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A Standard Model at Planck Scale

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Abstract

To extend the standard model to Planck scale energies I propose a phenomenological model of quantum black holes and dark matter. I assume that at the center of any black hole there is a core object of length scale L_{Planck}. The core replaces the singularity of general relativity. A simple phenomenological schematic model is presented for the core. In the high curvature $t \sim 0$ universe a core is spontaneously created in a false vacuum. Subsequently it tunnels into the true vacuum causing an inflationary process in the universe. A survey is made of calculational models that could support the present scheme and of theoretical frameworks for future work.

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1 Introduction and Summary

The motivation behind the model described here is to find an economic way to go beyond the standard model (BSM), including mini black holes, inflation and the model of renormalization group improved quantum gravity. This short note is hoped to be a step forward in exploring the role of Planck scale gravity in particle physics and inflationary universe while a complete theory of quantum gravity remains beyond the scope of this note.

I made earlier a gedanken experiment of what might happen when exploring a mini black hole deep inside with a probe. In [1] I made two assumptions

(**1**) Inside any black hole there is a three dimensional integral part core of spin 0 ($\frac{1}{2}$). The core has an associated length scale of the order L_{Planck}. The core is called here the gravon, and gravion if its a fermion.

(**2**) The black hole singularity of general relativity is replaced by the core.

Einstein equations hold outside black holes, but in the inner region of the hole a different picture for the core is proposed. Let us start from the vacuum. At $t \sim 0$ the core is a field of large amount of energy that is spontaneously created in a false vacuum. From there it tunnels to the true vacuum. Next it goes through an inflationary process leading to black holes and dark matter. For the tunneling a single bubble inflationary model is assumed. The core has an applicable lifetime on the inflation time scale (between 10^{-33} and 10^{-32} sec). The gravon has no horizon and it decays to gravitons which couple to classical objects like black holes and the Higgs. From the high temperature side, the core is the $T = 0$ limit remnant of a thermally end-radiated black hole [2].

As to cosmic microwave background (CMB) measurements, this model is not designed to give new predictions - most current models compare very well with all available data. The purpose of the model is to take a new look inside black holes. An illustrative, though not very good, comparison might be to see the core as the hydrogen atom nucleus and a black hole as the whole atom.

With the Planck scale having its the conventional value 10^{19} GeV finding a gravon is hard. Gamma-ray signals from the sky may be a promising way. A gamma-ray, or particle, with energy half the Planck mass would be a favorable signal for the model.

In this note I disclose the physical motivation and description of the model. In section 2 I discuss the core qualitatively and in subsection 2.2 alternative candidates for modeling quantum black holes are being searched. From this subsection on this note is a survey of literature. Section 3 is devoted to inflation mechanisms and the Starobinsky model of gravity. In 4 I give some hints of what may come after a simple model turns inadequate. I consider higher derivative gravity, issues of conformity in extra dimensions and a model in string theory. I finish in section 5 with conclusions. What is not discussed here is the horizon, which has been extensively treated in the literature after the AMPS paper [3]. [2] Dark energy is left for future considerations.

[2] Their paper introduced the field to this author.

2 The Black Hole Core

2.1 Qualitative Properties

Apart from the assumption of the existence the core the model makes use of known physical processes, supported by calculations, and is largely under the control of present day technology.

Properties of the gravon model include:

(**1**) at $t \sim 0$ in the tiny very early spacetime the curvature value R was very high, near singular, and a quantum fluctuation produced a gravon field of an associated length scale of the order L_{Planck},

(**2**) the created gravon is in a false vacuum with energy higher than the true vacuum energy. The subsequent processes started the inflationary phase of the universe. [3]

(**3**) the gravon is a horizonless remnant, either stable or with some lifetime, of a thermally end-radiated black hole. Remnants have no singularity or information loss problems, see the recent review [4],

(**4**) dark matter consists of neutral matter around a core, i.e. black holes.

