when those resistant forces, due to the resistence of medium where motion happens, are actually present. Besides we will consider the variation of mass with the speed relative to those physical systems that present this characteristic during motion.

1. Introduction

Two main static fields of force with central symmetry are the gravitational field and the electrostatic field. Force fields inside nuclei, atoms and molecules are above all interesting with regard to structure of matter[1] rather than to dynamics of motion and consequently they are not considered here. It is convenient still to specify the concept of field here considered is the vector model that is strictly related to the concept of vector force and to the concept of scalar potential. The central symmetry derive from the fact that in these force fields there is always a central pole that determines the symmetry. This pole is a mass in the gravitational field and a charge in the electrostatic field.
Motions in force fields with central symmetry, in concordance with different typologies of gravitational fields[2] are in three shapes:

1. Linear movement in the field of first type
2. Orbital, circular or elliptic, movement in the field of second type
3. Curvilinear movement in the field of third type.

In this paper let us will examine in detail the linear movement of first type whether in the gravitational field or in the electrostatic field. We will consider in both cases the influence of both, medium where motion happens and the prospective variation of mass with the speed, on physical characteristics of movement.

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2. General law of motion in the gravitational field of first type

In the type I gravitational field[2] the general law of motion for any physical system, body or particle, in free fall is given by the following equation[3][4] (fig.1)

$$m_o \frac{dv}{dt} + kv = \frac{G\, M_o\, m_o}{r^2} \qquad (1)$$

in which m_o is the resting mass, k is the resistant coefficient of medium where the fall happens, M_o is the central mass that generates the gravitational field, t is time of the reference frame supposed at rest S[O,r,t], r is the space coordinate with central symmetry where $r^2=x^2+y^2+z^2$, G is the gravitational constant, v is the variable speed of falling body, O is the origin of the reference frame S that coincides with the barycentre of the central body M_o. Considering, for instance, the initial or bordering condition $v(t=0)=v(r_o)=0$ and solving the general law of motion (1) we obtain the speed v of motion, where t=0 represents the initial instant of falling of physical system and $r=r_o$ represents the initial point P of falling at a range r_o from the origin O.
Integration of the equation (1) represents a complex problem of mathematical analysis that we will search for solving through two subsequent steps.
In the first step we will suppose k=0, i.e. the free falling of physical system happens in the absence of a resistant medium for which reaction forces are represented only by inertial force.

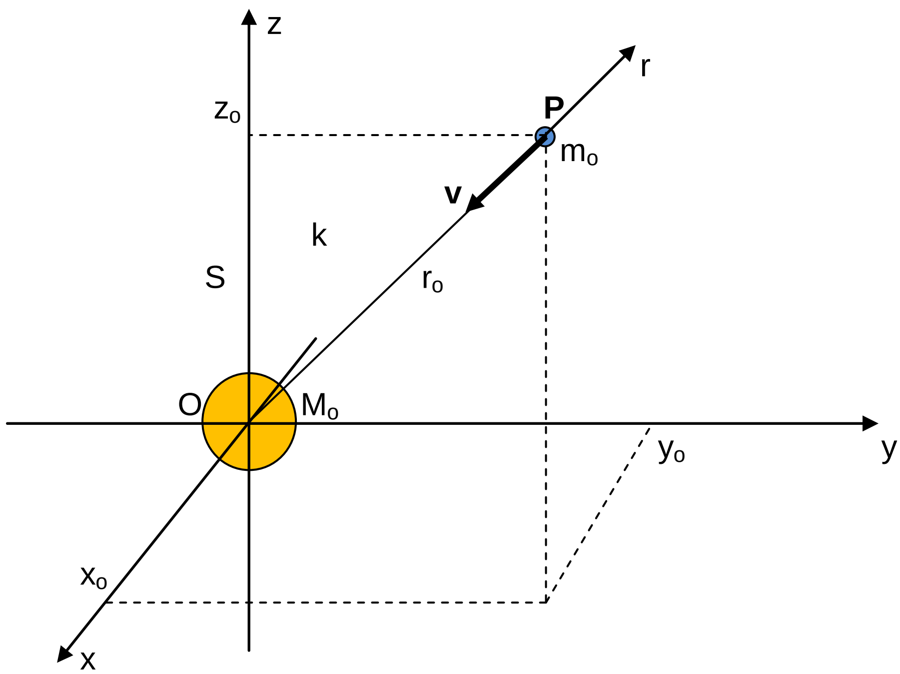

Fig.1 Graphic representation of free falling of a physical system. The falling system is initially (t=0) at a distance r_o from the origin O of the reference frame S supposed at rest. The barycentre of central mass that generates the gravitational field of first type is placed in O.

