A Concise Proof of Beal's Conjecture[1]

ABSTRACT. This paper offers a concise proof of Beal's conjecture using the identity.

1 Introduction

Beal's conjecture states that no pairwise coprimes x,y,z satisfy $x^a + y^b = z^c$ for positive integers $a,b,c > 2$. This paper will offer a concise proof of Beal's conjecture using the identity.

2 Proof

$$x^a + y^b = z^c; 2 < a,b,c \in \mathbb{Z}^+; x,y,z : \text{pairwise coprime}; \mathbb{Z}^+\text{: positive integer} \tag{1}$$

2.1 For the case at least one of a,b,c : odd prime ($a \neq b \neq c$)

Let a be an odd prime, and suppose that there exist pairwise coprimes x,y,z satisfying (1), then from (1) it follows that,

$$(x^a + y^a) + (y^b - y^a) = z^c, \tag{2}$$

$$x^a + y^a = z^c - (y^b - y^a). \tag{3}$$

Now, let A= $\{x,y,z : x,y,z$ satisfy (3); $x,y,z \in \mathbb{Z}^+\}$, B= $\{x,y,z : x,y,z$ satisfy (3); $x,y,z \in \mathbb{R}\}$. Then, A $\subset$ B. This means that (3) can be an identity. Then, (3) can be satisfied also in the case $x + y = 0$. Hence,

$$0 = z^c - [(-x)^b - (-x)^a]. \tag{4}$$

(4) means that z,x must have at least a common prime factor when $a \neq b$. The same applies to the case b or c: odd prime, with x^a and y^b, or: with $(-x)^a$ and $(-z)^c$, replaced by each other.

Consequently, no pairwise coprimes x,y,z satisfy (1) for at least one of a,b,c: odd prime ($a \neq b \neq c$). Hence, according to the laws of exponents no pairwise coprimes x,y,z satisfy $x^{l_1 a} + y^{l_2 b} = z^{l_3 c}$ (where $l_1, l_2, l_3 \in \mathbb{Z}^+$). This means that no pairwise coprimes x,y,z satisfy (1) for $2 < a,b,c \in \mathbb{Z}^+$, unless $a = 2^{m_1}, b = 2^{m_2}, c = 2^{m_3}$, where $2 \leq m_1, m_2, m_3 \in \mathbb{Z}^+$ ($a \neq b \neq c$) or $a = b = c$.

2.2 For the case $a = 2^{m_1}, b = 2^{m_2}, c = 2^{m_3}$ ($a \neq b \neq c$)

$$x^4 + y^4 = z^4 \tag{5}$$

That no positive integers x,y,z satisfy (5) was proven by Fermat.([1]) Hence, according to the laws of exponents no positive integers x,y,z satisfy (1) for $a = 2^{m_1}, b = 2^{m_2}, c = 2^{m_3}$ ($a \neq b \neq c$).

2.3 For the case $a = b = c$

That no positive integers x,y,z satisfy (1) (for $a = b = c$) was proven as Fermat's Last Theorem.(cf. [2])

3 Conclusion

No pairwise coprimes x,y,z satisfy $x^a + y^b = z^c$ for any positive integer $a,b,c > 2$. QED.

References

[1] Freeman, L., Fermat's One Proof, http://fermatslasttheorem.blogspot.kr/, Retrieved 2015-04-18.

[2] Wiles, A., Modular elliptic curves and Fermat's Last Theorem, *Ann. Math.* **142**(1995), 443–551.

[1] Yun, J., Daegu Univ., 712-714, South Korea; jmyun@daegu.ac.kr