2.2 Modeling the Core

The core is a finite lifetime bunch of energy, originating from vacuum or black hole decay, and obeying the Klein-Gordon equation as a free particle. When enough matter falls into the core it becomes a black hole and the wave function makes a transition into a different state with general relativity outside the hole.

I consider a few different model cases below which might give insight into the quantum nature of the core. There is an large amount of models and calculations in the literature on the general title of quantum gravity, and it may not be too optimistic that a selective synthesis of progress can be made in the near future. The modern view is that general relativity forms a quantum effective field theory at low energies upon which models can be built. The point of view advocated in this note gives an extremely minimal time interval before the big bang for any major effect of quantum gravity.

2.2.1 Einstein-Dirac Cosmology

The singularity of general relativity is a property independent of the size of the system, whether the whole universe or a mini black hole. I start with an example from the large scales. The work of ref. [6] gives indication of singularity avoidance in Friedmann-Robertson-Walker (FRW) cosmology. Their analysis leads to the formation of a fermion condensate, instead of the singularity, and a bouncing scale function. I summarize [6] as follows.

The authors study Einstein-Dirac (ED) equations

[3] The common multiverse picture of bubbles as universes is not excluded but it does not change conclusions for this model. The bubble collision rate can be made small by the vacuum tunneling potential height.

$$R^i_j - \frac{1}{2}R\delta^i_j = 8\pi\kappa T^i_j \tag{1}$$

$$(\mathcal{D} - m)\Psi = 0 \tag{2}$$

where T^i_j is the energy-momentum tensor of the Dirac particles, κ is the gravitational constant, $\mathcal{D}$ is the Dirac operator and Ψ the wave function. For metric the closed Friedmann-Robertson-Walker is chosen

$$ds^2 = dt^2 - R^2(t)d\sigma^2 \tag{3}$$

where R is the scale function and $d\sigma^2$ is the line element on the unit 3-sphere

$$d\sigma^2 = \frac{dr^2}{1-r^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2 \tag{4}$$

where r, θ and ϕ are the standard polar coordinates. The Dirac operator in this metric is written as

$$\mathcal{D} = i\gamma^0 \left(\partial_t + \frac{3\dot{R}(t)}{2R(t)}\right) + \frac{1}{R(t)} \begin{pmatrix} 0 & \mathcal{D}_{S^3} \\ -\mathcal{D}_{S^3} & 0 \end{pmatrix}, \tag{5}$$

where γ^0 is the standard Dirac matrix, and D_{S^3} is the Dirac operator on the unit 3-sphere. The operator D_{S^3} has discrete eigenvalues $\lambda = \pm\frac{3}{2}, \pm\frac{5}{2}, \ldots$, corresponding to quantization of momenta of the particles. The Dirac equation is separate with the ansatz

$$\Psi_\lambda = R(t)^{-\frac{3}{2}} \left[\frac{8\pi\kappa}{3}\left(\lambda^2 - \frac{1}{4}\right)\right]^{-\frac{1}{2}} \begin{pmatrix} \alpha(t)\,\psi_\lambda(r,\vartheta,\varphi) \\ \beta(t)\,\psi_\lambda(r,\vartheta,\varphi) \end{pmatrix}, \tag{6}$$

where α and β are complex functions. For a homogenous system the components of the energy-momentum tensor simplify and the time component is

$$8\pi\kappa T^t_t = \left[m\Big(|\alpha|^2 - |\beta|^2\Big) - \frac{2\lambda}{R}\,\mathrm{Re}(\alpha\overline{\beta})\right]. \tag{7}$$

Substituting ψ and T^j_i into the Einstein-Dirac equation one gets

$$i\frac{d}{dt}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} m & -\lambda/R \\ -\lambda/R & -m \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \tag{8}$$

$$\dot{R}^2 + 1 = \frac{m}{R}\left(|\alpha|^2 - |\beta|^2\right) - \frac{\lambda}{R^2}\left(\overline{\beta}\alpha + \overline{\alpha}\beta\right). \tag{9}$$

With the ansatz (6) all single particle wave functions have the same time dependence thus they form a coherent macroscopic quantum state. The fermionic many-particle state is a spin condensate.