2.1 Integration with k=0 (absence of resistant medium to movement)

In these physical conditions (k=0), the motion equation is

$$\frac{dv}{dt} = \frac{G M_o}{r^2} \qquad (2)$$

Suppose that at the initial instant t=0 mass m_o is at a distance r_o from the pole with null initial speed $v(t=0)=v(r_o)=0$. The speed is given by

$$v = - \frac{dr}{dt} \qquad (3)$$

where the sign "-" indicates that for decreasing values of distance r (dr<0) with respect to time (dt>0) the scalar module of the speed v is positive. Time acceleration is given by

$$a = \frac{dv}{dt} = \frac{dv}{dr}\frac{dr}{dt} = - v \frac{dv}{dr} \qquad (4)$$

and therefore from (2)

$$v \frac{dv}{dr} = - \frac{G M_o}{r^2} \qquad (5)$$

where - dv/dr is the space acceleration[5].
Integrating we have

$$\int_o^v v\, dv = - G M_o \int_{r_o}^{r} \frac{dr}{r^2} \qquad (6)$$

$$\frac{v^2}{2} = - G M_o \left(- \frac{1}{r} \right)_{r_o}^{r} \qquad (7)$$

$$v^2 = \frac{2 G M_o}{r_o} \left(\frac{r_o - r}{r} \right) \qquad (8)$$

$$v(r) = \sqrt{\frac{2 G M_o}{r_o} \frac{r_o - r}{r}} \qquad (9)$$

Graph of the (9) is represented in fig.2. We observe in (9) the important property that the falling speed of physical system into the gravitational field of first type, in the absence of medium resistance, is independent of system mass and therefore the falling is the same for all physical systems (ordinary bodies and elementary particles).

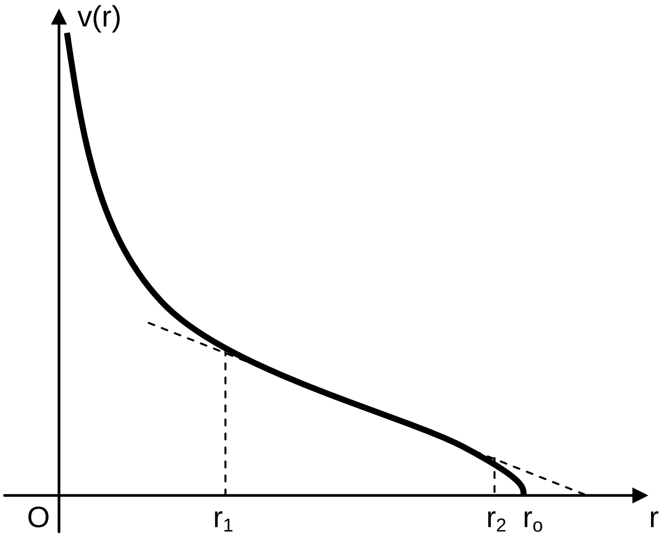

Fig.2 Graph of the speed of a body in free falling in the absence of resistant forces of medium

2.2 Integration with k≠0 (presence of resistant medium to motion)

The general equation of motion in that case is given by (1) that here we write again

$$m_o \frac{dv}{dt} + kv = \frac{G M_o m_o}{r^2} \tag{10}$$

and considering the (4) we have

$$-m_o v \frac{dv}{dr} + kv = \frac{GM_o m_o}{r^2} \tag{11}$$

In order to integrating the (11) it is suitable to consider the following equation derived from the (11)

$$-m_o v dv + kv dr = \frac{GM_o m_o dr}{r^2} \tag{12}$$

Considering the space acceleration

$$a_s = -\frac{dv}{dr} \tag{13}$$

we have

$$v dr = \frac{-v dv}{a_s} \tag{14}$$

Let us suppose that in first approximation the space acceleration is constant in the part of falling (r_1-r_2), in that case integrating the (12) we obtain

$$-m_o\int_o^v v\,dv - \frac{k}{a_s}\int_o^v v\,dv = GM_om_o\int_{r_o}^r \frac{dr}{r^2} \qquad (15)$$

and continuing in calculation we have

$$v(r) = \sqrt{\frac{2GM_om_o}{m_o + \frac{k}{a_s}}\;\frac{r_o - r}{r_or}} \qquad (16)$$

We observe the (16) for k=0 coincides with (9). We observe besides in (16), in the considered case of the presence of a resistant medium with coefficient k, the falling speed v depends on mass m_o of falling physical system.

