Frequency Modulation of 0S2-D

Herbert Weidner [a]

Abstract: Precision measurements of the 0S2 quintet after the 2004-12-26 earthquake show that the spectral line near 313.8 µHz is frequency modulated with a remarkably large frequency deviation of 0.15 µHz. By proper choice of the integration length, the center frequency can be determined with high precision.

Introduction

After earthquakes, the Earth vibrates like a bell at different frequencies. The lowest ones near 300 µHz are particularly interesting because of their relative proximity to the rotation frequency of the earth. The remarkably wide error bars of all previous measurements are probably caused by the overlooked frequency modulation of these natural frequencies. High precision can only be achieved when the integration period is adapted to the modulation frequency.

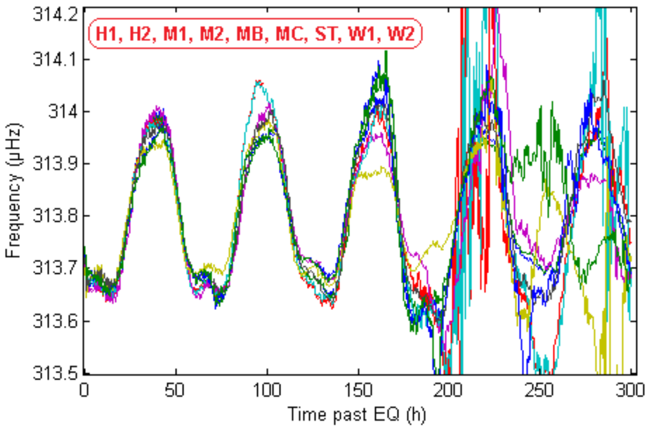
The Preparation of the data was described here[1]. The essential points: Before calculating the frequency, each data segment passed a narrow band Sinc filter with the bandwidth 0.8 µHz. A broadening of the bandwidth hardly changes the results. However, a reduction causes severe distortion.

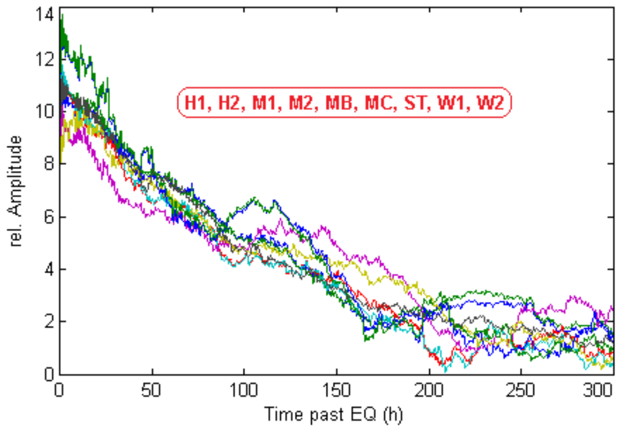
To distort the data as little as possible and to increase the frequency resolution, a new method[2] has been developed, eliminating the need for a window function and zero padding. In this way, any data corruption by the arbitrary window function is avoided. FFT is replaced by the faster Goertzel algorithm, because this allows period lengths that are not a power of 2.

The Frequency of 0S2-D (European Stations)

The instantaneous frequency differs almost always from the average frequency. The striking frequency deviation (± 0.15 µHz) far exceeds the deviation[3] of the other lines of the ${}_0S_2$ quintet[4].The frequency deviations of the two spectral lines at 304.6 µHz and 313.8 µHz are in phase opposition, as expected[5].

This frequency modulation was discovered a long time ago in a completely different way[6] (see Figure 5). The same lowest modulation period as now (60 hours) was also measured. All European stations consistently show the same frequency modulation of the spectral line ${}_0S_2$ -D. Due to the small mutual distances, the phase shift is nearly negligible.





Since the average frequency of a FM-oscillation is strongly influenced by the length of the integration period[1], a random choice of the integration time will surely generate an incorrect result.

