Possible Common Solution to the Problems of Dark Energy and Dark Matter in the Universe

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Abstract

We present here a model for a common explanation of the phenomena of the dark energy and dark matter, which is lying outside the General relativity theory, yet still compatible or, perhaps, even complementary to it.

We discuss the principal results of the method of the *Causal dynamical triangulations*, when applied under the assumption of topology S^3 of our world, *i.e.*, assuming the closedness of the Universe. Then it can be concluded that the resulting space-dimensionality three, being equal to what we consider to be the naturally optimal dimensionality of the real space, implies the existence of a certain deviation from this optimal value on the cosmological scale-level (*i.e.* a deviation from the space-'Euclidicity' there), since a fourth space-dimension is explicitely or implicitely necessary (depending only on what form of the spacetime-metric one has used) in order to allow the Universe to be closed. As a consequence, the bent space (considered to be the component of the curved spacetime), together with the real cosmic stratum there, struggles to arrive to the state with the optimal dimensionality, *i.e.*, it struggles to expand, while the 'pseudo-pressure' is the carrier of the 'dimensionally-elastic' energy, which appears as the *dark energy* (on the global cosmological scale) and the *dark matter* (on the scale-level of cosmic inhomogenities). The basic rules for their appearance are presented, as well as the pertaining questions are discussed: the feedback of the proposed mechanism, the problem of the entropy and self-organization of the cosmic stratum, and the evolution of the phenomenon.

Keywords: cosmology: dark energy – dark matter – Causal dynamical triangulations – pseudopressure – accellerating expansion – dimensionally-elastic energy – feedback – entropy – self-organization – entropy-course and expansion

1. Introduction

A plethora of more or less speculative solutions to the problems of the dark energy and dark matter in the Universe were published during recent years. The authors of the proposed solutions, however, do not take into consideration the fact that the expansion of the Universe has a very specific character. It is not generally regarded that the expansion is not a classical physical motion and it has not the conventional dynamics. For the expansion, together with its apparent acceleration ascertained by means of observations of the distant supernovae of type Ia (Riess *et al.*, 1998 and Perlmutter *et al.*, 1999), only the common dynamical interpretation is presumed. Yet, other options for explanation of the acceleration exist, which will be presented in a subsequent section.

We start from the assumptions (motivated in Voráček, 2008) that:

(i) The Universe is more or less marginally closed and Machian, with the trivial topology and the metric of the 3-sphere:

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + r^2(\tau)\,[\,\mathrm{d}\psi^2 + \sin^2\psi\,(\,\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\,)\,]\ . \tag{1}$$

Here τ is the *cosmic time* of the *fundamental cosmological observer* (observing – by definition – the cosmic background radiation being isotropic) and ψ, θ, and ϕ are the angular coordinates on the 3-sphere[1]. Further,

$$\mathrm{d}\tau = \frac{r(\tau)}{r(\tau_0)}\,\mathrm{d}\tau_0\ , \tag{2}$$

where r is the radius of the Universe, τ is a cosmic epoch in general, and τ_0 is the actual cosmic epoch. (Then, $\mathrm{d}\tau_0$ is an element of the York time.)

(ii) For the Universe, the Law of mass-energy conservation is valid, while its *global* (*i.e.*, abolutely total) energy is equal to zero (Voráček, 2008).

(iii) The Universe either arose in the *world-start* (big bang) from nothing or it is cyclic (*i.e.*, the *world-end* (big crunch) of the actual cycle becomes the world-start of the subsequent cycle).

(iv) There exists only one form of the Friedmann equation (FE), valid both for the radiation-dominated and mass-dominated Universe:

$$\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)^2 - \frac{Rr_0}{r^2} = -1 \tag{3}$$

(Voráček, in preparation), where R and r_0 is the radius of the Universe at its maximal expansion and at the actual epoch, respectively. Except for situations where the derivative of equation (3) is taken (when the model parameters, but no observational parameters, are considered), it is possible to put $r_0 = r$; then the equation, according to the currently established opinion considered to be the FE valid only for the matter-dominated (closed) Universe, is obtained:

$$\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)^2 - \frac{R}{r} = -1\ . \tag{4}$$

(v) Under conditions quoted above and – namely – when the FE is fulfilled, an equilibrium exists between (the negative) *cosmological space tension* Σ and (the positive) *cosmological pseudo-pressure* P, with a, for the time being, unknown origin and character, so that the total *cosmological space stress* is equal to zero,

$$P + \Sigma = 0\ , \tag{5}$$

(Voráček, 1986a; p. 339), with possible exception for relatively small deviating oscillations.

[1] We avoid the notion 'co-moving coordinates', since – as already mentioned – the expansion of the Universe is not a classical physical motion.

2. Spacetime curvature and *space-bending* in the *demonstration diagrams*

It is necessary and important to realize the principal difference between the spacetime curvature in the vicinity of a source of the local static gravitational field (we consider – for the sake of simplicity – the Schwarzschild field) and the curvature of the spacetime in the expanding/compressing[2] closed Universe:

(a) In the local static field the space-derivatives of the metric coefficients are non-zero, while their time-derivatives equal to zero.

(b) In the cosmic gravitational field the situation is the opposite: At a given cosmological epoch the space-derivatives of metric coefficients are equal to zero, while the same coefficients are changing with the cosmic time in the expanding/compressing Universe.

As we have serious objections against the relevance of the *embedding diagrams* frequently used in the GRT for the local gravitational fields (introduced by Misner *et al.*, 1973; pp. 613–615), we prefer to use so called *demonstration diagrams*, where the space-component of a null-geodesic is presented depending on how it appears for the distant (coordinate) observer; the relevant influence of the time metric coefficient (g_{tt}) is thus regarded implicitly[3]. In such a way, *e.g.*, the photon horizon is represented by a 2-sphere with its incident radius and the center at the origo of the Schwarzschild coordinate system.

For the demonstration diagram of the Universe we use the 4-dimensional Euclidean space, which, however, necessarily must be dimensionally restricted for the practical use. Usually, the 2-sphere in the 3-dimensional Euclidean space is applied for such a purpose. There, for a given cosmological epoch, the *bending of the cosmic space*, being qualitatively different from the curvature of the spacetime in the local gravitational field, is sufficient; it is because in the local field the non-zero derivatives $\partial g_{tt}/\partial r$ and $\partial g_{rr}/\partial r$ [4] evidently are also deciding the shape of the formations in the demonstration diagrams, while the same derivatives are zero for the Universe at a given cosmological epoch τ. It is why we introduce the notion *bending of the cosmic space*, being relevant on the cosmological scale; in the dimensionally restricted demonstration diagram the bending of the space is easily and didactically represented by the simple classical bending of the respective 2-sphere. In Figure 1, a section through such a 2-sphere by the plane passing through its center and the place of the fundamental cosmological observer is shown.

Since our Universe is – as assumed – closed with trivial topology (*i.e.*, it is an expanding 3-sphere), its bending is intrinsic. It should be emphasized that the dimensional analogy we use is exactly relevant to the real cosmological situation only for a universe with the trivial topology being closed (alternatively open and Euclidean), but not for open non-Euclidean models or for models with non-trivial topologies. The radius of the Universe (the quantity denoted r in the FE) is thus equivalent to the notion of the radius of the 2-sphere in the demonstration diagram. Most frequently, a form of the Friedmann-Robertson-Walker (FRW) metric of spacetime with three non-Euclidean space-coordinates is used for the Universe (Lawden, 1982; Sections 60 and 61), but such a metric is equivalent to the one we use here (1).