The (16) may be graphed like in fig.3, where the dotted red curve represents the general falling, independent of mass, in the absence of resistant forces of medium, and continuous curves represent the free falling of the physical system for two different values of mass in the presence of resistant forces.

We observe still in the presence of resistant medium the speed decreases when mass decreases (curve 1) Besides when mass m_o of falling body decreases, the resistant coefficient being equal, in the considered part of falling (r_1-r_2), the speed tends to become almost constant and accelerated motion tends to become an almost-uniform motion.

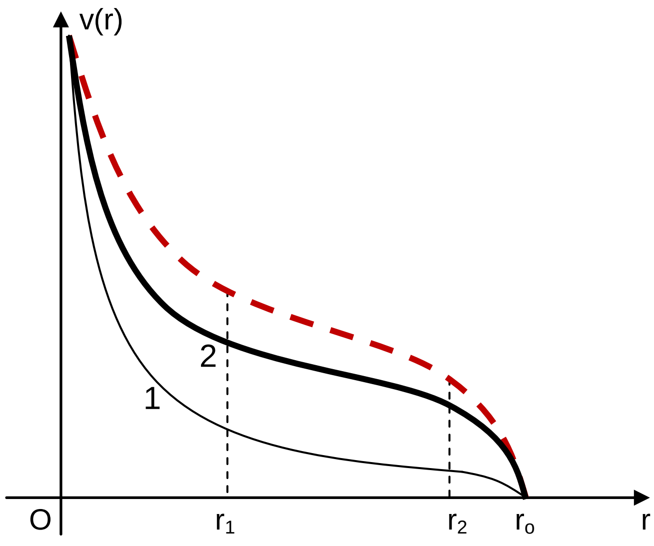

Fig.3 Speeds of a physical system in free falling in the presence of resistant forces for two different values of falling mass (black curves: $m_1<m_2$). The red dotted curve represents the falling in the absence of resistant forces and in that case fall is independent of mass.

If the physical system in free falling is an electrodynamic massive elementary particle the result is the same in the absence of resistant forces of medium, while in the presence of those forces the result is different because particle's electrodynamic mass depends on the speed. In that case, neglecting quantum behaviors of particles[6], because

$$m = m_o\left(1 - \frac{v^2}{2c^2}\right) \tag{17}$$

from (12) it is

$$vdv + \frac{2kc^2}{a_s m_o}\frac{vdv}{2c^2 - v^2} = -\frac{GM_o m_o dr}{r^2} \tag{18}$$

and integrating between r_o, where the speed is zero, and r, where the speed is v, we have

$$v^2 - \frac{2kc^2}{a_s m_o}\ln\left|1 - \frac{v^2}{2c^2}\right| = \frac{2GM_o}{r_o}\frac{r_o - r}{r} \tag{19}$$

For k=0 we have still the (9).

3. General law of motion in the electrostatic field of first type

Similarly the falling law of a charged electrodynamic massive particle -q into the electrostatic field generated by a pole charge +Q, is given by the following equation[2][3]

$$m_o\frac{dv}{dt} + kv = \frac{Qq}{4\pi\varepsilon_o r^2} \tag{20}$$

in which the field force is the Coulomb force, m_o is the resting electrodynamic mass of particle. Also here we distinguish two cases:

3.1 Absence of external resistant forces (k=0)

Let's suppose that external resistant forces are null (k=0), we have

$$m_o\frac{dv}{dt} = \frac{Q\,q}{4\,\pi\,\varepsilon_o\,r^2} \tag{21}$$

Let's suppose still that at the initial instant t=0 the particle is placed at distance r_o from the central charge with null initial speed $v(0)=v(r_o)=0$. Solving[2[3] the (21) we obtain for every distance r the speed

$$v(r) = \sqrt{\frac{Q\,q}{2\,\pi\,\varepsilon_o\,m_o\,r_o}\frac{r_o - r}{r}} \tag{22}$$

