Understanding Social-Force Model in Psychological Principles of Collective Behavior

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To well understand crowd behavior, microscopic models have been developed in recent decades, in which an individual's behavioral/psychological status can be modeled and simulated. A well-known model is the social-force model innovated by physical scientists (Helbing and Molnar, 1995; Helbing, Farkas and Vicsek, 2000; Helbing et al., 2002). This model has been widely accepted and mainly used in simulation of crowd evacuation in the past decade. A problem, however, is that the testing results of the model were not explained in consistency with the psychological findings, resulting in misunderstanding of the model by psychologists. This paper will bridge the gap between psychological studies and physical explanation about this model. We interpret this physics-based model from a psychological perspective, clarifying that the model is consistent with psychological studies on stress, including time-related stress and interpersonal stress. The simulation result of the model actually reflects Yerkes–Dodson law, explicating how stress could improve or impair human performances in a collective sense. Herding effect is further discussed in detail where the social force is renewed by integrating attractions. Based on the conception of stress, we further link the model parameters (e.g., the desired velocity) with certain environmental stressors (e.g., guidance, hazard and surrounding people), and explain how such stressors function in collective motion of people based on psychological principles.

I. About the Social-Force Model

The social-force model presents psychological forces that drive pedestrians to move as well as keep a proper distance with others. In this model an individual's motion is motivated by a self-driven force $\boldsymbol{f}_i^{self}$ and resistances come from surrounding individuals and facilities (e.g., walls). Especially, the model describes the social-psychological tendency of two individuals to keep proper interpersonal distance (as called the social-force) in collective motion, and if people have physical contact with each other, physical forces are also taken into account. Let $\boldsymbol{f}_{ij}$ denote the interaction from individual j to individual i, and $\boldsymbol{f}_{iw}$ denote the force from walls or other facilities to individual i. The change of the instantaneous velocity $\boldsymbol{v}_i(t)$ of individual i is given by the Newton Second Law:

$$m_i \frac{d\boldsymbol{v}_i(t)}{dt} = \boldsymbol{f}_i^{self} + \sum_{j(\neq i)} \boldsymbol{f}_{ij} + \sum_{w} \boldsymbol{f}_{iw} \tag{1}$$

where m_i is the mass of individual i. Furthermore, the self-driven force $\boldsymbol{f}_i^{self}$ is specified by

$$\boldsymbol{f}_i^{self} = m_i \frac{\boldsymbol{v}_i^0(t) - \boldsymbol{v}_i(t)}{\tau_i}\,, \tag{2}$$

This force describes an individual tries to move with a desired velocity $\boldsymbol{v}_i^0(t)$ and expects to adapt the actual velocity $\boldsymbol{v}_i(t)$ to the desired velocity $\boldsymbol{v}_i^0(t)$ within a certain time interval τ_i. In particular, the desired velocity $\boldsymbol{v}_i^0(t)$ is the target velocity existing in one's mind while the actual velocity $\boldsymbol{v}_i(t)$ characterizes the physical speed and direction being achieved in the reality. The gap of $\boldsymbol{v}_i^0(t)$ and $\boldsymbol{v}_i(t)$ implies the difference between the human subjective wish and realistic situation, and it is scaled by a time parameter τ_i to generate the self-driven force. This force motivates one to either accelerate or decelerate, making the realistic velocity $\boldsymbol{v}_i(t)$ approaching towards the desired velocity $\boldsymbol{v}_i^0(t)$. This mathematical description of the self-driven force

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could be dated back to the Payne-Whitham traffic flow model (Payne, 1971; Whitham, 1974). Sometimes $\boldsymbol{v}_i^0(t)$ is rewritten as $\boldsymbol{v}_i^0(t) = v_i^0(t)\boldsymbol{e}_i^0(t)$, where $v_i^0(t)$ is the desired moving speed and $\boldsymbol{e}_i^0(t)$ is the desired moving direction. In a similar manner, we also have $\boldsymbol{v}_i(t) = v_i(t)\boldsymbol{e}_i(t)$ where $v_i(t)$ and $\boldsymbol{e}_i(t)$ represent the physical moving speed and direction, respectively.

