The General Lorentz Transformations

(General Special Relativity)

(draft)

In this paper I introduce the General Lorentz transformations which are a generalization of the original Lorentz transformations. These new transformations are the foundations of a new theory of relativity that I call: General Special Relativity (or Special Relativity based on the General Lorentz transformations). In this paper (a) I derive the formula for time dilation, (b) the formula for length contraction, (c) a new relativistic velocity addition formula which encompasses Einstein's counterpart, and finally (d) I prove that Newton's law of Universal Gravitation is invariant under a General Lorentz transformation.

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1. Introduction

Despite the fact the Lorentz transformations are generally attributed to H. A. Lorentz, they were discovered by W. Voigt in 1887 and published in 1904. This means that the transformations were discovered before the Special Theory of Relativity [1]. These transformations reduce to the Galilean equations when the velocity of the body, v, is small compared to the speed of light, c. A law of physics could be invariant under a given transformation but it could turn out to be not invariant under a different set of equations. Maxwell's equations of electromagnetism, for example, are invariant under a Lorentz transformation, but they are not invariant under a Galilean transformation. In contrast, acceleration is invariant under a Galilean transformation but is not invariant under a Lorentz transformation. The Lorentz transformations are given in **Table 1**.

Lorentz Transformations *Direct transformations* (Values of system *S′* in terms of values of system *S*)		Lorentz Transformations *Inverse transformations* (Values of system *S* in terms of values of system *S′*)	
$x' = \frac{x - v\,t}{\sqrt{1-\beta^2}}$	(1.1)	$x = \frac{x' + v\,t'}{\sqrt{1-\beta^2}}$	(1.5)
$y' = y$	(1.2)	$y = y'$	(1.6)
$z' = z$	(1.3)	$z = z'$	(1.7)
$t' = \frac{t - \frac{v\,x}{c^2}}{\sqrt{1-\beta^2}}$	(1.4)	$t = \frac{t' + \frac{v\,x'}{c^2}}{\sqrt{1-\beta^2}}$	(1.8)

Table 1: *The Lorentz transformations.*

I shall call *direct transformations* to the set of equations containing minus signs (first column of Table 1), and *inverse transformations* to the set containing the plus signs (second column of Table 1). Two problems affect the Lorentz transformations:

(a) The first problem is that the transformations are classical, meaning that they cannot be applied to the quantum world which is characterized by a minimum quantum distance, L_P , and a minimum quantum time, T_P . I have corrected this problem in three articles I published recently [2, 3, 4].

(b) The second problem is that the transformations do not consider the general case when the velocity vector, $\vec{v}$ (relative velocity between two inertial reference systems: *S* and *S′*), is not parallel to any of the Cartesian coordinates axes of system *S*.

Therefore the Lorentz transformations are useful but incomplete transformations. In the next section I shall fix the second problem mentioned above by introducing a new set of transformation equations. This new set of equations are the General Lorentz Transformations (GLT). **Appendix 1** contain the nomenclature used in this paper.

2. The General Lorentz Transformations

The General Lorentz Transformations are a generalization of the Lorentz transformations when the direction of the velocity vector, $\vec{v}$, does not coincide with any of the coordinate axes of system *S*. Let us consider two Cartesian coordinate systems: system *S* and system *S′*. We assume that system *S′* moves at a constant speed *v (yellow arrow)* with respect to system *S* along an arbitrary direction with respect to system *S* (along the *green dash line*). We also assume that, in general, this direction does not coincide with any of the coordinate axes of system *S*. See **Figure 1**. We also have two observers: an observer in system *S* called John (also known as observer *O*) and another observer in system *S′* called Sophia (also known as observer *O′*). These two observers will describe the same event differently.