In the FRW-metric, the curvature radius is considered to be just a parameter, not a vector, which is a logically necessary consequence in the (3+1)-dimensional curved spacetime where a

[2]We avoid here the notion *contraction* in order to respect the necessity to differentiate between the complex character of the phenomenon of *compressing* Universe and the usual relativistic contraction. Further, we differentiate between the notions *relaxation/constriction of the space* in the Universe and the *expansion/compression of the Universe*, but the difference will not be applied here.

[3]A *perfect demonstration diagram*, where the influence of metric coefficient g_{tt} would be taken into consideration, would necessarily have to be kinematic.

[4]. . . for the anisotropic Schwarzschild metric; for an isotropized metric: "... non-zero derivatives $\partial g_{ij}/\partial r$, where $i = j$, ..."

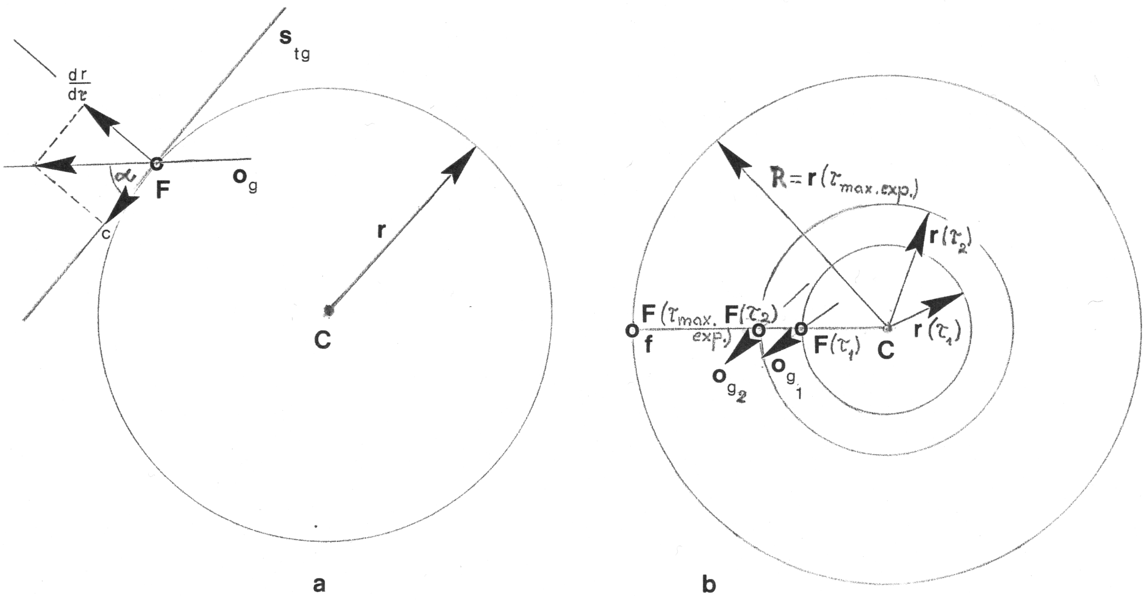

Figure 1: (a) The section through the restricted demonstration diagram of the Universe at given epoch τ by a plane through its center C and fundamental cosmological observer F. The center is beside the space of the universe, both in the diagram and in our real closed Universe. For observer F, the closest local part of the circle, being the *space component* of a null-geodesic with tangent o_g, is coinciding with the Euclidean tangential straight line s_{tg}; for $c = 1$, $\mathrm{d}r/\mathrm{d}\tau = \tan\alpha$. (A planar section through the center of our real Universe would be identical to the one presented here, despite the different number of dimensions!) In an analogous 3-D demonstration diagram it would be possible to draw the planar pencil of the tangential straight lines forming the Euclidean tangential plane, touching the pencil of great circles in point F, *i.e.*, touching the 2-sphere of the diagram. In the real space of the Universe, the spatial pencil (bundle) of the tangential straight lines can be drawn through the place of the fundamental observer, creating in such a way the Euclidean tangential space, touching in the point F the spatial pencil (bundle) of great circles in the *bent space*, *i.e.*, touching the 3-sphere of the Universe with a common central point beside it. The section shown in the diagram is usually called the *world-map* of the Universe. Then, even though it is just a section through the 3-D demonstration diagram, it is – consequently seen – the very world-map of the real Universe as well.
(b) The 2-D set of all world-maps at different epochs we then call the *world-scheme* of the Universe. There, the straight radial line f is the world-line of fundamental observer F, while o_{g1} and o_{g2} are tangents to the null-geodesics of the photons passing in his vicinity at epochs τ_1 and τ_2, respectively. (Another – more didactic – manner would be to design the world-scheme in 3-D, with the single world-maps (planar sections) for different cosmic epochs being circles at planes parallel with the plane of projection (here, the paper), having their centers on the straight line of coordinate τ (*i.e.*, on the τ-axis) passing through point C and being orthogonal to the plane of projection. Lines o_g, o_{g1} and o_{g2} in diagrams (a) and (b) are – more correctly – the time-parametrized orthogonal projections of the tangents to the null-geodesics from such a 3-D world-scheme onto the plane of the diagrams.)

fourth space dimension, relevant to the curvature radius, is a purely mathematical abstraction, while – in the Euclidean 4-space of the demonstration diagram – the radius-vectors of the Universe is orthogonal everywhere to its space, which is bent in the fourth dimension (*i.e.*, the radius-vector is not 'inside our Universe'). The hypersphere radius presented above and curvature radius used in the concept avoiding the Euclidean 4-space are identical.

3. Character of the *cosmological space tension*

Cosmological space tension Σ has a very good analogy in the surface tension on the soap bubble. Except for the number of dimensions, the principal difference consists in the fact that the mediating interaction in the Universe is the gravitation, while, on the soap bubble the surface tension is caused by the intermolecular forces. Both for the soap bubble and the Universe, the rule holds that the smaller the radius of the bending, the stronger the '(self-)compressing tendency' of the formation (Voráček, 1986a).

In the opposite situation, the analogy of the necessary bending is very illustrative and adequate: An infinitely extended soap membrane can not shrink, and – in an infinitely extended Euclidean (*i.e.* marginally open) universe – the gravitation can not act dynamically on a cosmological scale. It is possible to show that the Euclidean universe, with a positive value of the critical mass-density, has zero *global mass-energy* density entering the field-equation; such a universe is then *formally* (*i.e.*, in terms of mathematics) *empty* and thus the original static solution of Einstein (actually considered to be erroneous) can be taken as a relevant model (Voráček, 2008).

4. Character of the *cosmological pseudo-pressure*

In the epoch of the Universe dominated by the matter a cosmologically significant classical pressure does not exist in the cosmic stratum.