The interaction force of pedestrians consists of the social-force $\boldsymbol{f}_{ij}^{soc}$ and physical interaction $\boldsymbol{f}_{ij}^{phy}$. i.e., $\boldsymbol{f}_{ij}=\boldsymbol{f}_{ij}^{soc}+\boldsymbol{f}_{ij}^{phy}$. The social-force $\boldsymbol{f}_{ij}^{soc}$ characterizes the social-psychological tendency of two pedestrians to stay away from each other, and it is given by

$$\boldsymbol{f}_{ij}^{soc} = A_i \exp\left[\frac{(r_{ij}-d_{ij})}{B_i}\right]\boldsymbol{n}_{ij} \tag{3}$$

where $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$ are positive constants, which affect the strength and effective range about how two pedestrians are repulsive to each other. The distance of pedestrians i and j is denoted by d_{ij} and the sum of their radii is given by r_{ij}. $\boldsymbol{n}_{ij}$ is the normalized vector which points from pedestrian j to i. The geometric features of two pedestrians are illustrated in Figure 1.

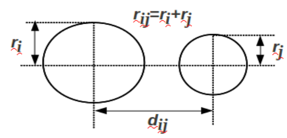


Figure 1. A Schematic View of Two Pedestrians

The physical interaction $\boldsymbol{f}_{ij}^{phy}$ describes the physical interaction when pedestrians have body contact, and it is composed by an elastic force that counteracts body compression and a sliding friction force that impedes relative tangential motion of two pedestrians. Both of them are valid only when $r_{ij}>d_{ij}$. The interaction of a pedestrian with obstacles like walls is denoted by $\boldsymbol{f}_{iw}$ and is treated analogously. In Helbing, Farkas and Vicsek, 2000 the interaction force is repulsive. The model may also include an attraction force in its original version (Helbing and Molnar, 1995, Korhonen and Hostikka, NIST, 2010).

By simulating many such individuals in collective motion, blocking was observed as people pass a bottleneck doorway, and this phenomenon is named by the "faster-is-slower" effect in Helbing, Farkas and Vicsek, 2000. Especially, it demonstrates that increasing desired velocity $\boldsymbol{v}_i^0$ can inversely decrease the collective speed of passing through the doorway.

In the past decade, the social-force model has generated considerable research on evacuation modeling (Helbing and Johansson, 2010), and it has been incorporated into several egress simulators, such as Fire Dynamics Simulator with Evacuation (Korhonen and Hostikka, 2010) and Maces (Pelechano and Badler, 2006). The model has been partly validated based on data sets from real-world experiments. The method of validation involves comparing the simulation of the model with associated observations drawn from video-based analysis (Johansson et al., 2008).

II. Psychological Explanation of Self-driven Force

A. Stress and Panic

One problem about the social-force model is that most of the testing results were explained by "panic" behavior of people (Helbing, Farkas and Vicsek, 2000; Helbing et al., 2002; Helbing and Johansson, 2010) while existing egress research clarifies the psychological state of panic occurs relatively rarely in real-world evacuation events (Sime, 1980; Proulx, 1993; Ozel, 2001; Rogsch et al., 2010), and this could cause misunderstanding of the model by social psychologists. Defined psychologically, "panic" means a sudden over-whelming terror which prevents reasoning and logical thinking, and thus results in irrational behavior. Based on Equation (1) and (2), we see that the equations do not imply any irrational behavior aroused by fear, but describe a kind of rational mechanics that govern an individual's motion. Thus, we think that the general use of the term panic is not essential to the social-force model.

By searching in literature of social psychological studies in emergency egress, we think that "stress" is more accurate conceptualizations of the social-force model than "panic." (Selye, 1978, Sime, 1980; Ozel, 2001). Psychological stress can be understood as the interaction between the environment and the individual (Staal, 2004), emphasizing the role of the individual's appraisal of situations in shaping their responses. In Stokes and Kite, 2001, such stress is the result of mismatch between psychological demand and realistic situation, and Equation (2) characterizes the mismatch in terms of velocity: the psychological demand is represented by desired velocity $\boldsymbol{v}_i^0$ while the physical reality is described by the physical velocity $\boldsymbol{v}$.