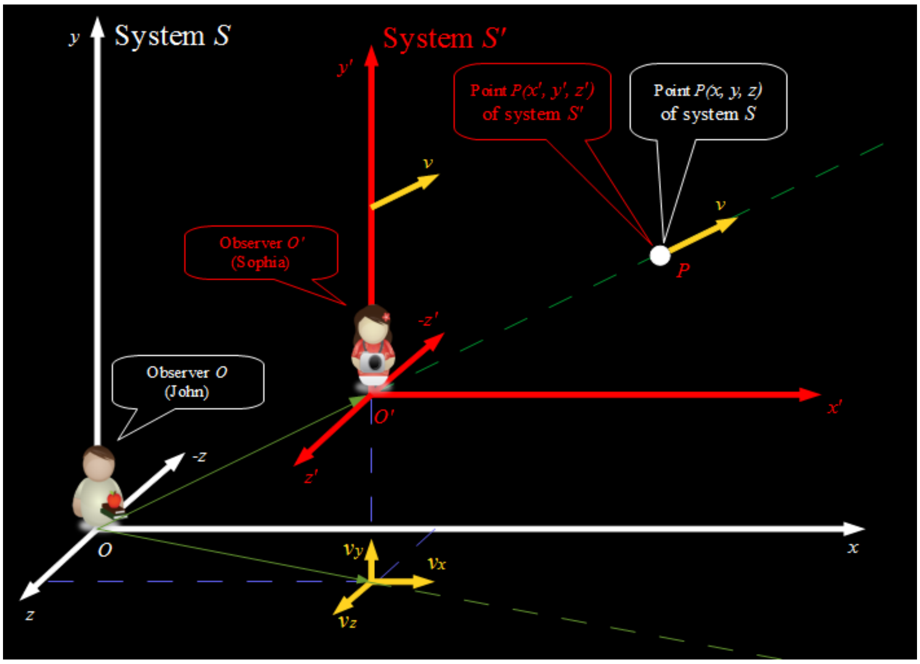

Figure 1: *Reference system S′ moves along the straight line OO′ (green dash line) at a constant speed v in an arbitrary direction with respect to system S.*

The velocity $\vec{v}$ is the velocity of system *S′* with respect to system *S*. We assume that this velocity is constant and its module, *v*, can be expressed in terms of its components: v_x, v_y and v_z as follows

$$v=\sqrt{v_x^2+v_y^2+v_z^2} \tag{2.1}$$

These components are constant because we have assumed that $\vec{v}$ is a constant vector. It is worthwhile to observe that, in general, the direction of the vector $\vec{v}$ does not coincides with any of the coordinates axes of system *S*.

In order to simplify the equations and the derivations I shall also use two standard parameters: β , which is defined as

$$\beta=\frac{v}{c} \tag{2.2}$$

and γ , which is defined as

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\beta^2}} \qquad (2.3)$$

Let us assume that Sophia (system *S'*) describes an event though a set of numbers: (*x', y', z', t'*). Let us also assume that John (system *S*) describes the same event though another set of numbers (*x, y, z, t*). The problem is to find the relationship between these two set of numbers when these two observers are in relative uniform motion, so that, Sophia moves in any arbitrary direction with respect to John as shown in **Figure 1**. The relationships we are looking for are the General Lorentz Transformations. These transformations are given in **Table 2**.

General Lorentz Transformations *Direct transformations* (Values of system *S'* in terms of values of system *S*)	General Lorentz Transformations *Inverse transformations* (Values of system *S* in terms of values of system *S'*)
$x' = \frac{x - v_x t}{\sqrt{1-\beta^2}} = \gamma\left(x - v_x t\right)$ (2.4)	$x = \frac{x' + v_x t'}{\sqrt{1-\beta^2}} = \gamma\left(x' + v_x t'\right)$ (2.8)
$y' = \frac{y - v_y t}{\sqrt{1-\beta^2}} = \gamma\left(y - v_y t\right)$ (2.5)	$y = \frac{y' + v_y t'}{\sqrt{1-\beta^2}} = \gamma\left(y' + v_y t'\right)$ (2.9)
$z' = \frac{z - v_z t}{\sqrt{1-\beta^2}} = \gamma\left(z - v_z t\right)$ (2.6)	$z = \frac{z' + v_z t'}{\sqrt{1-\beta^2}} = \gamma\left(z' + v_z t'\right)$ (2.10)
\ $t' = \gamma\left(t - \frac{v^2 x}{c^2 v_x}\right)$ (2.7a) $t' = \gamma\left(t - \frac{v^2 y}{c^2 v_y}\right)$ (2.7b) $t' = \gamma\left(t - \frac{v^2 z}{c^2 v_z}\right)$ (2.7c)	$t = \gamma\left(t' + \frac{v^2 x'}{c^2 v_x}\right)$ (2.11a) $t = \gamma\left(t' + \frac{v^2 y'}{c^2 v_y}\right)$ (2.11b) $t = \gamma\left(t' + \frac{v^2 z'}{c^2 v_z}\right)$ (2.11c)