In the currently accepted standard description of the closed FRW-model universe, one more or less tacitly presumes that the gravitational tension classically dynamically decelerates its expansion, *i.e.*, that in such a process the adjacent gravitational force is in equilibrium with a cosmological inertia force originating in the matter of the cosmic stratum thanks to the deceleration. It is, however, a quite irrelevant description of the situation since:

(i) The expansion of the Universe is not a classical physical motion; it has the direction orthogonal to the physical cosmic space and actually still its *rate* $\mathrm{d}r/\mathrm{d}\tau > 1\ (= c)$.

(ii) The concept of a *cosmological inertia force* related to the *deceleration rate* $\mathrm{d}^2r/\mathrm{d}\tau^2$ of the cosmic expansion does not exist. The cosmic matter determining the origination of the inertia force in the sense of the Mach principle[5] (Voráček, 1983 and 1985) is relativistically at rest relative to each of its own elements in the expanding cosmic stratum, since the matter itself participates in the expansion of the Universe. (Here it should be emphasized that the matter on the 'opposite side of the bubble' is influencing a given mass-particle gravitationally only along its hypersurface – not 'directly through the space of the bubble', which, in cosmology, means not directly through the Euclidean 4-dimensional hyperspace 'inside' the Universe being limited by its 3-sphere; the 4-dimensional interior does not belong physically to the Universe.)

The relevant description of the dynamics of the cosmic expansion is – in our opinion – that the gravitational space tension is in equilibrium with a cosmologically effective pressure P. Referring to the law of mass-energy conservation we can perform the following deduction:

[5]The Mach principle is not inconsistent with the role of the Higgs boson.

For the mass-energy $m(\tau)$ of the cosmic stratum at epoch τ, defined as

$$m(\tau) = \rho(\tau)V(\tau) , \tag{6}$$

where $V(\tau)$ is the volume of the 3-sphere of the closed Universe

$$V(\tau) = 2\pi^2 r^3(\tau) \tag{7}$$

and $\rho(\tau)$ is the average density of the matter and energy in the Universe at the same epoch, holds that

$$-\,\mathrm{d}m = P\mathrm{d}V \tag{8}$$

(Misner *et al.*, 1973; pp. 726–730), which is the Law of mass-energy conservation in a closed system (for $c = 1$) as used in thermodynamics: The energy necessary for the expansion equals the decrement of the total mass-energy of the cosmic stratum. In the Law, the pressure has still the classical character and a performed physical work is usually presented as the work made by means of a force of pressure pushing on the surface of a piston. In the cosmology, the physical character of pressure P is not known *a priori*, but if it were unveiled that it is formally (*i.e.* mathematically) identical with the classical pressure, the respective mass-decrement dm necessarily remains to be determined in a quite unconventional way. Since 'no cosmic piston is pumping up' the Universe, the decrement of the bending of the space is offered as a hint when looking for something that fulfills its analogous function.

If the matter-energy of the cosmic stratum behaves in a Machian way (as presumed), *i.e.*, as the energy of the ideal photon gas, then – recalling equation (7) – the numerical density of the photons

$$\rho_n(\tau) \propto V^{-1}(\tau) \propto r^{-3}(\tau) , \tag{9}$$

while the representative frequency ν of a photon in the gas – and naturally its respective average energy as well – depends on $r(\tau)$ as

$$\nu(\tau) \propto r^{-1}(\tau) , \tag{10}$$

being a consequence of relation (2). Further, evidently,

$$\rho(\tau) \propto \rho_n(\tau)\nu(\tau) . \tag{11}$$

From the last three formulae follows that

$$\rho(\tau) \propto r^{-4}(\tau) \propto V^{-4/3}(\tau) . \tag{12}$$

Our aim is now to find the value of w in the Equation of state

$$P = w\rho . \tag{13}$$

From relations (6), (8), and (13) now follows that

$$\rho(\tau) \propto V^{-(w+1)}(\tau) , \tag{14}$$

which, compared with relation (12), results in the conclusion that $w = 1/3$ or:

$$PV^{4/3} = const. \tag{15}$$

It means that the cosmic stratum apparently behaves as an ideal adiabatic gas with $\kappa = 4/3$ in spite of the very physical character of effective pressure P obviously cannot be considered as conventional, and it is why we call such a quantity *(cosmological) pseudo-pressure.* The FE as it is presented by means of relation (4) is then an energy-equation (compatible with relation (10)) valid for the whole evolution-history of the Universe, in spite of the currently generally accepted opinion is, that such a form of FE is *not* valid for the radiation-dominated phase of the Universe. Nevertheless, in the era of dominating radiation, a physically significant part of pressure P can be identified as a conventional pressure P_0 in the cosmic stratum (measurable as the pressure on the wall of a theoretically absolutely cold vacuum cell, considered that effects of the zero-point radiation can be neglected). It means that, in the early Universe, the conventional component P_0 of total pressure P plays a synergic role in the expansion, acting directly dynamically within its bent space filled with cosmic stratum. (In the soap-bubble analogy, pseudo-pressure P could be represented by a *fictive* 'surface pressure' determined by the horizontal elasticity of the bubble membrane, where the pressure is working against the surface tension, still under the condition that the bubble membrane is bent; *horizontal* here means: ... being in every point of the bubble orthogonal to its radius.)

Anyway, the cosmological pseudo-pressure – in its prevalent part $(P - P_0)$ – seems to have an unknown *abstract physical character*.

On this place it would be emphasized that not even the conventional pressure-component P_0 can act dynamically on the expansion of the Universe if its infinitely extended space were not bent; such a situation would be identical to the situation with cosmological space tension in a Euclidean universe, as mentioned in Section 3. The conventional pressure-component P_0 can perform cosmological physical work only in the bent space, which is why it is possible to include it – by definition – into the notion of *cosmological pseudo-pressure*, in spite of its *concrete physical character*.

5. Alternative explanations of the apparent acceleration of the cosmic expansion

First option explaining in an unconventional way the apparent acceleration of the expansion of the Universe, as ascertained from the observations of the distant supernovae of type Ia, is the possibility that it is a manifestation of the cosmic expansion combined with relatively small oscillations. (The idea of the Universe with oscillating expansion rate appeared already long ago; Morikawa, 1991.)

The expansion is modulated by the oscillations going on within the whole volume of the 3-sphere simultaneously (*i.e.*, being the oscillations with modulus $l = 0$). We use the notion 'simultaneously' in place of the expression 'on the hypersurface of the homogeneity', considering that the expansion of the Universe *is not* a physical motion in the sense of the SRT: The *recession* is not any cause of the SRT-time dilation because the recession rate is not a conventional physical velocity; two cosmologically distant fundamental observers can – in principle – simply introduce a common cosmic time (τ) and establish their cosmologically synchronous cosmic clocks (although it is possible to make of them the common time etalons only after a mutual contact has been established and rather complicated connection arrangements have been made).[6] These conclusions are just implications of Weyl's postulate (Lambourne, 2010; p. 241). The quasi-contemporaneity of the above mentioned oscillations on the globally cosmological scale is determined by the common initial conditions in the process of the world-start, while the problem of the causal horizon is possible to overcome rather by means of the existence of the EPR-phenomenon already performed in laboratories than by the speculative mechanism of cosmic inflation. It is not excluded that higher modulating harmonics related to the considered oscillations exist (Karlsson, 1971).