a) 17. June 2015, email: herbertweidner@gmx.de

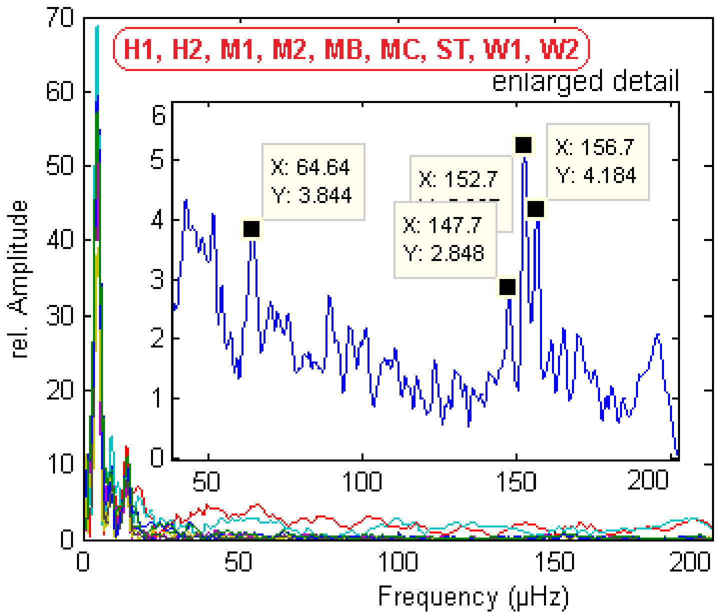
The following table shows how the mean value depends on the integration time (first 250 hours after the earthquake). As shown below, the modulation period length is $T = 60.47$ hours.

Int. Time	$1\cdot T$	$1.5\cdot T$	$2\cdot T$	$2.5\cdot T$	$3\cdot T$	$3.5\cdot T$	$4\cdot T$
f in µHz	313.79977	313.77640	313.80292	313.78957	313.80403	313.78884	313.80130

If we restrict ourselves to the fields *without* systematic error (highlighted in yellow), the jackknife method provides the mean frequency (313.80201 ± 0.00093) µHz.

The Modulation Frequencies (European Stations)

Another way to calculate the precise average frequency is to compensate for the frequency deviation by an opposite phase signal. As with the "detiding", the amplitudes and phases of all significant modulation frequencies must be determined. The spectral line $_0S_2$ -D is modulated with two different frequency groups. The lower group includes three individual frequencies: $f_1 = 4.59$ µHz, $f_2 = 9.02$ µHz and $f_3 = 13.90$ µHz. The higher group near 150 µHz is much weaker and may be neglected.

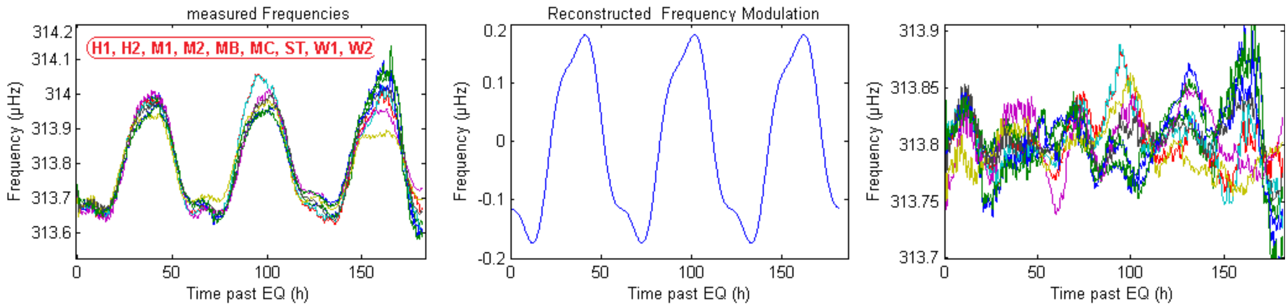


The strongest modulation frequency f_1 determines the period ($T = 60.47$ hours) of the very large frequency deviation.

With sine waves of these three frequencies, the actual time-dependent frequency course of each station can be reconstructed with high accuracy. The required amplitudes (in µHz) and phases (0 .. 2π) are tabulated below.