Table 2: *The General Lorentz transformations of coordinates. Note that equations (2.7a), (2.7b) and (2.7c) yield the same value of time t'. Similarly, equations (2.11a), (2.11b) and (2.11c) yield the same value of time t.*

4. Derivation of the Time Dilation Formula from the General Lorentz Transformations

For this derivation I shall use the reverse General Lorentz transformation (2.11a) of **Table 2**.

$$t = \gamma \left(t' + \frac{v^2 x'}{c^2 v_x} \right) \qquad \text{s(Equation 2.11a)}$$

It is worthwhile to observe that we could have also used either transformation (2.11b) or transformation (2.11c) to get the same result. From equation (2.11a) we calculate the beginning of the time interval, t_1

$$t_1 = \gamma \left(t'_1 + \frac{v^2 x'_1}{c^2 v_x} \right) \qquad \text{(beginning of the time interval)} \qquad (4.1)$$

and the end of the time interval, t_2

$$t_2 = \gamma \left(t'_2 + \frac{v^2 x'_2}{c^2 v_x} \right) \qquad \text{(end of the time interval)} \qquad (4.2)$$

Then we calculate the time interval $t_2 - t_1$ measured by an observer of system *S*:

$$t_2 - t_1 = \gamma \left[t'_2 + \frac{v^2 x'_2}{c^2 v_x} - \left(t'_1 + \frac{v^2 x'_1}{c^2 v_x} \right) \right] \qquad (4.3)$$

$$t_2 - t_1 = \gamma \left[t'_2 - t'_1 + \left(x'_2 - x'_1 \right) \frac{v^2}{c^2 v_x} \right] \qquad (4.4)$$

Because the time measurements are made at the same location, the difference of the space coordinates must be zero. Consequently, we write

$$x'_2 - x'_1 = 0 \qquad (4.5)$$

With this simplification, equation (4.4) reduces to

$$t_2 - t_1 = \frac{t'_2 - t'_1}{\sqrt{1-\beta^2}} \qquad (4.6)$$

But the time difference $t_2 - t_1$ is the time interval, *t*, measured by an observer of system *S*

$$t_2 - t_1 = t \qquad (4.7)$$

And the time difference $t'_2 - t'_1$ is the time interval, t_0 , measured by an observer of system *S′*

$$t'_2 - t'_1 = t_0 \qquad (4.8)$$

From equations (4.6), (4.7) and (4.8) we finally get

$$t = \frac{t_0}{\sqrt{1-\beta^2}} \tag{4.9}$$

This formula is the Einstein's time dilation formula. Thus Special Relativity and General Special Relativity produced the same result with respect to time dilation.

5. Derivation of the Length Contraction Formula from the General Lorentz Transformations

Let us assume that Sophia has a sphere of diameter $d' = d_0$ which is at rest with respect to her reference system (*S'*). The problem is to find a formula for the sphere's diameter, *d,* measured by John (an observer of system *S*) for whom the sphere travels through space with a relatively high velocity *v.* Let us begin by finding the diameter in the direction of motion (direction of the velocity vector $\vec{v}$, green dash line of **Figure 1**) measured by Sophia. The diameter Sophia measures is

$$d'^2 = d_0^2 = \left(x'_2 - x'_1\right)^2 + \left(y'_2 - y'_1\right)^2 + \left(z'_2 - z'_1\right)^2 \tag{5.1}$$