The gap of two variables describes how much stress people are bearing in mind, and thus are motivated into certain behavior in order to make a change in reality. Such behavior is formulated as the self-driven force in Equation (1) and (2).

Furthermore, velocity is a time-related concept in physics and the gap of velocities actually describes a kind of time-related stress, which is generally considered as time-pressure. Such a kind of stress is caused by insufficient time when people are dealing with a time-related task. Here time is the critical resource to implement the task. In sum, although the social-force model is labeled with the term "panic," its mathematical description is not related to the true "panic" in a psychological sense and the self-driven force critically characterizes the psychological concept of stress and time-pressure. This also explains why the model can be well used in simulation of emergency egress because "emergency" implies shortage of time in a process.

B. Yerkes–Dodson law and Faster-is-Slower Effect

Next, we will explain the simulation results of the social-force model from the psychological perspective. In particular, the simulation of the model reiterates an existing psychological knowledge: moderate stress improves human performance (i.e., speeding up crowd motion); while excessive stress impairs their performance (i.e., disorders and jamming), and this theorem is commonly known as Yerkes–Dodson law in psychological study (Yerkes and Dodson, 1908; Teigen, 1994; Wikipedia, 2016).

Yerkes–Dodson law states the relationship between arousal level and performance: performance increases with arousal, but only up to a point. Here the arousal level indicates the motivation level of behavior and it depends on stimulus strength from environment. Beyond the point the intensity of motivation becomes excessive and the situation is much stressful such that performance diminishes. In the social-force model, the arousal is represented by desired velocity $\boldsymbol{v}^0$. The performance is measured in the simulation by pedestrian flow ρv, describing how many individuals pass through a doorway of unit width per time unit (See Figure 2). Here v^0 and v are the average of desired velocity and physical velocity at the crowd level. The passage capacity is the maximal pedestrian flow that people are able to realize in collective motion (Wang et al., 2008), and it determines whether the collective motivation is excessive or not. By comparing ρv with the passage capacity, there are two scenarios as introduced below.

When the passage capacity is sufficient, v increases along with v^o while ρ can be adjusted such that the physical distance among people is psychologically comfortable. As a result, people are able to move as fast as desired while still keep proper interpersonal distance. This scenario corresponds to the increasing segment of the curve in Figure 2. If the passage is saturate, the physical speed v and density ρ reach the maximum and the pedestrian flow ρv is the maximal. In this case, further increasing v^o will compress the crowd and increase the repulsion among people. As the repulsion increases, the risk of disorder and disaster at the bottleneck increases correspondingly (e.g., jamming and injury). If such disastrous events occur, the moving crowd will be significantly slowed down and the faster-is-slower effect comes into being, and this corresponds to the decreasing segment of the curve in Figure 2. In sum, the relationship between arouse v^0 and performance ρv is depicted by an inverted-U curve as shown in Figure 2.

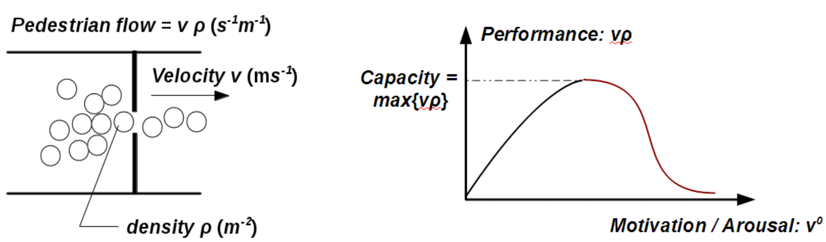


Figure 2. About crowd movement in a passageway

From the perspective of stress Yerkes–Dodson law is also understood by dividing stress into eustress and distress (Selye, 1975): stress that enhances function is considered eustress. Excessive stress that is not resolved through coping or adaptation, deemed distress, may lead to anxiety or withdrawal behavior and degenerate the performance.