One might object that a 'Universe with oscillations' is not described by the FE and thus the idea is only an unjustified speculation, but the similar problem arises considering that the Universe is not ideally homogeneous while the FE is commonly applied. It is quite possible that

[6] The SRT-Lorentz transformation can be easily modified in such a way that it becomes relevant for the cosmology in the expanding Universe (Voráček, in preparation).

both aspects – the oscillations and the deviation from the ideal homogeneity – are mutually determined.

The second option is similar to the possible explanation published by Ellis (2011).

Our modification of the mechanism presented there is founded on the Horák gravitational law in fluids (Horák, 1984). The law is logically explaining the mechanism behind the formation of the 'honeycomb-structure' of the cosmic matter with agglomerations forming isles in the centra of the cells. In addition to this explanation, according to the same law, the test particle (a galaxy) inside a spherical void, not having the central position there, must be expelled radially with an acceleration increasing with the distance from the center of the void; for more details, see the subsequent text. If our peculiar velocity (relative to a momentary co-local fundamental observer) were interpreted as a statistical fluctuation, and/or as a consequence of our only slightly non-central position in the void, then there would not be a conflict with the Cosmological principle[7] (Blanchard, 2010; pp. 628–631). The problem is, however, the absence of more observationally founded indications. Anyway, we would prefer such an explanation before the option explaining the acceleration of the cosmic expansion by means of oscillations, presented above.

Here it is important to mention that the article published by Cárdenas *et al.* (2013) is consistent with the possibility of the existence of a cosmic void with a (roughly) central agglomeration of matter, in combination with our recently ascertained position in the Laniakea supercluster. The authors claim that they have found "evidence for a low-redshift transition of the deceleration parameter indicating that the acceleration has passed a maximum around $z \approx 0.2$ and now evolves towards a deceleration phase in the near future."

The presented results are qualitatively compatible with the mentioned Horák gravitational law in fluids, since:

(i) The central region of the cosmic honeycomb cell with the agglomeration of matter is characterized by infall accelerations/velocities being opposite to the cosmic expansion. (The dispersion owing to peculiar velocities is high.)

(ii) Farther out, when the average density of the matter and energy becomes lower than the average density of the cosmic stratum, the gravitational attraction is changing to the effective radial repellence increasing with the radial distance from the center of the cell, thus apparently contributing to the cosmic expansion with the resulting phenomenon of its acceleration.

(iii) Approaching the cell wall, the dynamic influence of neighboring cells causes the successive weakening of the repellance.

(iv) At the cell-wall the galaxies from a neighboring cell are penetrating into our cell and then oscillate relative to the wall. The oscillations are strongly damped thanks to high viscosity of the matter in the wall. A weak, still significant, deceleration with the top at $z \approx 0.6$ to 0.7 is apparent. Clearly, such a damped oscillative movement in the direction towards us can be the cause.

We realize, however, that the qualitative concordance does not mean that the theoretical explanatory model is a guarantee for a quantitative match with reality as well: With the $z \approx 0.7$ for the cell wall, it means that its proper distance from us is 10.7×10^9 l.y. (= 3.3 Gpc). Is a void with such a huge radius possible? Nevertheless, with the actual radius of the Universe r_0 greater than

[7] The recently published Letter (Tully *et al.*, 2014) about the Laniakea supercluster of galaxies yields a new view on our cosmological position. The impression is that we are situated roughly centrally in a local dense conglomeration with a diameter of about 160 Mpc (= 520×10^6 light years) in a huge void. Any conflict with the Cosmological principle need not exist, since the highest probability for us as observers is to be situated in the central agglomeration or – rather less probably – within the wall of a cell in the honeycomb-structure of the cosmic matter.

340×10^9 l.y. (*i.e.* with $\Omega_{crit.} < \Omega_{tot} < 1.0046\ \Omega_{crit.}$, or $\mathrm{d}r/\mathrm{d}\tau > 14.8\ c$) such 'megavoids' would still present less than one percent of the circumference of the Universe, and then the criterium of its cosmological homogeneity could still be fulfilled.

Nonetheless, even if the question of the acceleration of the cosmic expansion were eliminated, the problem of the dark energy and the dark matter in the Universe still persists.

6. Cosmological pseudo-pressure as a manifestation of the existence of the *dimensionally-elastic energy* of the bent cosmic space, filled with the homogeneous matter and energy of the cosmic stratum, appearing as the *dark energy*

In 2004 Ambjorn, Jurkiewicz, and Loll published an essay where, under assumption of the existence of the (3+1)-spacetime in the local Lorentz frame of reference and of the validity of the Causality principle (including the absolute – *i.e.* light – velocity) on the Planck-scale, together with the chosen topology of a three-sphere (S^3), a model of the macroscopic world is deduced with the use of the *Causal dynamical triangulations* (CDT). The resulting dimensionality of its spacetime is then (3+1), which is not a self-evident consequence of the assumed (3+1)-dimensionality on the Planck-level; anyway, it is equal to that one of the real spacetime of our Universe. We consider that it is reasonable to assume that the space-dimensionality on the medium-macroscopic scale-level in the Universe is an optimal natural solution/choice under the really existing conditions. The result of the above quoted essay is thus putting the dimensionality choice, made on the Planck-scale-level, into the context with the physical reality characterized by the naturally optimal dimensionality of our world on the medium-macroscopic scale-level. As the chosen topology (S^3) is the same both for the quantum and macro worlds (inclusive the cosmological scale), the conclusion can be made that the used CDT-method is at least self-consistent. The assumption of the S^3-topology can however be legitimized as well, and thus, the status of the CDT can change from self-consistency to compatibility with reality. The justification of the assumption is based on theoretical deduction, which has been made by Horák (1963a,b) and successively developed and modified by us (Voráček, 1985 and 2008)[8], together with the result of the measurements of the Planck space telescope, revealing that no evidence for a multiply-connected topology has been found (Planck Collaboration, 2014b). Moreover, as the authors of the essay mention, the Euclidean dynamical triangulations always gave results inconsistent with reality. It is important that the CDT-deduction also yields two other results: The first of them means that the pertinent model universe must be expanding, which is in accordance with the GRT (giving the Friedmann equation, which describes the expansion of the Universe), as well as with the observed reality. The second result has a quite new impact. It states that there exists the positive cosmological 'constant' of matter-energy quality, being 'consumed' in the process of the expansion. Such a conclusion is permitted by the GRT, yet it is not *obligatory* there, while the CDT makes it *necessary.*

We developed the explanatory model for the dark energy and the dark matter that uses the quoted results, namely the possible relation between the closedness of the Universe and the positive cosmological 'constant'.