The ED equations further reduce to ordinary differential equations involving the scale function $R(t)$ and the complex functions $\alpha(t)$ and $\beta(t)$. In the limits $\lambda = 0$ and $m = 0$ the equations reduce to the Friedmann equations for dust and radiation universes, respectively.

For large R the universe behaves classically as in the dust case. But near the singularities big bang and big crunch quantum effects change the situation. Under certain conditions $\dot{R}$ can become zero and change sign even for small values of R. Thus the formation of a big bang or big crunch is prevented. This effect is called the bouncing scale function.

2.2.2 Asymptotically Free Quantum Gravity

Building on higher derivative terms in the Einstein-Hilbert action, super-renormalizable and asymptotically free theories of gravity have been discussed in the literature [7], see also [8]. Asymptotic freedom removes the singularity. Secondly, asymptotic freedom due to higher derivative form factor causes an effective negative pressure. Repulsive gravity at high density produces a bounce of a black hole. Black holes in fact never form. A distant observer sees a long lifetime for the trapped surface and interprets it as a black hole. The bounce is not given by Heisenberg uncertainty but follows from the dynamics of the system.

In [7] the following non-polynomial extension of the quadratic gravitational action of [9] has been considered

$$\mathcal{S} = \int d^4x \frac{2\sqrt{|g|}}{\kappa^2} \Big[R - G_{\mu\nu} \frac{V(-\Box/\Lambda^2)^{-1} - 1}{\Box} R^{\mu\nu} \Big] , \tag{10}$$

where $\kappa^2 = 32\pi G_N$ and Λ is the Lorentz invariant energy scale. Its value is of the order of Planck mass. The form factor, an entire function V contains the non-polynomial property of the theory. V cannot have poles in the complex plane to ensure unitarity and it must have at least logarithmic behavior in the UV to give super-renormalizability at the quantum level. The theory reduces to general relativity in the low energy limit since all the corrections to the Einstein-Hilbert action are suppressed by the factor Λ^{-1}.

The form factor is related to the propagator and to the effective potential of the theory. An example of a form factor is

$$V(z)^{-1} = exp(z^n) \tag{11}$$

where $z = -\Box/\Lambda^2$ and n is a positive integer. String theory suggests n = 1. These theories have only the graviton pole. There are no ghosts or tachyons. The UV is dominated by the bare action, counterterms are negligible. Further details of these theories are discussed in [7].

It is known that if one adds all quadratic curvature invariants to the Einstein-Hilbert action the resulting theory is renormalizable at the price of ghost modes [9]. In string theory the Einstein-Hilbert action is the first term of an infinite series containing powers of the curvature tensor and its derivatives.

According to Narain and Anishetty [10] the behavior of running coupling constant in the coupled system of higher derivative gravity and gauge fields is renormalizable to all order loops. The leading contribution to the gauge coupling beta function comes entirely from quantum gravity effects and it vanishes to all order loops.

In [10] the authors study fourth order higher derivative gravity which is claimed to be renormalizable to all loops [9] and unitary [11]. The motivation for their study came from the realization that at one loop four kinds of divergences appear $\sqrt{-g}$, $\sqrt{-g}R$, $\sqrt{-g}R_{\mu\nu}R^{\mu\nu}$

and $\sqrt{-g}R^2$. They consider the following higher derivative gravity action in dimensions $2 \leq d \leq 4$

$$S = \int \frac{d^4x\sqrt{-g}}{16\pi G}\left[-R - \frac{1}{M^2}\left(R_{\mu\nu}R^{\mu\nu} - \frac{d}{4(d-1)}R^2\right) + \frac{(d-2)\omega}{4(d-1)M^2}R^2\right] \tag{12}$$

where M has dimension of mass and ω is dimensionless. There are negative norm states, the propagator of the spin 2 massive mode appears with wrong sign violating unitarity at tree level. It was found though that in a certain domain of coupling parameter space, large enough to include known physics, the one loop running of gravitational parameters makes the mass of spin 2 massive mode behave in such a way that it is always above the energy scale being studied.