Using the General Lorentz Transformations we rewrite equation (5.1) in terms of x_1, x_2, y_1, y_2, z_1 and z_2 . This yields

$$d_0^2 = \left[\gamma\left(x_2 - v_x t_2\right) - \gamma\left(x_1 - v_x t_1\right)\right]^2 + \left[\gamma\left(y_2 - v_y t_2\right) - \gamma\left(y_1 - v_y t_1\right)\right]^2 + \left[\gamma\left(z_2 - v_z t_2\right) - \gamma\left(z_1 - v_z t_1\right)\right]^2 \tag{5.2}$$

working algebraically we get

$$d_0^2 = \gamma^2\left[\left[x_2 - x_1 - v_x\left(t_2 - t_1\right)\right]^2 + \left[y_2 - y_1 - v_y\left(t_2 - t_1\right)\right]^2 + \left[z_2 - z_1 - v_z\left(t_2 - t_1\right)\right]^2\right] \tag{5.3}$$

Because all measurements are made at the same time we have

$$t_2 - t_1 = 0 \tag{5.4}$$

Thus equation () reduces to

$$d_0^2 = \gamma^2\left[\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2\right] \tag{5.5}$$

But the quantity within the square bracket is the square of the diameter, d^2 , measured by an observer of system *S.*

$$d^2 = \left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2 \tag{5.6}$$

Thus from equations (5.5) and (5.6) we can write

$$d_0 = \gamma\, d \quad (5.7)$$

$$d = \frac{d_0}{\gamma} \quad (5.8)$$

$$d = d_0 \sqrt{1-\beta^2} \quad (5.9)$$

This is the General length contraction formula of this formulation. It is worthwhile to remark that the sphere will contract in all directions and not only in the direction of motion. In contrast, Special Relativity cannot predict this effect since only contemplates motion along one and only one coordinate axis. Thus, Sophia's sphere (system *S'*) will still appear to be a sphere to John (from system *S*), although he will see a smaller sphere due to the relativistic effect caused by the relative motion between the two observers. This can be easily deduced by carrying out a similar analysis in any other direction of space (different to the direction of motion).

6. General Relativistic Velocity Addition

In non-relativistic physics the velocities are added according to the formula: $u = u' + v$. But in both Special Relativity and General Special Relativity the velocities must be combined using a more complex formula. The purpose of this section is to find these formulas.

Let us start by defining the velocities of the point *P* (the coordinates of *P* could represent the coordinates of a spaceship travelling trough space) with respect to each coordinate system, *S'* and *S*. Thus, if (x', y', z', t') and (x, y, z, t) refer to a moving point P, their velocities with respect to system *S'* and system *S* are *u'* and *u,* respectively (See **Figure 1**). The components of these two velocities are shown in **Table 3**.

Velocity *u'* of point *P* with respect to system *S'*	Velocity *u* of point *P* with respect to system *S*
$u'_x = \frac{dx'}{dt'}$ (6.1a)	$u_x = \frac{dx}{dt}$ (6.2a)
$u'_y = \frac{dy'}{dt'}$ (6.1b)	$u_y = \frac{dy}{dt}$ (6.2b)
$u'_z = \frac{dz'}{dt'}$ (6.1c)	$u_z = \frac{dz}{dt}$ (6.2c)

Table 3: *Components of the velocity of the point P with respect to both system S' and S.*

Now we shall find the expression for the velocity, u_x , of the point *P* with respect to system *S*

$$u_x = \frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \tag{6.3}$$

$$\Delta x = \gamma \left(\Delta x' + v_x \Delta t' \right) \tag{6.4}$$

$$\Delta t = \gamma \left(\Delta t' + \frac{v^2}{c^2 v_x} \Delta x' \right) \tag{6.5}$$

$$u_x == \lim_{\Delta t \to 0} \frac{\gamma \left(\Delta x' + v_x \Delta t' \right)}{\gamma \left(\Delta t' + \frac{v^2}{c^2 v_x} \Delta x' \right)} \tag{6.6}$$

Dividing numerator and denominator by $\Delta t'$ yields

$$u_x = \lim_{\Delta t \to 0} \frac{\left(\frac{\Delta x'}{\Delta t'} + v_x \right)}{\left(1 + \frac{v^2}{c^2 v_x} \frac{\Delta x'}{\Delta t'} \right)} \tag{6.7}$$

Finally we get the relativistic addition of velocities of General Special Relativity