By jointly using the concept of stress and Yerkes–Dodson law, we reinterpret the faster-is-slower effect as above. Though faster-is-faster effect exists, it has been deemphasized in Helbing, Farkas and Vicsek, 2000. In addition, whether faster-is-slower effect emerges also depends on whether people tend to compete or cooperate with each other. If interpersonal force is not significant increased, the faster-is-slower phenomenon will not emerge in the simulation (Høiland-Jørgensen et al., 2010). In other words, the competition or cooperation of people is another vital issue here, and we will elaborate it in the following section and reiterate Yerkes–Dodson law based on two-dimensional stressors.

III. Psychological Explanation of Social Force

A. Proxemics and Interpersonal Distance

As above we mainly discuss stress and arousal with respect to desired velocity. There is another kind of stress originating from social relationship and leads to competition or cooperation during crowd movement, and such stress is characterized by the social-force. Next, we will mainly discuss such stress and its relationship with interpersonal distance.

People surround themselves with a "bubble" of personal space that they claim as their own region, and they normally feel stressed when their personal space is invaded by others. Our personal space protects us from too much arousal and helps us keep comfortable when we interact with surrounding people. In Hall, 1963 the study of interpersonal distance was named by proxemics, and it was defined as "the interrelated observations and theories of man's use of space as a specialized elaboration of culture."

There are four interpersonal distances mentioned in Hall, 1966: intimate, personal, social, and public. The public distance is usually greater than 3.7m, and is often used for one-way public speaking. The social distance is from 1.2m to 3.7m, and it is used for formal social interactions among acquaintances (e.g., business). Personal distance is from 46cm to 122cm, and this is the distance to interact with our friends or family, and normal conversations can take place easily at this range. Intimate distance is smaller than 122cm, such as whispering and embracing. In general, the interpersonal distance is object-oriented. For example, we usually keep smaller distance to a friend than to a stranger, and such distance implies friendship. Entering somebody's personal space is normally an indication of familiarity and sometimes intimacy. Besides, such distance further depends on the culture and social occasions. For example, male and female commonly keep larger distance in Middle East culture than in modern western culture. Also, when in a crowded train, elevator or street, although such physical proximity is psychologically disturbing and uncomfortable, it is accepted as a fact of modern life. In sum, proximal distance origins from basic human instincts, and it is also widely redefined in different social norms and cultures.

B. About Social Force

Proxemics implies that when the interpersonal distance is smaller than the desired, people feel stressed. Repulsion comes into being in this situation, and repulsion increases when the distance further decreases. This theory justifies the assumption of repulsive social-force in Equation (3). However, the repulsion is not related to physical size of two people (i.e., r_{ij}), but the social relationship, culture and occasions. Comparing social force with self-driven force, we suggest that there should be a subjective concept of desired distance d_{ij}^0 in the social force, and it replaces r_{ij} in Equation (3). Here d_{ij}^0 is the target distance that individual i expects to maintain with individual j. This distance describes the desired interpersonal distance when people interact, and it is a function of the social relationship of individual i and j as well as the culture and social occasions. If we keep using the exponential form in Equation (3), the social force is rewritten as

$$\boldsymbol{f}_{ij}^{soc} = A_i \exp\left[\frac{(d_{ij}^0 - d_{ij})}{B_i}\right]\boldsymbol{n}_{ij} \tag{4}$$

Similar to desired velocity $\boldsymbol{v}_i^0$, the desired distance d_{ij}^0 is the target distance in one's mind, specifying the distance that one expects to adapt oneself with others. The physical distance d_{ij} is the distance achieved in the reality. The gap of d_{ij}^0 and d_{ij} implies the difference between the subjective wish in one's mind and objective feature in the reality. Similar to $\boldsymbol{v}_i^0 - \boldsymbol{v}_i$, as an indication of time-related stress concerning emergencies, $d_{ij}^0 - d_{ij}$ is an indication of interpersonal stress related to the social composition of crowd. Such stress depends on the intrinsic social characteristics of the crowd, not directly related to the emergency situation. Here A_i and B_i are parameters as introduced before, and $\boldsymbol{n}_{ij}$ is the normalized vector which points from pedestrian j to i. The social force also functions in a feedback manner to make the realistic distance d_{ij} approaching towards the desired distance d_{ij}^0. A difference is that $\boldsymbol{v}_i^0$ and $\boldsymbol{v}_i$ are vectors while d_{ij}^0 and d_{ij} are scalars.