The (22) tell us an elementary particle into an electrostatic field has a different behavior from a particle into a gravitational field[2]. In fact in the electrostatic field, also in the absence of external resistant forces (k=0), the particle's speed depends on the electrodynamic mass for which particles with different mass have different speeds, unlike what happens into the gravitational field. Besides we know ordinary charged bodies have constant mass with the speed while charged electrodynamic particles have variable mass. Because the relativistic electrodynamic mass of particle changes with the speed according to the relationship

$$m = m_o \left(1 - \frac{v^2}{2c^2}\right) \tag{23}$$

repeating calculations and considering the (23), the (21) becomes

$$m_o \left(1 - \frac{v^2}{2c^2}\right) \frac{dv}{dt} = \frac{Q\,q}{4\,\pi\,\varepsilon_o\,r^2} \tag{24}$$

from which, being as per (4) dv/dt=-vdv/dr, and integrating between 0 and v with respect to v and between r_o and r with respect to r, we have

$$v(r) = \sqrt{\frac{Q\,q}{2\pi\varepsilon_o r_o m_o \left(1 - \frac{v^2}{4c^2}\right)} \; \frac{r_o - r}{r}} \tag{25}$$

The graph of (25) is represented in fig.4, in which it is possible to see when r decreases the mass dependence on the speed involves an increase of the speed (blue curve).

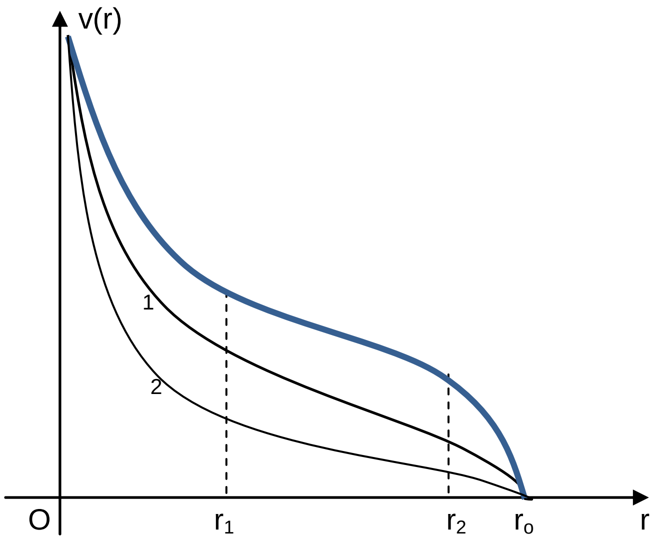

Fig.4 Two graphs in black represent the falling of two ordinary charged bodies with different values of mass (curve 1 for smaller mass). The blue graph represents the falling of a charged electrodynamic particle with velocity-dependent mass.

3.2 Presence of external resistant forces (k≠0).

Supposing k≠0 and assuming space acceleration is again a_s=-dv/dr, the equation (20) becomes

$$\left(m_o + \frac{k}{a_s}\right) v dv = - \frac{Qq}{4\pi\varepsilon_o} \frac{dr}{r^2} \qquad (26)$$

Supposing still space acceleration is constant in the part of falling (r_1-r_2), the solution of (26) is given by

$$v(r) = \sqrt{\frac{Qq}{2\pi\varepsilon_o \left(m_o + \frac{k}{a_s}\right)} \frac{r_o - r}{r_o r}} \qquad (27)$$

For k=0 we have again the (22).
Considering then mass of electrodynamic particle changes with the speed, in concordance with (23), the equation (20) becomes

$$\left(m_o \left(1 - \frac{v^2}{2c^2}\right) + \frac{k}{a_s}\right) v dv = - \frac{Qq}{4\pi\varepsilon_o} \frac{dr}{r^2} \qquad (28)$$

Integrating the (28) we have

$$v(r) = \sqrt{\frac{Qq}{2\pi\varepsilon_o \left(m_o \left(1 - \frac{v^2}{4c^2}\right) + \frac{k}{a_s}\right)} \frac{r_o - r}{r_o r}} \qquad (29)$$

For k=0 we obtain again the (25).
Quantum aspects of motion relative to electrodynamic particles, with emission of energy quanta at the speed of light and at the critical speed, have been examined in the Ref. [6] and they have been neglected in this paper.

References

[1] D. Sasso, Basic Principles of Deterministic Quantum Physics, viXra.org, 2011, id: 1104.0014
[2] D. Sasso, Physics of Gravitational Fields, viXra.org, 2014, id: 1405.0028
[3] D. Sasso, Dynamics and Electrodynamics of Moving Real Systems in the Theory of Reference Frames, arXiv.org, 2010, id: 1001.2382
[4] D. Sasso, On Physical Behavior of Elementary Particles on Force Fields, viXra.org, 2014, id: 1409.0013
[5] D. Sasso, Physico-Mahematical Fundamentals of the Theory of Reference Frames, viXra.org, 2013, id: 1309.0009
[6] D. Sasso, Quantum Dynamics of Elementary Particles, viXra.org, 2015, id: 1504.0166