On the medium-macroscopic scale-level, the (3+1)-spacetime of our Universe is practically flat; of course, only if the local gravitational fields are not considered. The S^3-topology has no

[8] The closedness of the Universe can also easily be logically deduced from the quantum concept of the photon, which, however, is not generally sustained: A photon can not be emitted if its future reception is not guaranteed (by entanglement?). Then, if our Universe were open, the engineers at any municipal street illumination central would take notice of that the electricity consumption is significantly lower at the nights with a clear sky than during nights when the sky is covered by clouds.

measurable impact on the space geometry; the space on that level is thus practically perfectly Euclidean. On the cosmological level, however, as previously pointed out, in the closed Universe with the trivial topology, the necessity of the existence of the fourth spatial dimension for its adequate description appears explicitly (as the radius of the Universe) or implicitly (as the curvature radius of its space). Followingly, it means that the bending of the cosmic space into the fourth space-dimension is a deviation from the optimal number of space dimensions, which is three. (In other words, it is a deviation from the space-'Euclidicity'.) If the space of the Universe is filled with the matter and energy of the cosmic stratum, the situation necessarily leads to the origin of an elastic energy (the cosmological 'constant' of the CDT) and to a tendency to 'rectify' such a space in order to make it 3-dimensional and Euclidean. Such a tendency thus apparently has a character of cosmological pseudo-pressure P considered above. The positive *dimensionally-elastic energy* of the bent cosmic space filled by the matter and energy of the cosmic stratum, related to the pseudo-pressure is – as a consequence – the source of the gravitational field. On the scale-level of the homogeneity of the cosmic stratum it is possible to identify this energy as the enigmatic *dark energy* in the Universe.

In the 3-D demonstration diagram this energy would be represented as an energy of deformation of a matter-rich plane into the spherical shell. Such a practically technically hardly performable deformation is much more easily possible to represent by a slice being closely similar to its section by the plane (Fig. 1a), *i.e.* by an elastic blade closed into a circle (similar to a clock spring, but only with one winding – not a spiral), which struggles to rectify itself.

The situation in the smooth elastic bent blade, corresponding to the state in the phase of the early (radiation-dominated) Universe being highly homogeneous, is possible to demonstrate in such a manner that a certain part of the energy of the deformed spring is determined by a conventional pressure (caused, *e.g.*, by its high temperature) that is isotropic in the horizontal plane of the blade. (Recall the fictive 'surface pressure' in the bubble model.) The adjacent pressure-energy in its hot material is analogously relevant to P_0, while its part relevant to pressure-component $(P - P_0)$ is caused by the very deformation, *i.e.* the bending of the blade into the circle in the cold state.

7. Appearance of the *dimensionally-elastic* energy as the *dark matter* in the Universe having a hierarchic structure

In the era of the matter-dominated Universe, characterized by the hierarchic heterogeneity, the omnipresent bending of the cosmic space is interacting with the hierarchic deviations of the local density of the matter from the average density of the Universe, *i.e.* with deviations from its ideal homogeneity. The adjacent hierarchic dimensionally-elastic energy is then identifiable as the *dark matter*.[9] The mechanism of its behavior is not determined just by the magnitude of the deviation of the local matter and energy density from the average density of the Universe; it is much more complex, since the only possible link can be the local gravitational potential energy of the hierarchic deviation, which is determined by four – hitherto more or less unknown – rules:

(i) *The gravitational potential energy is related to that locality in the gravitational field, which is logically physically relevant to a given situation* (Voráček, 1979).

The conclusion follows from the theory of GR-Hamiltonian (*i.e.* the total energy) of a test particle applied for mechanics of the free particle in a local gravitational field, where

[9] The idea that the phenomena of dark energy and dark matter have one common explanation appears also in the essay of Capozziello (2012). Consistently with our solution presented here, Capozziello proposes the opinion that the cause need not be conventional energy and matter having the form of some "exotic ingredients", but that both are the curvature effects, and that the solution to the problem (being presented as well in the mentioned article) does not pertain into the frame of the actually presented form of the General relativity theory. In the rest of the essay, however, the key ideas of our solutions of the enigma are mutually quite different.

the potential energy would be related to the locality of a standard observer. In the case of a hierarchic matter-formation, being the deviation from the ideal homogeneity of the cosmological stratum, the potential gravitational energy of the matter in such a formation, with a density deviating from the cosmic average value, would be related to its center of gravity[10]; consequently, it is positive, with a value of zero in its center.

(ii) *The potential energy of a test particle in a gravitational field is localized in the particle* (Cooperstock, 2009; p. 86, Section 6.4 – as a conjecture, or Voráček – as a hitherto unpublished proof).[11]

(iii) (a) *The hierarchic dimensionally-elastic energy – i.e., the dark matter – manifests first on such a distance-scale from the center of gravity of a hierarchic formation, which is significant in the context with the scale of effectivity of the determining bending of the cosmic space.*

(b) *On the other side, for the identical reason, the same energy manifests significantly locally in a sub-region of the hierarchic formation only if the sub-region has a suffiently large space-extension.*

Sub-rule (a) is explaining why the dark matter manifests only outside the central regions of galaxies, while sub-rule (b) excludes the possibility that single stars, solar systems, or small star-groups could be connected with the dark matter in spite of their high baryon-density. Thus, both sub-rules together can answer the question why the lumps of baryonic matter forming the relatively small galactic satellites and more external parts of spiral arms, both being more distant from the center of gravity of a galaxy, yet having enough significant space-extension, usually are rich in dark matter. The rules can also explain – quite logically – why the great sub-galactic agglomerations of apparently thin baryonic gas at great distances from the mother-galaxy become the dark matter-abuntant formations. On the contrary, sub-rule (a) is explaining why the dark matter is not appearing gravitationally in the central regions of galaxies, despite the matter density being highest right there (Fig. 5) [12]. The query why the clusters of galaxies are – on average – evenly penetrated by the dark matter can be answered in such a way that it is generated by galaxies mutually gravitationally interacting on the huge distance-scale, while the intergalactic space in the cluster is practically baryonic matter-empty (picture reference 1: *El Gordo*). The gravitational potential in the group of galaxies has a high absolute value (when compared to the average cosmic value), but its space-gradient is low, namely in the central part of the group (Sarli *et al.*, 2014). The single galaxies in the galaxy-clusters evidently have a much higher dark-matter density than the clusters on average; the cause is the value of underlying baryon-density and their sufficient space-extension.

(iv) *The basic explanation of the phenomenon of dark matter indicates that the mechanism of its origin is working in a convergent feedback.*

Such a feedback is local, which means it *seemingly* increases the local baryon-density at its locality, while the *cosmological bending of space is not influenced on the local scale-level.* (The feedback-influence on the bending of the space on the cosmological scale-level will be considered in the subsequent text.) At this place, the enigma of the Bullet Cluster (Harvey *et al.*, 2015), (picture reference 2: *Bullet Cluster*) and of the Musket Ball Cluster (picture reference 3: *Musket Ball Cluster*) is easily solvable: During the penetrative meeting

[10] We consider such an application of the rule above being a logically well-motivated conjecture.

[11] The rules (i) and (ii) together give a simple logical solution to the 'arch-problem' of localization of gravitational energy in the GRT.

[12] The apparent absence of dark matter in the galactic region of the Sun (Bidin *et al.*, 2012) might be explained by rules (a) and (b) together, considering that the distance from the center of the Galaxy is not too great and that the considered regions are not too large.

of two groups of galaxies, the thin intergalactic gas in the groups collided, while their 'dark matter', together with the galaxies generating the same 'dark matter' passed without collisions. According to our view of the substance of the 'dark matter' in the galactic clusters, presented above, the absence of its interaction is quite logical.