Here we have several remarks as below.

The exponential form of social-force is not well justified. Existing psychological theory does not provide enough evidence to justify the exponential description as above. However, data from observation seems to support this assumption. So we will keep the assumption in this paper. Further justification is still necessary.

In addition, Equation (4) implies that d_{ij}^0 may be different from d_{ji}^0. As a result, the social-force between two individuals is not balanced, i.e., $d_{ij}^0 \neq d_{ji}^0$ and $f_{ij}^{soc} \neq f_{ji}^{soc}$. Thus, Newton third law does not hold for social force.

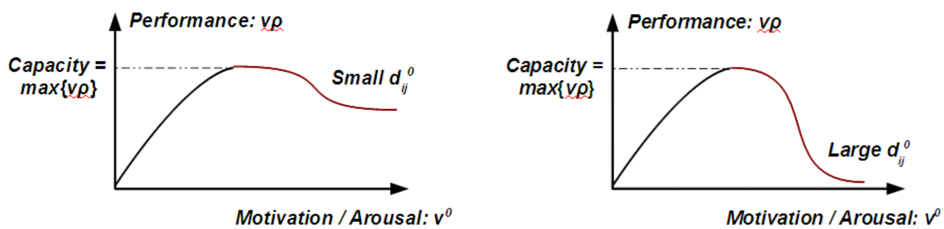
Although desired interpersonal distance d_{ij}^0 in Equation (3) is also affected by the culture and social occasions, we will not discuss culture difference in the following sections, and we simply assume that the social occasion is emergency evacuation in this paper. Thus, d_{ij}^0 is considered as an indication of familiarity of individual i and individual j in the following discussion.

Also, similar to the desired velocity, when a group of people are investigated, we will write d^0 as the average of desired interpersonal distance within the group. In other words, d^0 represents a kind of crowd characteristic in the collective sense.

C. Faster-Is-Slower Effect and Social Relationship

In brief, d_{ij}^0 critically represents the social relationship of individual i and individual j. The smaller d_{ij}^0 is, the closer is the relationship of individual i and individual j. In crowd evacuation, small value of d_{ij}^0 implies familiarity of evacuees and they tend to cooperate rather than compete with each other. As a result, when they pass through a bottleneck, even if they get close to each other, repulsion will not significantly increase. The faster-is-slower effect is thus mitigated and the relationship of motivation (i.e., $\boldsymbol{v}^o$) and the pedestrian flow (i.e., ρv) should be replotted as shown in Figure 3(a). In contrast, large value of d_{ij}^0 implies people are mainly composed of strangers and it is more likely for them to compete than cooperate at the bottleneck, resulting in higher probability of faster-is-slower effect at a bottleneck (See Figure 3b).

According to the above analysis, whether the faster-is-slower effect occurs also depends on the social-force. If the social-force does not sufficiently increase when individuals are close to each other, the faster-is-slower effect will not emerge. This standpoint has been partly justified in Høiland-Jørgensen et al., 2010 where the faster-is-slower effect was not observed when the interaction force is not properly given.



(a) (b)

Figure 3. About Social Force and Faster-Is-Slower Effect

Based on aforementioned discussion, we summarize effect of two kinds of stress on the risk of blocking at bottlenecks as shown in Figure 4. Here x-axis indicates the interpersonal stress as given by $d_{ij}^0 - d_{ij}$, and y-axis represents the impatience level of evacuees as given by $v^0 - v$. Here z-axis is the risk of disorder and blocking at the bottleneck when the bottleneck is saturate. The relationship of two kinds of stress and blocking risk is demonstrated as below, and it characterizes the Yerkes–Dodson law regarding two-dimensional stress during crowd motion.