General Special Relativity's velocity addition formula

$$u_x = \frac{\left(u'_x + v_x \right)}{\left(1 + \frac{v^2 u'_x}{c^2 v_x} \right)} \tag{6.8}$$

In similar way we get

$$u'_x = \frac{\left(u_x - v_x \right)}{\left(1 - \frac{v^2 u_x}{c^2 v_x} \right)} \tag{6.9}$$

Special Case: Special Relativity

Let us examine the special case when the components of v along the y and the z axes are zero If $v_y = v_z = 0$, then according to equation (2.1) the direction of the velocity of system S' with respect to system S turns out to be along the x axis. Thus

$$v = v_x \tag{6.10}$$

This means that equation (6.8) can be simplified as follows

$$u_x = \frac{\left(u'_x + v\right)}{\left(1 + \frac{v^2}{c^2 v} u'_x\right)} \tag{6.11}$$

Einstein's Special Relativity's velocity addition formula

$$u_x = \frac{\left(u'_x + v\right)}{\left(1 + \frac{v u'_x}{c^2}\right)} \tag{6.12}$$

This is the formula for the addition of velocities of Einstein's Special Relativity.

7. Invariance of Newton's Law of Universal Gravitation before a General Lorentz Transformation

Newton's law of Universal Gravitation has been superseded by Einstein's general relativity. Like all scientific theories, Einstein's theory may one day be superseded by quantum gravity and/or other more accurate and sophisticated gravitational models. In this section I shall show that Newton's Gravity Law is invariant under a General Lorentz transformation.

Let us consider two masses, M' (depicted as *The Earth*) and m' (depicted as the *Moon*), at rest with respect to system S'. Due to relativistic effects, John, measures M and m, respectively. The masses are general and they are depicted as the Earth and the Moon for illustrative purposes only.

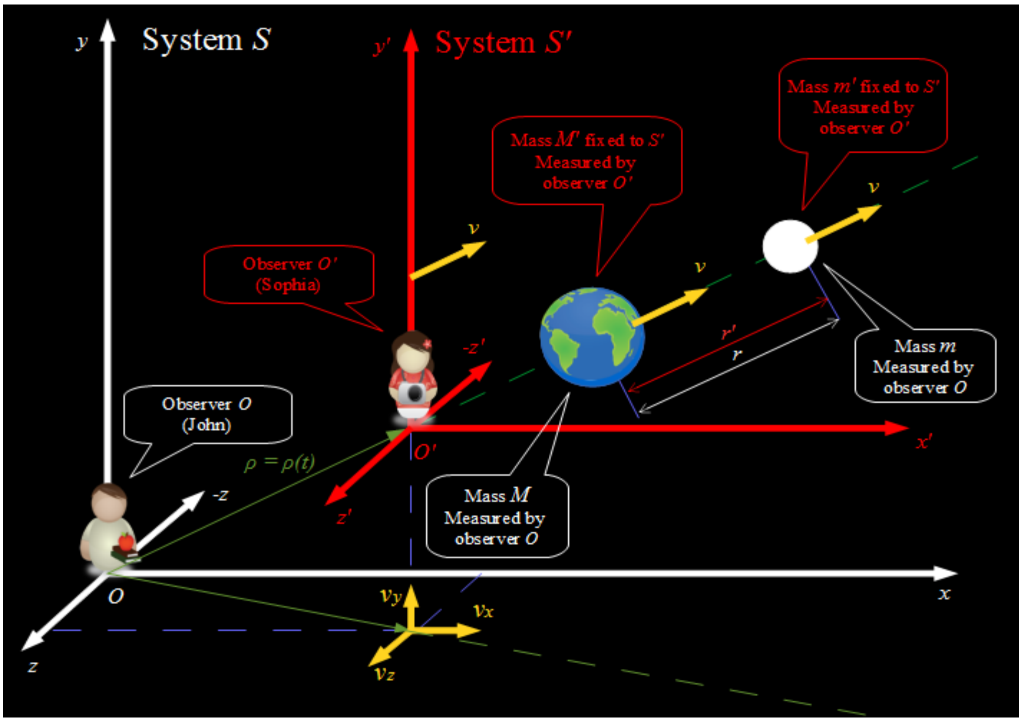

Figure 2: *Reference system S' moves at a constant speed v along an arbitrary direction with respect to system S (moves along the straight line OO', see green line). The distances, r' and r, between the masses, are the distances measured by Sophia and John, respectively.*

