# The Correct Derivation of Kepler's Third Law for Circular Orbits Reveals a Fatal Flaw in General Relativity Theory

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**Abstract**

In this paper the Kepler's third law is derived for circular orbits using the two different metrics. The resulting formulas are compared with the formula for the Kepler's third law derived from the Newtonian physics. The derivation is using the Lagrange formalism, but comments are made on error in derivation that has appeared in previous publication. It is found that the Kepler's third law derived using the Schwarzschild metric results in an identical formula obtained from the Newtonian physics of a flat spacetime geometry. This clearly illustrates a problem for the Schwarzschild metric and consequently for the General Relativity Theory.

**Key words:** Lagrange formalism, Kepler's third law, **r**elativistic Kepler's third law, Schwarzschild metric, metric derived in the Metric theory of gravity, errors in the General Relativity Theory

## 1. Introduction

The Kepler's third law is a very important law for astronomers, which is used to determine the mass of planets and stars based on the gravitational constant measured here on Earth and on the time of the planet's full orbit completion. The discovery of this law played an important role in the past in advancing the knowledge about our Solar system neighborhood and in convincing astronomers that the planets orbit the Sun and that the Moon orbits Earth. Because the time of the orbits can be measured with a high precision and the radius of the orbits is also reasonably well known, the mass of the centrally gravitating bodies can thus be found very accurately.

This law is easily derivable from the Newton inertial and gravitational laws for a circular orbit by equating the inertial centrifugal force with the gravitational force as follows:

$$\frac{v^2}{r} = \frac{\kappa M}{r^2} \tag{1}$$

where $M$ is the mass of the centrally gravitating body and $\kappa$ the gravitational constant. A hidden assumption used in this formula derivation is the absolute equality of inertial and gravitational masses, which is not strictly true [1]. By realizing that the average velocity is the length of the orbit circumference divided by the time of the orbit completion Equation 1 can be rewritten in the familiar third Kepler's law form:

$$t_{nt}^2 = \frac{4\pi^2 r^3}{\kappa M} \tag{2}$$

With many advances in the theory of gravity from the Newtonian to Einstein's General Relativity Theory (GRT) and further to more general Metric Theories of Gravity (MTG) it is thus natural to ask how is this law changed and is it accurate enough to determine, for example, the mass of our Sun with enough precision so that no large trajectory errors are generated when the space probes are sent to investigate other planets of our Solar system.

It is fascinating to see that this law plays again an important role in showing that the GRT is not the correct theory of gravity, similarly as the old planetary epicycle theory was shown incorrect, and that the GRT thus needs to be fundamentally changed.

## 2. The derivation of Kepler's third law for a general metric of a centrally gravitating mass

Several derivations of this law have been already published in the literature [2, 3]. The references given here are for the comparison purposes of various assumptions used in GRT derivations and in MTG derivation and the conclusions obtained from them. The derivation presented in this paper is very basic and more importantly it does not depend on the validity of the GRT. The Kepler's third law will therefore be derived first in a general form and then applied to two key cases: the Schwarzschild metric spacetime and the new metric spacetime derived previously by the author [1]. It is, of course, possible to apply the derived formulas to other metrics that can be found published in the literature, but the new metric satisfies the same four observational tests of GRT for the weak gravitational fields as the Schwarzschild metric does so it is interesting to make a comparison only between these two.

The general differential metric line element of a spacetime of a non-rotating centrally gravitating body is as follows:

$$ds^2 = g_{tt}(cdt)^2 - g_{rr}dr^2 - g_{\varphi\varphi}d\Omega^2 \tag{3}$$

where: $c$ is the local intergalactic speed of light, $d\Omega^2 = d\vartheta^2 + sin^2\vartheta d\varphi^2$, $g_{tt} = \exp(2\varphi_v)$, $g_{tt}\ g_{rr} = 1$, and where the metric coefficients depend only on the radial coordinate. This form of metric assumes that according to the Riemann hypothesis the motion can be represented by a curved spacetime in which the test bodies move in a free fall along geodesic lines and are not experiencing any forces in contrast to a flat spacetime with fields and forces that guide the motion. This concept forms the basis for all MTG theories and has also been adapted by Einstein in his derivation of general relativity. The Einstein's GRT, however, includes additional assumptions related to the Ricci tensor that led to the derivation of Einstein field equations with the Schwarzschild metric as a solution. The Riemann principle is thus more general in comparison to the GRT and allows derivation of other metrics describing the spacetime not only the Schwarzschild metric. In the new metric derived previously [1, 5] the metric coefficients are: $g_{tt} = \exp(-R_s/\rho)$, $g_{tt}\ g_{rr} = 1$, and $g_{\varphi\varphi} = \rho^2 g_{tt}$, while for the Schwarzschild metric they are: $g_{tt} = (1-R_s/r)$, $g_{tt}\ g_{rr} = 1$, and $g_{\varphi\varphi} = r^2$. The Schwarzschild radius $R_s$ is defined as usual as follows: $R_s = 2\kappa M/c^2$. Using the well-known and ages tested Lagrange formalism, considering for simplicity motion only in the equatorial plane, the Lagrangian describing such motion of a small test body in this spacetime is then as follows:

$$L = g_{tt}\left(\frac{cdt}{d\tau}\right)^2 - g_{rr}\left(\frac{dr}{d\tau}\right)^2 - g_{\varphi\varphi}\left(\frac{d\varphi}{d\tau}\right)^2 \tag{4}$$

The first integrals of Euler-Lagrange (EL) equations corresponding to the time and angle coordinates derived from the variational principle $\delta\int_\tau Ld\tau = 0$ are thus:

$$g_{tt}\left(\frac{dt}{d\tau}\right) = k \qquad g_{\varphi\varphi}\left(\frac{d\varphi}{d\tau}\right) = \alpha \tag{5}$$

where $k$ and $\alpha$ are arbitrary constants of integration. The EL equation of motion corresponding to the radial coordinate is as follows:

$$-\frac{d}{d\tau}\left(2g_{rr}\frac{dr}{d\tau}\right) = \dot{g}_{tt}\left(\frac{cdt}{d\tau}\right)^2 - \dot{g}_{rr}\left(\frac{dr}{d\tau}\right)^2 - \dot{g}_{\varphi\varphi}\left(\frac{d\varphi}{d\tau}\right)^2 \tag{6}$$

where the dot represents the partial derivative with respect to the radial coordinate. Since for the circular orbits the radial coordinate is constant with: $dr/d\tau \Rightarrow 0$, and $d^2r/d\tau^2 \Rightarrow 0$, Equation 6 simplifies to read:

$$\left(\frac{d\varphi}{dt}\right)^2 = c^2\frac{\dot{g}_{tt}}{\dot{g}_{\varphi\varphi}} \tag{7}$$

In this formula the first integral corresponding to the time coordinate shown in Equation 5 was used to eliminate the non-observable variable $\tau$. Considering now that the coordinate orbital time $t_o$, which is the observable quantity referenced to the central mass coordinate system, is found when the angle is set to: $\varphi = 2\pi$, the following equation is obtained:

$$t_o = \frac{2\pi}{c}\sqrt{\frac{\dot{g}_{\varphi\varphi}}{\dot{g}_{tt}}} \tag{8}$$

This is the general formula that can be used for any metric describing the spacetime of a non-rotating centrally gravitating body that conforms to a form given in Equation 3. For the Schwarzschild metric the result is:

$$t_{os} = \frac{2\pi}{c}\sqrt{\frac{2r^3}{R_s}} = 2\pi\sqrt{\frac{r^3}{\kappa\ M}} \tag{9}$$

Surprisingly this result is identical with the Newtonian case derived in Equation 2, which indicates that the Schwarzschild metric spacetime with the Ricci curvature tensor equal to zero does not have any effect on the planetary orbital period. Apparently even the event horizon does not seem to pose any problems for the orbital time. For example, inside of the Black Hole (BH) at $r = R_s/2$ the test bodies should whiz around at the vacuum speed of light and at the smaller radius even faster. This does not seem reasonable and therefore this metric does not describe the reality correctly as already discussed elsewhere [4, 5]. For the new metric, however, the result is:

$$t_{oh} = 2\pi\sqrt{\frac{\rho^3}{\kappa M} + \frac{\rho^2}{c^2}} \tag{10}$$

where the physical distance $\rho = \rho(r)$ is a function of the natural coordinate distance $r$ and is calculated using the following differential equation obtained from the metric:

$$d\rho = e^{R_s/2\rho}dr \tag{11}$$

For more clarity in understanding of these differences in orbital time formulas the results are for convenience summarized in a table T1.

The result in Equation 10 indicates that the orbital time has a limit for large $M$ equal to the physical length of the path divided by the speed of light. This is reasonable and easily understandable for the new metric since the orbital motion cannot exceed the speed of light. For the Schwarzschild metric formula, however, there is no such limit, which presents a significant problem for this metric and consequently for the GRT. The Schwarzschild metric describes the reality only approximately and should not be used to model the spacetimes with strong gravitational fields.