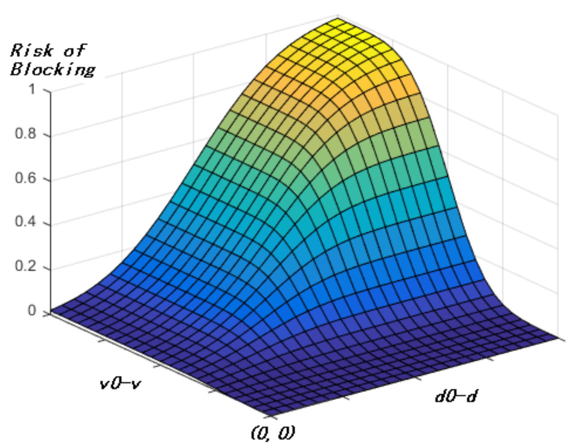


Figure 4. Two kinds of stress and the risk of blocking at bottlenecks

Although a major difference exists between the concepts of r_{ij} and d_{ij}^0, most of the simulation results in Helbing, Farkas and Vicsek, 2000 are still valid. In fact, the coding framework of social-force model is not affected if r_{ij} is replace by d_{ij}^0. When realizing the model by computer programs, r_{ij} and d_{ij}^0 are exactly at the same position in coding work. Here the difference is mainly at the quantitative assignment.

In sum, mismatch of psychological demand and physical reality results in a stressful condition. In emergency egress, such stress is aroused from environmental factors such as alarm or hazard conditions, resulting in impatience of evacuees and time-pressure. Another kind of stress is aroused from surrounding people, resulting in interpersonal stress in collective behavior. Stress could either improve or impair human performance. Traditionally, this psychological theorem mainly refers to performance at the individual level, such as class performance of a student or fight-or-flight response of an evacuee. The simulation of social-force model reiterates this well-known psychological knowledge in the sense of collective behavior. In brief, the testing result of social-force model agrees with Yerkes–Dodson law and it provides a new perspective to understand this classic psychological principle.

IV. Social Cohesion and Grouping Dynamics

In this section we will briefly discuss another kind of social pattern demonstrated by the social-force model. The pattern refers to the herding and grouping dynamics in collective motion of crowd.

A. Herding

Many models have been constructed to investigate the herding behavior in biological systems such as bird flocks or fish schools. For human crowd, when an individual immerses himself into the crowd, they do not think or reason in an individual manner, but follow the collective pattern of crowd. This effect is called herding behavior and it describes that people are affected by the surrounding people and tend to do what others are doing instead of making independent decisions.

Herding is especially evident when people are responding to an emergency. Emergency implies time-pressure as mentioned before and excessive time-pressure weakens the ability of logical thinking and reasoning, and independent decision making is more difficult in stressful conditions. Thus, people are more inclined to follow others (e.g., neighbors' decisions) rather than make decisions by themselves. Based on the social-force model in Helbing, Farkas and Vicsek, 2000 and Helbing et al., 2002, the herding effect is modified as below.

$$\boldsymbol{e}_i^0(t+1) = Norm[(1-p_i)\boldsymbol{e}_i^0(t) + p_i[\boldsymbol{e}_j(t)]_i] \qquad [\boldsymbol{e}_j(t)]_i = \sum_{R_i} \boldsymbol{e}_j(t) \tag{5}$$

The above equation characterizes that an individual desired direction e^0_i is updated by mixing itself with the average direction $e_j(t)$ of his neighbors j within radius R_i. Norm[] represents normalization of a vector. Both options are weighted with some parameter $(1\text{-}p_i)$ and p_i, and two opinions follow two-point distribution with probability $(1\text{-}p_i)$ and p_i, and e^0_i is updated by the statistical average. As a consequence, individualistic behavior is dominant if p_i is low, but herding behavior emerges if p_i is high. In Helbing, Farkas and Vicsek, 2000 p_i is considered to indicate one's panic level. Similarly we can understand that p_i is a stress indicator and people are more inclined to follow others when they are stressed or under much time-pressure.

