

# Sorry Bohr, Ground State Energy of Hydrogen Atom is Not Negative

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## Abstract

The negative energy of the hydrogen atom ensuing from Bohr's theory has been widely accepted for over a century. However, the present author considered it strange and came to the conclusion about ten years ago that it is in fact positive and is the electromagnetic energy of a condenser with unit charges of opposite sign at a distance of Bohr radius. This resulted in the finding that the latter is divided at the point of electrical neutrality into two Golden sections pertaining to the electron and proton and enabled establishing the 'additivity of atomic and or ionic radii' rule in bond lengths in the atomic structures of molecules.

**Keywords.** Bohr model; New atomic model; Hydrogen; Golden ratio; Energy of Hydrogen; Atomic condenser.

## 1. Introduction

Bohr's atomic theory had its centennial recently [1]. A general introduction to the Bohr's model of the hydrogen atom can be found in [2a,b]. Basically, this model treats the ground state ionization energy ( $E_H$ ) as the sum of the kinetic energy of the electron of mass  $m_e$  in an orbit with velocity  $v$  at a distance of the Bohr radius ( $a_B$ ) from the proton and the coulombic/potential energy as shown,

$$E_H = (1/2)m_e v^2 - e^2/\kappa a_B = - e^2/2\kappa a_B \quad (1)$$

In Eq. (1),  $e$  is the charge and  $\kappa$  is the electrical permittivity of vacuum.

Note that although the absolute value of the energy has been found correct, the energy is negative. As 75% of the Universe consists of hydrogen [2c], this implies that the energy of the Universe is dominantly negative! However, the concept of negative energy has remained [3, 4], although it has been pointed out as a flaw in the Bohr model as e.g. in [5]. In the book by Rigden [6], one can see that models after models have come and gone, but hydrogen continues to amaze scientists. The present author's contribution to this field [7 - 9] is outlined in the sections below.

## 2. Bohr's ground state energy of H-atom reinterpreted

On rewriting Eq. (1) in the following way,

$$E_H = (1/2)(e^2/\kappa a_B) = (1/2)(e^2/C_B) \quad (2a)$$

$$C_B = \kappa a_B \quad (2b)$$

where  $C_B$  is the capacity,  $E_H$  can be considered [7 - 9] as the electromagnetic energy of a condenser [10] with two unit opposite charges separated by the Bohr radius. The energy is thus positive.

The ionization potential,  $I_H$  is given by,

$$I_H = E_H/e = (1/2)(e/\kappa a_B) \quad (3)$$

On considering  $I_H$  as the difference in absolute potentials of the electron and proton at the distance of the Bohr radius from each other, Eq. (3) can be written as [7 - 9],

$$I_H = (1/2)(e/\kappa a_B) = (1/2)(e/\kappa)[(1/a_{p+}) - (1/a_{e-})] \quad (4)$$

where  $a_B = a_{p+} + a_{e-}$ ,  $I_{p+} = (1/2)(e/\kappa a_{p+})$  and  $I_{e-} = (1/2)(-e/\kappa a_{e-})$ . From Eq. (4) one gets the relation,

$$(1/a_B) = 1/(a_{p+} + a_{e-}) = (1/a_{p+}) - (1/a_{e-}) \quad (5)$$

On multiplying throughout by  $a_{e-}$  and writing in terms of the ratio,  $(a_{e-}/a_{p+})$ , one obtains,

$$(a_{e-}/a_{p+})^2 - (a_{e-}/a_{p+}) - 1 = 0 \quad (6)$$

Eq. (6) is the Golden quadratic equation since its positive root is the Golden ratio ( $\phi$ ),

$$(a_B/a_{e-}) = (a_{e-}/a_{p+}) = (1 + 5^{1/2})/2 = 1.618... = \phi \quad (7)$$

$$a_{e-} = a_B/\phi \text{ and } a_{p+} = a_B/\phi^2 \quad (8)$$

It was thus found for the first time [7 - 9] that the Bohr radius is divided at the Golden point into two Golden sections,  $a_{e-} > a_{p+}$  pertaining to the electron and proton, and that their ratio is  $\phi$ , a mathematical constant, which appears in many spontaneous creations in the Universe [11]. Therefore, the atom is a unique construction of Nature!