It is possible to demonstrate the function of some of the rules above in a simple didactic manner using again the blade of the steel spring, where the thickness of its material represents the hierarchic density variations of the baryonic matter and energy in the cosmic stratum (Fig. 2, 3, 4, and 5).

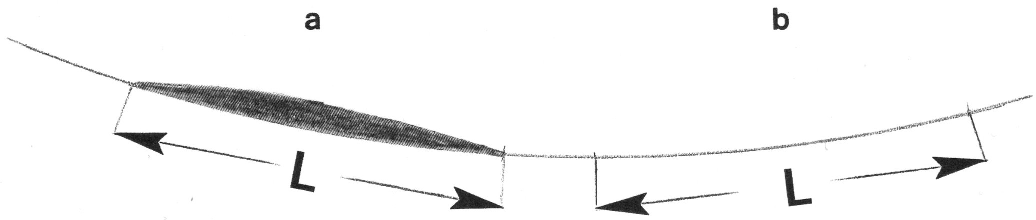

Figure 2: In situation (a) a much higher elastic energy is present on the length-scale L in the region with high local density of matter, determined by the hierarchic structure of cosmic stratum, than is the case on the same scale in situation (b) in the region with the homogeneous stratum having an average cosmological density. In both situations, the degree of the cosmological bending of the cosmic space is the same. (Yet, it must be pointed out that we realize that we are borrowing here the spatial hyperdimension in the demonstration diagram for an explanation of the role of the thickness of the steel blade.)

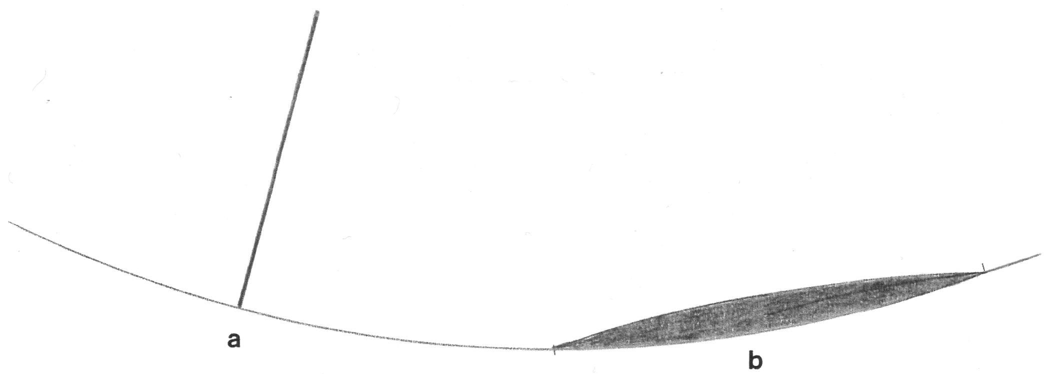

Figure 3: (a) The region of the sharply increased (needle-shaped) thickness of the spring blade contains less elastic energy than (b) the wide region with moderately increased thickness.

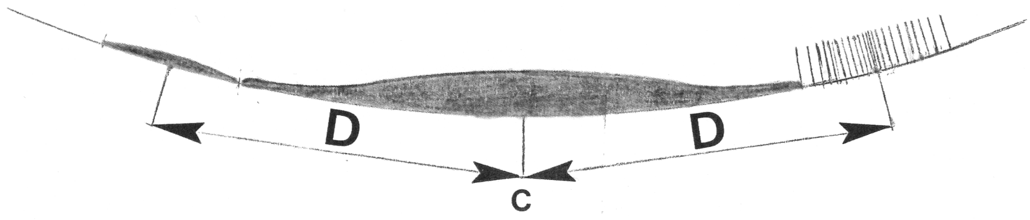

Figure 4: The galactic satellite with the hierarchic gravitational binding to a massive galaxy at great distance D from center of gravity C of the galactic system is demonstrated by a small, but still significant, increment of the thickness of the blade. The density of the baryonic matter in the galactic satellite is sometimes observed to be very low, while the dark matter-density is gravitationally significant (Geha *et al.*, 2009). Here it is also possible to explain (zoomed at right) why the stars in the galactic satellites (and – as a matter of fact – even the stars in galaxies) have not their adjacent dark matter components of their masses localized 'sharply' at their places: Following consequently such a (false) idea, it would be necessary to consider them as very long and thin needle-shaped isolated sources of gravitation as presented in Figure 3a; then, the single stars can not cause a 'sharp' appearance of a significant quantity of dark matter, and this is why they play a relevant role solely in great quantities and then merely as extended diffuse objects.

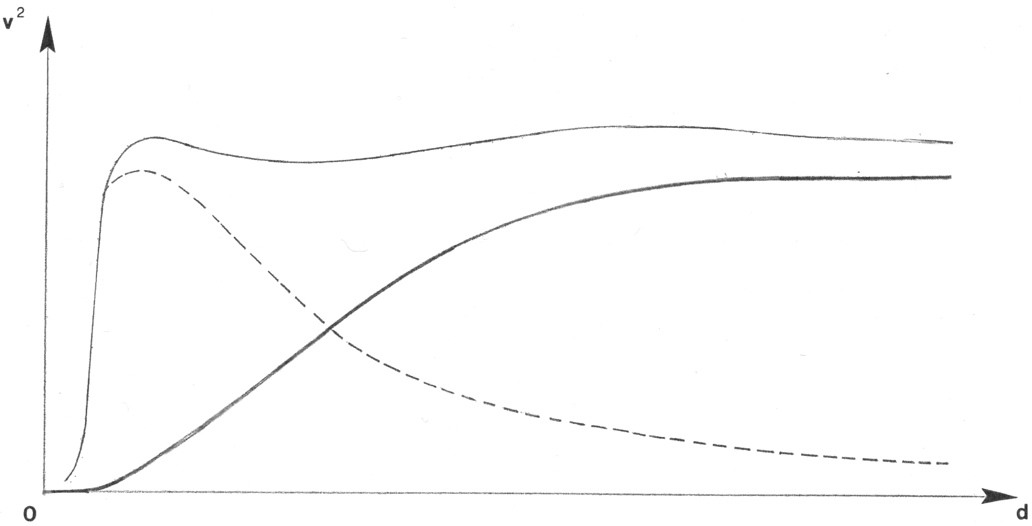

Figure 5: A qualitative diagram explaining the ascertained typical distribution of the total gravitational matter-energy in an 'average' galaxy. It is a composition of the appearance of a classical gravitational field of the baryonic matter and the influence of the cosmological bending of the space in the hierarchical environment of the galaxy appearing as the dark matter, with reference to rule (iii-a) above. In the diagram are plotted squares v^2 of the observed values of orbital velocities in the galactic disc (smooth line), their expected values (dashed line) and their respective differences caused by the presence of dark matter (bold smooth line), in relation to radial distance d from the galactic center.

In the qualitatively rough approach, it seems that the current knowledge of the galactic dynamics is compatible with the existence of an axially symmetric ring of gravitationally effective ring of the *apparent dark matter* extending in the galactic plane from a certain radial distance outwards (Binney and Tremaine, 1987; Section 3.3). Yet, a thorough complex analysis under the adequate angle of view is necessary in order to arrive at more exact conclusions.