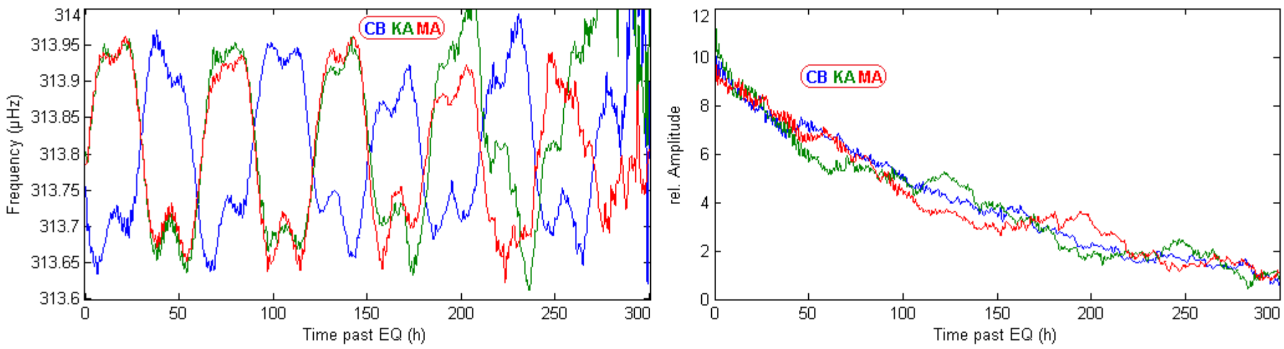
Station	$Ampl_1$	$Ampl_2$	$Ampl_3$	$Phase_1$	$Phase_2$	$Phase_3$
F (µHz)	4.64	9.22	13.80	4.64	9.22	13.80
H1	0.1871	0.0154	0.0227	3.84	5.97	5.96
H2	0.1828	0.0143	0.0227	3.83	5.83	5.93
M1	0.1891	0.0220	0.0241	3.90	0.91	6.12
M2	0.1910	0.0146	0.0248	3.92	0.54	6.11
MB	0.1692	0.0183	0.0169	3.78	0.69	6.05
MC	0.1293	0.0115	0.0213	3.98	0.03	5.85
ST	0.1755	0.0114	0.0220	3.83	0.35	6.05
W1	0.1724	0.0086	0.0276	3.90	0.88	0.18
W2	0.1740	0.0099	0.0297	3.90	0.27	0.13

The modulation function of each European station is individually calculated (one example is shown in the middle picture below) and subtracted from the measured values, resulting in the right picture. Since only the three lowest and most intense modulation frequencies were compensated, the difference (shown in the right picture) is not just noise, but still shows some regular structures.



The average of the nine difference curves was calculated with the jackknife method, yielding (313.80424 ± 0.00032) µHz. In earlier measurements[7], significantly larger error bands were given. This may have been caused by the ignorance of the frequency modulation with its consequences.

The Frequency of 0S2-D (CB, MA, KA non Europe)



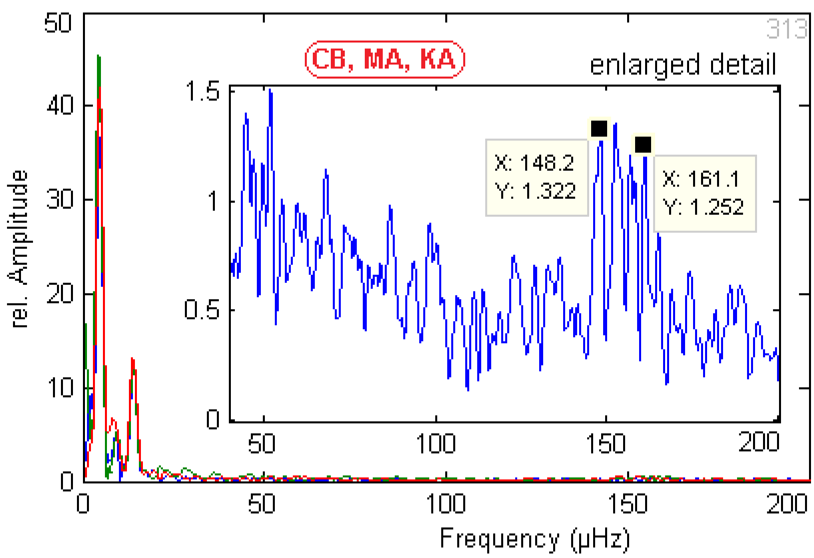
The stations CB, KA and MA measure a smaller frequency deviation (± 0.13 µHz) than the European stations. Data from S1 and S2 can not be evaluated due to strong interference. The following table shows how the mean value depends on the integration time (first 260 hours after the earthquake). As shown below, the modulation period length is $T = 61.2$ hours.

Int. Time	$1 \cdot T$	$1.5 \cdot T$	$2 \cdot T$	$2.5 \cdot T$	$3 \cdot T$	$3.5 \cdot T$	$4 \cdot T$
f in µHz	313.79994	313.81314	313.80197	313.81026	313.80157	313.80824	313.80452

If we restrict ourselves to the fields *without* systematic error (highlighted in yellow), the jackknife method provides the mean frequency (313.80200 ± 0.00095) µHz.