Since the capacity  $C_B = \kappa a_B$ , on using Eq. (5), one obtains,

$$(1/C_B) = 1/(C_{p+} + C_{e-}) = (1/C_{p+}) - (1/C_{e-}) \quad (9)$$

where  $C_{e-} = \kappa a_{e-} = C_B/\phi$  and  $C_{p+} = \kappa a_{p+} = C_B/\phi^2$  are the capacities of the electron and proton respectively and are the Golden sections of the capacity of the atomic condenser,  $C_B = \kappa a_B$ .

### 3. Confirmation of the above interpretation

The next step was to check whether the division of the Bohr radius into two sections pertaining to the electron and proton makes sense. This was done [7 - 9] as follows.

The bond length  $d(HH) = 0.74 \text{ \AA}$  in the hydrogen molecule [12] is the diagonal,  $2^{1/2}a_{B,H}$  of a square with the Bohr radius as a side. Since the latter is divided into two Golden sections,  $d(HH)$  also has two Golden sections,  $d(H^+) = d(HH)/\phi^2 = 2^{1/2}a_{p+} = 0.28 \text{ \AA}$  and  $d(H^-) = d(HH)/\phi = 2^{1/2}a_{e-}/\phi = 0.46 \text{ \AA}$ . The ions  $H^+$  and  $H^-$  correspond to the resonance forms in the HH molecule [12]. It was amazing to find that for the bond lengths of partially ionic hydrogen halide (HX) bonds, Pauling [12] suggests empirically the value,  $0.28 \text{ \AA}$  as the radius of H. This proves that  $d(H^+)$  and therefore  $a_{p+}$  are correct. Moreover, on subtracting  $d(H^+)$  from the observed values of  $d(HX)$ , the covalent radii,  $d(X) = d(HX) - d(H^+)$  were obtained. This shows that the 'covalent and ionic radii are additive'. Figs. 1 and 2 show the various radii and distances of significance in the H atom and  $H_2$  molecule.

Similarly, it was found [7 - 9] that, the observed bond lengths  $d(MH)$  of alkali metal (M) hydrides were the sums of  $d(H^+)$  and  $d(M^+) = d(MM)/\phi^2$ , the Golden ratio based cationic radii of alkali metals, where  $d(MM)$  is the edge length in the bcc