**T1**. Summary of the orbital time formulas for different metrics using coordinates referenced to the central mass:

| **Spacetime type** | **Metrics/Formula** | **Orbit time formula** |
|---|---|---|
| Flat | Newton-Kepler $m_g = m_i$ | $t_{nt} = 2\pi\sqrt{\frac{2r^3}{c^2 R_s}}$ |
| Curved | Schwarzschild metric | $t_{os} = t_{nt} = 2\pi\sqrt{\frac{2r^3}{c^2 R_s}}$ |
| | New metric | $t_{oh} = 2\pi\sqrt{\frac{2\rho^3}{c^2 R_s} + \frac{\rho^2}{c^2}}$ |

## 3. The derivation errors that are often made

In order to shorten the calculations it may be tempting to simplify the above presented derivation procedure and use the Lagrangian itself as the first integral. The fact that the Lagrangian is also a first integral equal to: $L = c^2$ can be found proven in many publications. The computation using this first integral and the first integral for the time coordinate with $k = 1$ as shown in Equation 5 would thus proceed as follows:

$$L_0 = g_{tt}\left(\frac{cdt}{d\tau}\right)^2 - g_{\varphi\varphi}\left(\frac{d\varphi}{d\tau}\right)^2 \tag{12}$$

resulting in the formula:

$$t_0 = \frac{2\pi}{c}\sqrt{\frac{g_{\varphi\varphi}}{g_{tt}(1-g_{tt})}} \tag{13}$$

which is markedly different from the correct formula shown in Equation 10. The approach similar to this one was used by Hynecek [6] and it is unfortunately incorrect. The correct calculation is available in another publication by

Hynecek [7], but this publication is not easily accessible and for this reason it is repeated here. The derivation error results from an incorrect imposition of constraint $dr/d\tau \Rightarrow 0$ on the Lagrangian in Equation 4 before the variations are carried out. The Lagrangian $L_0 = c^2$ in Equation 12 is, therefore, not the correct Lagrangian and consequently results in an incorrect first integral.

## 4. The dire consequences for the GRT

The fact that the correct derivation of the GRT Kepler's third law leads to the same formula as the formula derived from the Newtonian physics of flat spacetime geometry is well known to many mainstream relativists. They can even derive the Schwarzschild metric from the Kepler's third law [8]. The typical excuse that is often used is that this is due to the lucky choice of coordinates. This, of course, cannot be true. The formula presented in Equation 10 is an invariant. It does not matter what coordinates are used, because the physical coordinates are always the same and are not affected by the gravity. The formula also clearly includes the curvature of spacetime, which is described by the relationship between the physical coordinate $\rho$ and the natural coordinate $r$. The curvature of spacetime is clearly not included in Equation 9, which is an obvious and glaring problem. It is thus clear that the Schwarzschild metric does not correspond to reality and consequently the GRT is the incorrect theory of gravity and should be abandoned.

It is fascinating to see that the Kepler's third law for the circular orbits dispels again the myths of "relativistic epicycles" that the mainstream relativists so desperately adhere to. Unfortunately the facts do not matter here; it is the religion of GRT and its ideology that is not permitted to be challenged.

## 5. Conclusions

In this article it was clearly shown that the Schwarzschild metric of GRT does not correspond to reality. This is a fatal problem for the theory. The derivation used the Kepler's third law for circular orbits to show this problem. It was also shown that the previously published derivation contains a subtle error. The error was analyzed and its origin clearly explained. It is thus necessary to always correctly use and correctly adhere to the well tested and proven Lagrange formalism that so beautifully describes the physics of curved spacetimes as it was introduced by Riemann and others. It is also necessary to mention that this derivation did not take into account the repulsive dark matter [9] that permeates all the space and manifests itself when cosmological distances are involved. The Kepler's third law will thus have to be modified for such distances.

One can only wonder when the mainstream relativistic scientists will realize this problem and abandon the GRT with its preposterous Black Holes and Big Bang theory. Perhaps this will take another 100 years before enough conflicting data and observations accumulate and the theory crumbles under the weight of this evidence [10, 11]. Unfortunately this will not happen during my lifetime, so I will not be able to enjoy this wonderful paradigm shift. For the time being I am only enjoying the discovery of a very small, but beautiful, piece of eternal truth. Similar papers criticizing the GRT can easily be accessed elsewhere [12, 13].

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