The presented explanation of the phenomenon of dark matter is also consistent with the existence of extremely diffuse galaxies (Van Dokkum *et al.*, 2015).

During the early epochs with a small radius of the Universe, *i.e.*, with the higher degree of the bending of its space, the dark matter – quite logically – was bound much more 'eagerly' to the localities with relatively less outstanding positive density deviations (from the average value) in the baryonic matter. It means that then it is possible to explain why the conglomerates of the dark matter observed at great distances are smaller and more rich in details than those relatively close to us. It also explains the possible mechanism of the transfer of the energy of pseudo-pressure P into the dark matter and the rate of such a process: The small hierarchic inhomogeneities had a great qualitatively effective capacity to appear as holders of the dark matter already at the first stages of the matter-dominated era with a small radius of the Universe. Later, the bending of the space was diminishing owing to the expansion of the Universe, but the hierarchic inhomogeneities were increasing with time, as well as the contrasts of their baryonic matter-density. The actually observed picture of the early development thus appears as rather complex in order to be interpreted simply in an adequate manner; yet it is possible that the morphology of great hierarchic formations at huge distances is observed to be rather fuzzy and – as a consequence – the sub-groups of small distant hierarchic inhomogenities with relatively low contrast, appear as coalesced into the huge 'monolithic' mega-formations. Another option is, that the ascertainment of huge monolithic conglomerations of dark matter at great distances is not charged by an observational fuzziness, but that it has a real ground in (i) their enormous space-extension and (ii) the high degree of the cosmic space-bending, while (iii) the deviation of their baryon-density from the average cosmological value were still very low (picture reference 4: *Three-Dimensional Distribution of Dark Matter in the Universe*).

8. The *cosmological feedback*

The dark matter and dark energy are the strong sources of the gravitational field of the Universe, *i.e.*, they are strongly contributing to the effective average density of the cosmic stratum, and thus, they also *globally* determine the bending of the space in the Universe. Consequently, in such a mechanism, they become the sources of the gravitational cosmic field in the feedback. Then, being both determined by the *same* bending of the cosmic space, it is possible to describe them mathematically together by means of a sum of an infinite geometric series with quotient q:

$$\overline{\rho}_{tot.} = \frac{\overline{\rho}_{bar.}}{1-q} \quad , \tag{16}$$

where $\overline{\rho}$ are the average cosmological relative values ($\overline{\rho}_{tot.} = 1$) of the respective densities. Since $\overline{\rho}_{bar.}$ for our epoch has the value 0.049 (which means that 4.9% is the relative density of the baryonic matter), we obtain for feedback-quotient q the value equal to 0.951.

Any retardation of information about the state of the homogeneous Universe is (according to the Standard model) eliminated as:

(i) The cosmological space bending is for given epoch τ the same in the whole Universe.

(ii) All hierarchic inhomogeneities have a scale lying under/within the causality horizon for every fundamental cosmological observer.

The dark energy and dark matter are determined by the bending of the cosmic space, *i.e.*– more generally – by the curvature of the spacetime of our Universe. Since the spacetime curvature is tightly related to the gravitation, it is possible to state that the gravitational interaction is the mediating agent in the phenomenon of appearance of both dark energy and dark matter.

9. Entropy of the dimensionally-elastic energy

In our previous paper (Voráček, 1991) the entropy of the dimensionally-elastic energy was denoted (in a rather intuitive manner) as the entropy appertaining to the expansion of the Universe. Now we can present the same idea more precisely: It is the entropy appertaining to the energy that is the direct cause of the expansion of our Universe, which is possible to claim since the matter and energy are merely two qualitatively different forms of the same substance. Also, the conclusion was made that the *total generalized entropy* of a closed universe, being a totally isolated reversible and cyclic system, is constant. Its reversibility is determined, besides the closedness of the Universe, exclusively by the Planck density of the matter-energy, as the only parameter characterizing the cosmic stratum at the world-start (big bang) and world-end (big crunch). In the cosmology it is possible and useful to normalize such entropy to zero, as the mentioned parameter is unique for the Universe[13].

Thus, in the same manner as total entropy $\mathscr{S}$, the *specific entropy*, defined as

$$\mathscr{S}_0 = \mathscr{S} V^{-1} \quad , \tag{17}$$

is consequently normalized. In the formula, V is the volume of a closed compact universe, as specified by relation (7). (In cosmology it is reasonable to relate specific entropy $\mathscr{S}_0$ to volume V as the sole – for the intended purpose available – parameter, being well-defined during the whole process of evolution of the Universe; thus, specific entropy $\mathscr{S}_0$ becomes the *generalized entropy-density.*) Actually, the total generalized entropy of the cosmic stratum filling the cosmic space consits only of two components, one being the entropy of the dimensionally-elastic energy and the second, the entropy of the baryonic matter with all possible conventional forms of energy pertaining to it. Their sum is the *generalized entropy*, which is the constant equal – as we choose – to zero, while the entropy of the baryonic matter-component is positive.

Then, the entropy of the dimensionally-elastic energy necessarily must be negative. Yet, from our perspective it is impossible to decide whether the absolute values of the entropy-components are increasing or decreasing with the cosmic expansion. (In Voráček 1991 it was assumed that they are decreasing, but today we realize that such an assumption was not reserved enough.) It is however plausible to claim that the absolute values of respective entropy-densities are decreasing. Such an outstanding character of those densities is determined by the fact – being special for cosmology – that for the rate of expansion of the Universe the condition $\mathrm{d}r/\mathrm{d}\tau \gg 1$ (where $c = 1$) is valid in the decisive stages of its evolution; actually, the rate is still greater than one[14]. Thus, since the positive entropy-density component of the cosmic stratum is decreasing owing to the expansion of the Universe, the conditions are given for the origination of local sources and sinks of energy, as well as for the formation of the hierarchic systems on subsequently increasing space scales, *i.e.*, of the process of *self-organization* of the baryonic matter filling our Universe.

[13]We consider that the volume of the Universe at its start has a value resulting from the laws of the GRT and quantum physics. A thorough explanation of the idea, however, lies outside the frame of this essay. (Here it can only be said that the validity of the Law of mass-energy conservation is the starting point of the explanation.)

[14]The value of $\mathrm{d}r/\mathrm{d}\tau$ can actually be equal, very roughly, to 21 ± 2, if the results from the Planck space telescope are regarded. Assuming a closed Universe, the result of the Planck space telescope by itself limits this value to be greater than 19 (Planck Collaboration, 2014a).

10. Course of the generalized total entropy and its components during the evolution of the Universe until now

(a) During the short era of the very beginning of the Universe, its stratum consisted exclusively of unified bosons, which can be called 'hyper-photons'; in terms of thermodynamics, it consisted of the ideal adiabatic photon-gas, which is extremely ultra-relativistic and homogeneous. The pressure of the gas during those short moments is possible to identify with concrete conventional pressure P_0, being at that time the only component of total cosmological pseudo-pressure P. The pressure-energy of the photon-gas may be of use in the process of expansion only in the positively bent cosmic space, *i.e.* in the closed Universe, while the same energy is determined by the pertinent bending. (For more details, see Voráček, 2008.) In this sense, the pressure-energy of the photonic gas could still be considered (which we will anyhow avoid in the subsequent text) as a kind of dimensionally-elastic energy, even though it can be specified to be the sum of the energies of the single photons; regarding the cosmic expansion, it is possible to claim that "the space, in such a special manner, struggled to establish its optimal dimensionality (*i.e.* the 'Euclidicity')". The pressure-energy of the photonic gas had been consumed during that era solely for the compensation of the negative[15] cosmological potential energy of (as assumed) closed Universe in the process of the cosmic expansion; the mechanism is quite similar to the gravitational redshift of the photons moving outwards in a local gravitational field, when a photon is moving from one place to another with a successively diminishing absolute value of the (negative) gravitational potential at the respective places.