In an addition, the mixture of two choices may not only refer to moving directions, but also the moving speed. Thus, the desired speed and physical speed can also get involved in Equation (5). For example, if one see that his neighbors move very quickly towards somewhere, he or she probably also wants to accelerate to carry on with the neighbors. In Lakoba, Kaup and Finkelstein, 2005, the speed was taken into account based on Helbing, Farkas and Vicsek, 2000. We present the following equation to describe how moving speed gets involved in herding behavior.

$$v_i^0(t+1) = (1-p_i)v_i^0(t) + p_i[v_j(t)]_i \qquad [v_j(t)]_i = (\sum_{R_i} v_j(t))/N \tag{6}$$

where the magnitude of velocity is the moving speed, i.e., $|\boldsymbol{v}_i^0| = v_i^0$ and $|\boldsymbol{v}_i| = v_i$.

Based on Equation (5) and (6) we know how an individual's velocity is affected by his neighbors in terms of both direction and speed. Here we have the following remarks.

One question regarding Equation (5) is whether people should copy their neighbors' desired moving directions e_j^0(t) or the actual moving directions e_j(t). In other words, whether to use e_j^0(t) or e_j(t) remains a question. The similar question exists for Equation (6) also. In brief, if we assume that an individual observes what others are doing and directly copy others' behavior, we have Equation (5) where e_i^0(t) is mixed with e_j(t). If we assume that emotion or mood can be affected by surrounding people (Le Bon, 1895), e_j(t) in Equation (6) will be replaced by e_j^0(t). We think that both equations are reasonable and useful to model crowd behavior.

Another key issue is that people can mix two choices in mind, but such mixture of options may not always be meaningful in the physical world. For example, you plan to go to Exit A while the surrounding people want to head for Exit B. You must choose definitely Exit A or Exit B, but cannot mix the two choices because you cannot go to both. Sometimes people must select a definite choice and cannot stay somewhere in between. In other words, you can mix two choices to get a mean value in a statistic sense, but in the realistic world you must flip a coin and get one definite result, but cannot mix both. To realize this idea we suppose p_i in Equation (5) and (6) are 0-1 random numbers that follow Bernoulli distribution. As a result we will get the number either 0 or 1 in practical computing, choosing to make an individual decision or follow others.

Very interestingly, Equation (6) implies that one affects the surrounding people and is also affected by surroundings. Such interaction is mutual in nature, but it is not symmetric, and thus does not obey Newton 3rd Law.

Finally, whether Equation (6) will result in convergence of desired motion in a collective sense is another topic to study. Current simulation results seem to be chaotic. However, existing psychological studies suggest that people's opinions may be unified when they interact in certain circumstances, and this means that everyone's desired moving direction and speed may converge to a common value and finally form a crowd opinion.

REFERENCES

[1] W. Daamen and S. P. Hoogendoorn, “Emergency Door Capacity: Influence of Door Width, Population Composition and Stress Level,” Fire Technology, Vol. 48, pp. 55-71, 2012.

[2] E. T. Hall. "A System for the Notation of Proxemic Behavior," American Anthropologist. Vol. 65, No. 5, pp. 1003–1026, 1963.

[3] E. T. Hall, The Hidden Dimension. Anchor Books. 1966.

[4] D. Helbing, I. Farkas, and T. Vicsek, “Simulating Dynamical Features of Escape Panic,” Nature, Vol. 407, pp. 487– 490, 2000.

[5] D. Helbing, I. Farkas, P. Molnar, T. Vicsek, “Simulation of pedestrian crowds in normal and evacuation situations,” in: Schreckenberg, M., Sharma, S.D. (Eds.), Pedestrian and Evacuation Dynamics, pp. 21–58. 2002.

[6] D. Helbing and P. Molnar, “Social force model for pedestrian dynamics,” Physical Review E, Vol. 51, No. 5, pp. 4282-4286, 1995.