For our scheme asymptotically free quantum gravity is very interesting but there may not be at the moment general consensus whether it works as hoped.

2.2.3 Asymptotic Safety

Asymptotic safety was proposed by Weinberg [12] in 1976 as a condition of renormalizability, for a thorough review see [13]. It is based on a nontrivial, or non-Gaussian, fixed point (NGFP) of the underlying renormalization group (RG) flow for gravity. It is nonperturbative in character and it guarantees finite results for measurable quantities. The method for investigation of this scenario is functional renormalization group equation (FRGE) for gravity. The FRGE defines a Wilsonian RG flow on a theory space which consists of all diffeomorphism invariant functionals of the metric $g_{\mu\nu}$ of the type occuring in the action of general relatvity. From this construction emerges a theory called Quantum Einstein Gravity (QEG). QEG is not a quantization of classical general relativity, but it is consistent and predictive theory within the framework of quantum field theory.

The nature of the fundamental degrees of freedom is of secondary importance. From the viewpoint of renormalization theory it is the universality class that matters, not the particular choice of dynamical variables. Once a functional integral picture has been adopted, even nonlocally and nonlinearly related sets of fields or other variables may describe the same universality class and hence the same physics.

The method of ref. [14] uses the effective average action Γ_k, which is background independent. The RG scale dependence is governed by the FRGE of ref. [15]

$$k\partial_k\Gamma_k[\Phi,\bar{\Phi}] = \frac{1}{2}\mathrm{Str}\left[\left(\frac{\delta^2\Gamma_k}{\delta\Phi^A\delta\Phi^B} + R_k\right)^{-1} k\partial_k R_k\right]. \tag{13}$$

where Φ^A is the collection of all dynamical fields and $\bar{\Phi}^A$ denotes their background counterparts. R_k is an infrared cutoff which vanishes for $p^2 \gg k^2$ and provides a k-dependent mass term for fluctuations with momenta $p^2 \ll k^2$. Solutions of the FRGE give families of effective field theories $\Gamma_k[g_{\mu\nu}], 0 \leq k < \infty$, labeled by the coarse graining scale k. The solution Γ_k interpolates between the microscopic action at $k \to \infty$ and the effective action $\Gamma_{k\to 0}$.

Suppose there is a set of basic functionals $P_\alpha[\cdot]$. Any functional can be written as a linear combination of the P_α's. The the solutions Γ_k of the FRGE have expansions of the form

$$A[\Phi,\bar{\Phi}] = \sum_{\alpha=1}^{\infty} \bar{u}_\alpha \, P_\alpha[\Phi,\bar{\Phi}] \, . \tag{14}$$

The basis $P_\alpha[\cdot]$ may include local field monomials and non-local invariants. The generalized couplings $\bar{u}_\alpha$ are used as local coordinates. Or use a subset of couplings, so called essential couplings, which cannot be absorbed by a field reparametrization. The method is non-perturbative but truncations have to be made to the expansions of solutions.

Expanding Γ_k as above and inserting into FRGE one obtains a system of infinitely many coupled differential equations for the $\bar{u}_\alpha$'s

$$k\partial_k \, \bar{u}_\alpha(k) = \overline{\beta}_\alpha(\bar{u}_1, \bar{u}_2, \cdots; k) \; , \quad \alpha = 1, 2, \cdots \, . \tag{15}$$

which can be solved using analytical or numerical methods.

A simple ansatz for action is the Einstein-Hilbert action where Newton's constant G_k and the cosmological constant Λ_k depend on the RG scale k. Let $g_{\mu\nu}$ and $\bar{g}_{\mu\nu}$ denote the dynamical and background metric, respectively. The effective action then satisfies in arbitrary spacetime dimension d

$$\Gamma_k[g,\bar{g},\xi,\bar{\xi}] = \frac{1}{16\pi G_k} \int d^dx \sqrt{g} \Big(-R(g) + 2\Lambda_k \Big) + \Gamma_k^{gf}[g,\bar{g}] + \Gamma_k^{gh}[g,\bar{g},\xi,\bar{\xi}] \tag{16}$$

where $R(g)$ is the scalar curvature from metric $g_{\mu\nu}$, Γ_k^{gf} denotes the gauge fixing action and Γ_k^{gh} the ghost action with the ghost fields ξ and $\bar{\xi}$.