From the viewpoint of thermodynamics, the energy consumed for the expansion of the Universe is analogous to the work done be the expanding ideal gas pushing on a piston in an adiabatic process. *If* the stratum of the Universe forever remained *only* in the form of homogeneous photon-gas, its pressure P_0 (with the physical character described above) would be non-negligible even at the epoch of maximal expansion, being still/constantly in equilibrium with the space tension Σ, while $\mathrm{d}r/\mathrm{d}\tau$ momentarily would be equal to zero. Neither dark energy nor dark matter would exist.

Knowing the character of the adiabatic photon-gas, as described in Section 4, its picture presented there is hereby completed; the usual concept of entropy of the cosmic stratum, being hitherto constant, need not yet be generalized. We put that entropy equal to zero, which was motivated in Section 9; its value is usually considered to be quite arbitrary, or it has another fixed value motivated in another way (Rubakov, in Gorbunov and Rubakov, 2011; p. 99). Anyway, from the Planck era the photonic gas is starting with an entropy, which is the maximal possible in the given situation.

(b) The character of the composition of the cosmic stratum, however, began to change already in the consecutive – still very early – stage of the expansion of the Universe. The original pressure-energy of the photonic gas (being *always* equal to the sum of the energies of the single photons), besides the overwhelming consumption of it by the above mentioned energetically 'voracious' expansion of the Universe, began to transform to *baryonic matter*. (The notion 'baryonic matter' – as it is used in this paper – comprises all non-photonic elementary particles[16] together with their kinetic energies relative to a fundamental cosmological observer, as well as their mutual binding energies.) Together with the origination of the baryonic matter the adjacent dark energy (together with its feedback component)

[15] Consistently with the FE (4), the cosmological potential energy is negative when it is related to the epoch of the maximal expansion, while the FE is considered to be the Law of mass-energy conservation applied to the Universe.

[16] The term 'non-photonic elementary particles' here refers to particles not moving at the absolute velocity c. The term 'baryonic matter' is used because we could not find a proper established adequate term.

was also arising, undoubtedly in a process of transformation from the pressure-energy of the photon-gas. Further, it is needed to take note that in the era we consider, the cosmic stratum was still highly homogeneous on the scale-level higher than that of elementary particles, and thus, the dark matter was not yet present; the very dimensionally-elastic energy consisted exclusively of dark energy. That era could possibly be called the 'era of mixed photon-baryon Universe'.

Pressure P_0 of the photon-gas was becoming less and less significant and the value of the constant on the right-hand side of equation (15) was diminishing, caused by the decrement of the total number of photons in the Universe. In spite of this, the remaining photon-gas was still adiabatic (as is the state of the CBR, which actually has a cosmologically quite negligible total energy). Then, the entropy of the remaining photonic gas is not changing, *i.e.*, it is constantly equal to zero, as originally chosen. It is so, because at any epoch, with a given value of the right-hand side of equation (15), the process is – at least potentionally – reversible (which means – if the conversion to baryonic matter was stopped at the same moment).

Now the notion of the total entropy of the Universe needs to be generalized. It split into three components with the sum equal to zero, as explained in Section 9. The concept of the (zero-)entropy of the remaining photon-gas need not be changed, as already indicated above, while the positive entropy of the baryonic matter must be generalized, because its form can differ very strongly from the aggregation state of a gas. The same conclusion is applicable for the dimensionally-elastic energy, since its form is quite abstract; anyway, its generalized entropy-component is necessarily negative.

(c) The end of the mixed photon-baryon era is characterized by the decrease of photon pressure P_0 to the values so low that the relevant energy-component becomes cosmologically negligible. After the establishment of the baryon-dominated Universe, the whole effective pressure P has the form of the abstract representative pseudo-pressure. The extremely high expansion rate $\mathrm{d}r/\mathrm{d}\tau$ of the Universe manifested itself in the earliest eras in form of enormous recession rate already on the scale of elementary particles, which probably made the process of photon transformation into baryonic matter quite necessary. The expansion, still having the high rate, caused rather later that statistical fluctuations in the density of the baryonic matter could not dissipate, yet — just oppositely – they diverged. It resulted necessarily in the dehomogenization of the cosmic stratum on a successively increasing scale of extension in the space, from the scale of elementary particles in the earliest beginning-epochs to the scale of super-groups of galaxies much more later.

In the angle of view of thermodynamics, the divergent fluctuations in the density of the baryonic matter created local sources of energy – the stars, together with the effective energy-sinks (for the radiation produced by the stars) in the form of cavities – the voids. In such a way, the local regions arose where the respective local thermodynamical systems became open and where the cosmic baryonic stratum were not in a state of thermodynamical equilibrium. As a consequence, the conditions for validity of the Second thermodynamical theorem were not fulfilled there, which means that deviations from the validity of the theorem could appear there, and thus, the entropy in the considered regions could locally decrease. The process (still actually) manifests as *self-organization* of baryonic matter. In conventional physics, such a process in one locality must be followed by an increase of entropy in another place. In cosmology, however, the process of the hierarchic self-organization of the cosmic stratum is determined by the very special condition of the high expansion rate of the Universe, together with the (not just locally, but on average) decreasing entropy-density of the baryonic matter. Thus, a local entropy-decrement at the given place, which is a manifestation of the process of self-organization in the baryonic matter of the cosmic stratum, need not have, as a consequence, any local entropy increment in some other place.

Nonetheless, the local entropy decrement (being practically the same notion as the decrement of the entropy-density) of the baryonic matter is compensated at a given place by the *co-local* increment of the *negative* entropy (being practically the same notion as the increment of the entropy-density) of the dark matter.

In the hierarchically dehomogenized cosmic stratum, the energy of pseudo-pressure P consists only of the dimensionally-elastic energy in the forms of the dark energy and dark matter together.

The last logical query at this moment is: From what source came the dark matter that arose in the era when the pool of energy of photon-gas, with photons of significant individual energies and of sufficient numerical density, was already empty? Only two possible sources existed: (i) The pool of dark energy, which was partially consumed in the pertinent process, and/or (ii) a part of the already existing baryonic matter. In spite of the fact that no mechanism of transformation of the baryonic matter into dark matter is mentioned in the – for us known – literature, we consider that the existence of such a mechanism is not completely excluded. For instance, a new description of the physics at the event horizon of black holes (Voráček, 1997), and/or the impact of the expansion of the Universe on peculiar velocities in its stratum (Voráček, 1986b; pp. 405–408 or – equivalently – Kolb and Turner, 1990; formula (2.30), p. 38), seems to us a possible, though rather complex, solution to the present query.

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