[7] T. Høiland-Jørgensen, M. Hartmann, D. Albrechtsen, M. Thrane, T. Christensen, W. Xiao, “Crowd Modelling,” Modelling project, Mathematics RUC, fall 2010.

[8] A. Johansson, D. Helbing, H. Z. A-Abideen, and S. Al-Bosta, “From crowd dynamics to crowd safety: A video-based analysis,” Advances in Complex Systems, Vol. 11, No. 4, pp. 497-527, 2008.

[9] T. Korhonen and S. Hostikka, Technical Reference and User’s Guide for Fire Dynamics Simulator with Evacuation, (FDS+Evac, FDS 5.5.0, Evac 2.2.1), VTT Technical Research Center of Finland, 2010.

[10] T. I. Lakoba, D. J. Kaup, N. M. Finkelstein, “Modifications of the Helbing-Molnár-Farkas-Vicsek Social Force Model for Pedestrian Evolution,” Simulation, Vol. 81, Issue 5, pp. 339-352, May 2005.

[11] G. Le Bon, The Crowd: A Study of the Popular Mind, 1895.

[12] F. Ozel, “Time Pressure and Stress as a Factor During Emergency Egress,” Safety Science, Vol. 38, pp. 95-107, 2001.

[13] H. J. Payne, Models of freeway traffic and control, in Mathematical Models of Public Systems, Vol. 1 of Simulation Councils Proc. Ser., pp. 51-60, 1971.

[14] N. Pelechano and N. I. Badler , “Modeling Crowd and Trained Leader Behavior during Building Evacuation,” IEEE Computer Graphics and Applications, Vol. 26, No. 6, pp. 80-86, 2006.

[15] G. Proulx, “A Stress Model for People Facing a Fire,” Journal of Environmental Psychology, Vol. 13, No. 2, pp. 137-147, 1993.

[16] C. Rogsch, M. Schreckenberg, E. Tribble, W.W.F. Klingsch, and T. Kretz, "Was it Panic? An Overview about Mass-Emergencies and their Origins all over the World for Recent Years", Pedestrian and Evacuation Dynamics 2008, Springer, pp. 743-755, 2010.

[17] H. Selye. "Confusion and controversy in the stress field". Journal of Human Stress. Vol. 1, No. 2, pp. 37–44, 1975.

[18] H. Selye, The stress of life (Rev. ed.). New York: McGraw-Hill. 1978.

[19] J. D. Sime, “The Concept of Panic,” Fires and Human Behavior, D. Canter (ed.), First edition, John Wiley & Sons, pp. 63-81, 1980.

[20] M. A. Staal, “Stress, Cognition, and Human Performance: A Literature Review and Conceptual Framework (NASA/TM – 204-212824),” August 2004, Hanover, MD: NASA Scientific and Technical Information Program Office.

[21] A. F. Stokes and K. Kite, “On grasping a nettle and becoming emotional.” In P.A. Hancock, & P.A. Desmond (Eds.), Stress, workload, and fatigue. 2001, Mahwah, NJ: L. Erlbaum.

[22] Stress (psychological), (n.d.). In Wikipedia. Retrieved 16 march 2016, https://en.wikipedia.org/wiki/Stress_(psychological).

[23] K.H. Teigen, “Yerkes-Dodson: A law for all seasons”. Theory & Psychology, Vol. 4, pp. 525-547, 1994.

[24] P. Wang, P. B. Luh, S. C. Chang and J. Sun, “Modeling and Optimization of Crowd Guidance for Building Emergency Evacuation,” Proceedings of the 2008 IEEE International Conference on Automation Science and Engineering (CASE 2008), Washington, D.C., pp. 328 – 334, August 2008.

[25] G. B. Whitham, Linear and nonlinear waves, John Wiley and Sons, New York, 1974.

[26] R. M. Yerkes, J. D. Dodson. "The relation of strength of stimulus to rapidity of habit-formation." Journal of Comparative Neurology and Psychology, Vol. 18, pp. 459–482, 1908.