As we did before, we shall consider the most general case in which the direction of the velocity vector $\vec{v}$ does not coincide with any of the coordinate axes of system *S*. Now let's see whether Newton's law of universal gravitation is invariant before a General Lorentz transformation. Let us begin by considering Newton's Gravity law from the point of view of an observer (John) of reference system *S*

For an observer of system *S*

$$F = \frac{G\,M\,m}{r^2} \tag{7.1}$$

Here *r,* the distance between the masses *M* and *m,* is measured by John. According to him the square of this distance is

$$r^2 = \left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2 \tag{7.2}$$

The inverse General Lorentz Transformation given in **Table 1** relate the values of the coordinates (*x, y, z, t*) measured by an observer of system *S* with the corresponding coordinates, (*x', y', z', t'*), measured by an observer of system *S'*.

Now I shall express the square of the distance *r* in terms of the coordinates of Sophia's reference system (*S'*). Now in equation (7.2) we substitute

(a) x_2 and x_1 with the corresponding values obtained through equation (2.8) of the GLT,
(b) y_2 and y_1 with the corresponding values obtained through equation (2.9) of the GLT; and
(c) z_2 and z_1 with the corresponding values obtained through equation (2.10) of the GLT. This gives

$$r^2 = \left[\gamma\left(x_2'+v_x t_2'\right)-\gamma\left(x_1'+v_x t_1'\right)\right]^2+\left[\gamma\left(y_2'+v_y t_2'\right)-\gamma\left(y_1'+v_y t_1'\right)\right]^2+\left[\gamma\left(z_2'+v_z t_2'\right)-\gamma\left(z_1'+v_z t_1'\right)\right]^2 \quad (7.3)$$

Now we take γ^2 as a common factor

$$r^2 = \gamma^2\left[\left[\left(x_2'+v_x t_2'\right)-\left(x_1'+v_x t_1'\right)\right]^2+\left[\left(y_2'+v_y t_2'\right)-\left(y_1'+v_y t_1'\right)\right]^2+\left[\left(z_2'+v_z t_2'\right)-\left(z_1'+v_z t_1'\right)\right]^2\right] \quad (7.4)$$

Rearranging the terms

$$r^2 = \gamma^2\left[\left[x_2'-x_1'+v_x\left(t_2'-t_1'\right)\right]^2+\left[y_2'-y_1'+v_y\left(t_2'-t_1'\right)\right]^2+\left[z_2'-z_1'+v_z\left(t_2'-t_1'\right)\right]^2\right] \quad (7.5)$$

Now considering that

$$t_2'-t_1'=0 \quad (7.6)$$

we can write

$$r^2 = \gamma^2\left[\left(x_2'-x_1'\right)^2+\left(y_2'-y_1'\right)^2+\left(z_2'-z_1'\right)^2\right] \quad (7.7)$$

but the expression within the square bracket is the square of the distance, r'^2 , (between the masses *M'* and *m'*) measured by Sophia (System *S'*):

$$r'^2 = \left(x_2'-x_1'\right)^2+\left(y_2'-y_1'\right)^2+\left(z_2'-z_1'\right)^2 \quad (7.8)$$

Thus r^2 and r'^2 are related by

$$r^2 = \gamma^2 r'^2 \quad (7.9)$$

Also according to Einstein's relativistic mass law we can write the following two equations

$$M = \frac{M'}{\sqrt{1-\beta^2}} = \gamma M' \quad (7.10)$$

$$m = \frac{m'}{\sqrt{1-\beta^2}} = \gamma m' \quad (7.11)$$

Now in equation (7.1) we can substitute r^2 with the second side of eq. (7.9), *M* by the third side of equation (7.10) and m by the third side of equation (7.11). This yields

$$F = G\frac{\gamma M'\gamma m'}{\gamma^2 r'^2} \tag{7.12}$$

which can be written as

$$F = G\frac{M'm'}{r'^2} \tag{7.13}$$

But, according to Sophia, the gravitational force between the masses in question is

For an observer of system S'

$$F' = G\frac{M'm'}{r'^2} \tag{7.14}$$

Because the second side of equations (7.13) and (7.14) are identical, the first sides must also be identical. Thus we can write

$$F = F' \tag{7.15}$$

Consequently, Newton's law of universal gravitation has exactly the same form for any two inertial observers in relative motion. Mathematically, this means that Newton's law of universal gravitation is invariant before a General Lorentz Transformation.