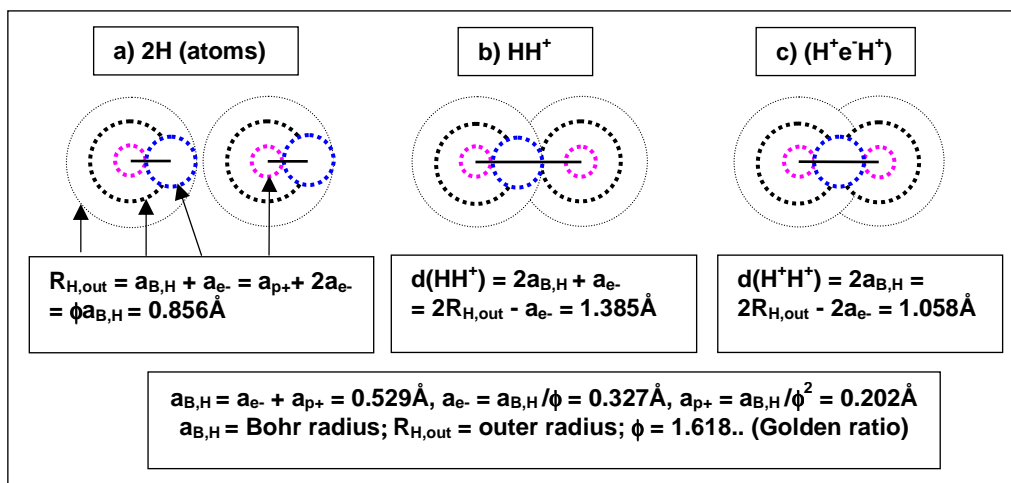


Fig.1. Two Hydrogen atoms (H,H),  $HH^+$  and  $(H^+e^-H^+)$  ions. The radii and various distances shown in the diagrams are drawn to scale and are related to  $\phi$ .

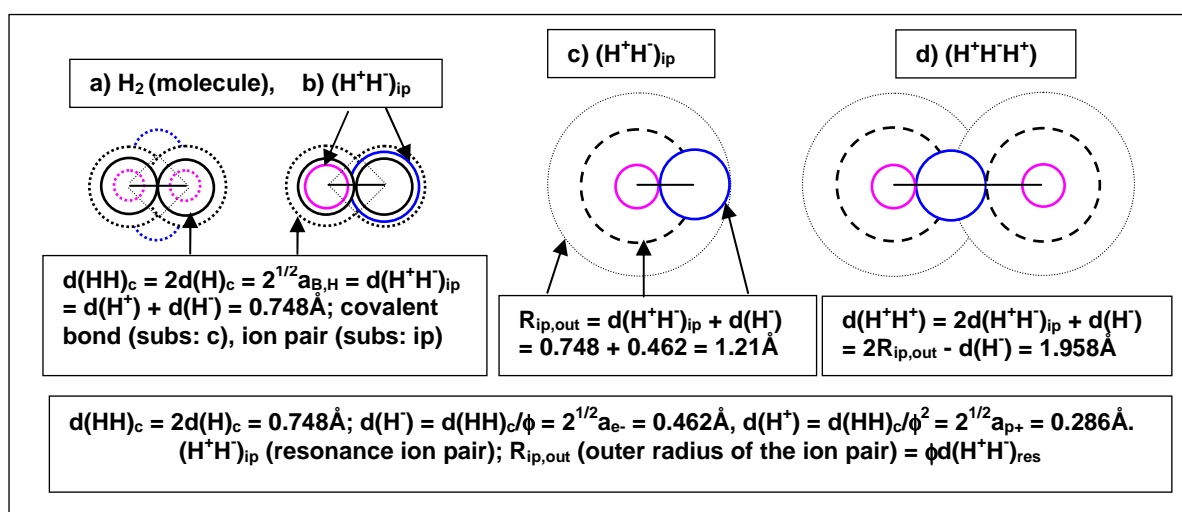


Fig. 2. Hydrogen molecule ( $H_2$ ), resonance ion pair  $(H^+H^-)_{ip}$ , and the  $(H^+H^-H^+)$  triple ion. The radii and various distances shown in the diagrams are drawn to scale and are related to  $\phi$ .

lattice of the metals. Therefore, the ionic character [12] of the MH bond is due to  $H^+$  and  $M^+$ . Moreover, the 'ionic radii are additive'.

Further, on subtracting the cationic radii,  $d(M^+)$  from the observed crystal ionic distances in alkali halides,  $d(MX)$ , it was a great surprise to obtain [7 - 9] exactly  $d(X^-) = d(MX)/\phi$ , which is the anionic radii of halogen atoms. This showed that Pauling's [12] radius ratio corrections are not necessary to account for the crystal ionic distance in alkali halides. Note that the 'Golden ratio based ionic radii are additive'.

During the last ten years, over twenty contributions were made to justify the above reinterpretation of Bohr's equation without the negative sign. These are listed in [13]. It is therefore concluded here that Eq. (2) with the positive sign for the energy is correct and hence the title of the paper.

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