The Modulation Frequencies (CB, KA, MA)

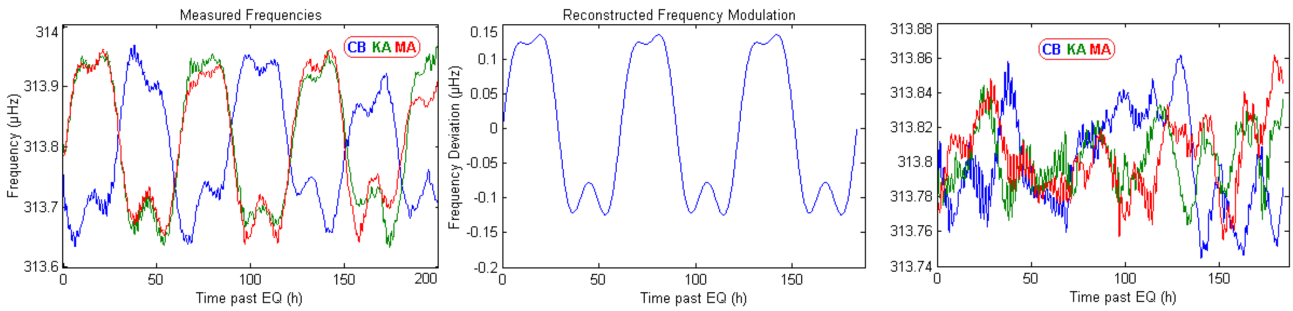
The low group includes three individual frequencies: $f_1 = 4.54$ µHz, $f_2 = 9.10$ µHz and $f_3 = 13.98$ µHz. The higher group near 150 µHz is much weaker and may be neglected.
The strongest modulation frequency f_1 determines the period ($T = 61.2$ hours) of the very large frequency deviation.



With sine waves of these three frequencies, the actual time-dependent frequency course of each station can be reconstructed with high accuracy. The required amplitudes (in µHz) and phases (0 .. 2π) are tabulated below.

Station	Ampl$_1$	Ampl$_2$	Ampl$_3$	Phase$_1$	Phase$_2$	Phase$_3$
F (µHz)	4.54	9.10	13.98	4.54	9.10	13.98
CB	0.1234	0.0144	0.0341	3.50	4.54	3.88
KA	0.1488	0.0242	0.0324	6.27	5.25	0.10
MA	0.1363	0.0224	0.0364	0.06	4.79	0.31

The modulation function of each European station is individually calculated (one example is shown in the middle picture below) and subtracted from the measured values, resulting in the right picture. Since only the three lowest and most intense modulation frequencies were compensated, the difference (shown in the right picture) is not just noise, but still shows some regular structures.



The average of the three difference curves was calculated with the jackknife method, yielding (313.80158 ± 0.00043) µHz.
The average phase-1 measured by European stations is in good agreement with the phase-1, which was measured by CB. In contrast, the signal measured by both Japanese stations was antiphase.

Amplitude Decay and Q-Factor of 0S2-D

The amplitude reduction of the $_0S_2$ -D frequency 313.8 µHz is expected to follow an exponential law. The decay during the first 220 hours past the earthquake may be described by the exponential function

$$A = A_0 \cdot e^{\frac{-t}{T}}$$

The time constant for the European stations is $T_{0S2-D} = (143.20 \pm 3.25)\, hours$.

The quality factor Q may be computed using the equation

$$A = A_0 \cdot e^{\frac{-t}{T}} \sin(\omega t + \varphi) = A_0 \cdot e^{\frac{-\omega t}{2Q}} \sin(\omega t + \varphi)$$

For $f_{0S2\text{-}D}$ =313.8 µHz, this equation yields

$$Q_{0S2-D} = 508.22 \pm 11.53$$

Acknowledgments

Thanks to the operators of the GGP stations for the excellent gravity data. The underlying data of this examination were measured by a net of about twenty SG distributed over all continents, the data are collected in the Global Geodynamic Project[8].

[1] H. Weidner, Frequency Modulation of 0S2-E, 2015, http://viXra.org/abs/1506.0084
[2] H. Weidner, A New method for High-resolution Frequency Measurements, 2015, http://vixra.org/abs/1506.0005
[3] The term *frequency deviation* is sometimes mistakenly used as synonymous with frequency drift, which is an unintended offset of an oscillator from its nominal frequency.
[4] H. Weidner, Frequency Modulation of 0S2-C, 2015, http://viXra.org/abs/1506.0099
[5] H. Weidner, Frequency Modulation of 0S2-B, 2015, DOI: 10.13140/RG.2.1.2789.3927, https://www.researchgate.net/publication/278404787_Frequency_Modulation_of_0S2-B
[6] H. Weidner, Envelope reconstruction of the 0S2 – free oscillation of the earth, 2010, http://arxiv.org/ftp/arxiv/papers/1012/1012.3259.pdf
[7] R. Häfner, R. Widmer-Schnidrig, Signature of 3-D density structure in spectra of the spheroidal free oscillation 0S2, *Geophys. J. Int.,2012*
[8] The "Global Geodynamics Project", http://www.eas.slu.edu/GGP/ggphome.html