It is a well known fact that Einstein's theory of General Relativity superseded Newton's law of Gravitation. However, we have just found that Newton's Gravity Law is invariant under a General Lorentz transformation. This means that we have proved that Lorentz invariance is a necessary but not a sufficient condition for a law of nature to be true. Because all Lorentz transformations, including the new formulation presented in this paper, can be considered Meta-laws, there must be other equally or more important Meta-laws that, when mathematically "invoked", should yield the true laws of nature. One of the physicist's and scientist's mission, in general, is to find these yet unknown Excluding Meta-laws.

8. Conclusions

In summary, the new theory of relativity presented in this paper have a number of implications. Some of these implications are:

(1) Lorentz contraction affects bodies such as spheres that move with respect to system S (in any arbitrary direction), so that they appear to be smaller to observers of system S. However, if the direction of the relative velocity v does not coincide with any of the

coordinates axis of system *S*, then spheres conserve their spherical shape.

(2) The formula for the addition of velocities turned out to be similar but not identical to that derived from the original Lorentz transformations. However, if the velocity vector coincides with any of the coordinate axes of system *S*, then both transformations yield the same formula for the addition of velocities. Thus, Special Relativity's velocity addition formula is a special case of General Special Relativity's counterpart.

(3) Newton's law of universal gravitation turned out to be invariant before a General Lorentz Transformation.

This list is very incomplete and, as further research continuous in this field, it is likely that the list will grow in the future. Finally, I would like to say that, this formulation could explain some phenomena that Einstein's Special Theory of Relativity is unable to predict.

Appendix 1
Nomenclature

The following are the symbols, abbreviations and terminology used in this paper

Symbols

$c=$ speed of light in vacuum
$G=$ Newton's gravitational constant
$L_P=$ Planck length
$T_P=$ Planck time
$S=$ reference system *S* (Cartesian coordinate reference system)
$S'=$ reference system S' (Cartesian coordinate reference system)
$l_0=$ proper length
$l=$ "contracted" length
$d'=$ proper diameter (diameter with respect to system *S'*)
$d_0=$ proper diameter
$d=$ "contracted" diameter (diameter with respect to system *S*)
$t_0=$ proper time
$t=$ "dilated" time
$(x', y', z', t')=$ coordinates of a given event in system *S'*
$(x, y, z, t)=$ coordinates of the same event in system *S*
$P=$ point of coordinates (x', y', z') in system *S'* and (x, y, z) in system *S*
$\vec{v}=$ relative velocity vector between system *S* and system *S'*
$v=$ module of the relative velocity vector between system *S* and system *S'*
$u'=$ velocity of the point *P* with respect to system *S'*
$u=$ velocity of the point *P* with respect to system *S*
$\beta=$ ratio of the speed, *v*, of a massive body to the speed of light, *c*
$\gamma=$ Lorentz transformations' factor (Lorentz transformations' scale factor or scaling factor)
$M'=$ rest mass of a body relative to system *S'*

$m' =$ rest mass of a body relative to system S'
$M =$ relativistic mass of a body relative to system S
$m =$ relativistic mass of a body relative to system S
$r' =$ distance between M' and m' relative to system S'
$r =$ distance between M and m relative to system S
$F' =$ gravitational force between M' and m' measured by an observer of system S'
$F =$ gravitational force between M and m measured by an observer of system S

Abbreviations

LT = Lorentz transformations
SR = Special Relativity
GLT = General Lorentz Transformations
GSR = General Special Relativity

Terminology

The term General Special Relativity (*) (or Special Relativity based on the General Lorentz transformations) refers to the new theory of relativity presented in this paper which is based on the General Lorentz Transformations.

(*) Not to be confused with Einstein's General Relativity.

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