The corresponding β-functions describing the evolution of the dimensionless Newton constant $g_k = k^{d-2} G_k$ and dimensionless cosmological constant $\lambda_k = k^{-2} \Lambda_k$, were derived the first time in [14] for any value of the spacetime dimensionality. The most important result is the existence of a non-Gaussian fixed point suitable for asymptotic safety. It is UV-attractive both in g- and λ-directions (roughly $\lambda \approx .35$ and $g \approx .4$).

In the study of [16] it was shown that for $r \to 0$ the RG improved black hole metric approaches that of de Sitter space. This means that the quantum corrected spacetime is completely regular, free from any curvature singularity. The improved regularity comes because the 1/r-behavior of $f_{class} = 1 - 2G_0M/r$ is tamed by very rapidly vanishing of the Newton constant at small distances.

A heavy black hole obeys the classical relation $T_{BH} \sim 1/M$. The mass of the hole is reduced by the radiation the temperature increases. This tendency is opposed by quantum effects. Once the mass is as small as $M_{cr} \sim M_{Planck}$ the temperature reaches its maximum value $T_{BH}(M_{cr})$ [16]. For even smaller masses it drops very rapidly and vanishes at or below the M_{Planck}. In the present model the microscopic black hole is supposed have a remnant which does not Hawking radiate any more.

Asymptotic safety is an important theoretical tool for quantum gravity. The methods used to derive the result are relevant to our scheme, even though the analysis does not support asymptotic freedom. On the other hand, the FRGE analysis necessitates approximations, like series truncations with unknown accuracy, and contains a number of field theory subtleties.

2.2.4 Sub-Planckian Black Holes

In [17] an approach with a more extensive length scale, including sub-Planckian, is considered although with the same type of goals as in this note. The authors discuss the concept of mass using the Komar integral and find that this provides a useful way of linking black holes and elementary particles. Their definition of mass suggests that gravity is effectively 2-dimensional near the Planck scale.

The Compton wave length $R_{\rm C} = \hbar/(Mc)$ and Schwarzschild radius R_S of a black hole are equal at the Planck scale. As one approaches the Planck point from the left in Fig. 1, it has been argued [19] that the Heisenberg uncertainty principle (HUP) should be replaced by a generalized uncertainty principle (GUP) of the form

$$\Delta x > \frac{\hbar}{\Delta p} + \left(\frac{\alpha L_{\rm Planck}^2}{\hbar}\right)\Delta p \tag{17}$$

where α is a dimensionless constant (usually assumed positive) which depends on the particular model and the factor of 2 in the first term has been dropped.

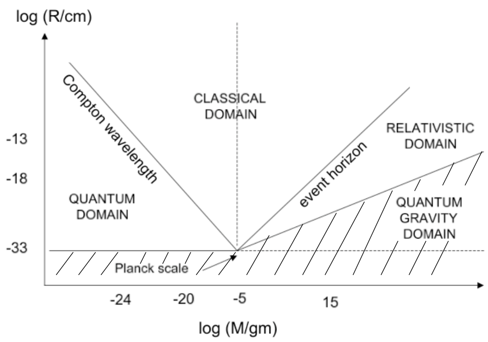

Figure 1: The division of the (M, R) diagram into the classical, quantum, relativistic and quantum gravity domains. - This figure is from [17].

If one rewrites (17) using the substitution $\Delta x \to R$ and $\Delta p \to cM$ one gets

$$R > R_C' \equiv \frac{\hbar}{Mc} + \frac{\alpha G M}{c^2} = \frac{\hbar}{Mc}\left[1 + \alpha\left(\frac{M}{M_{\rm Planck}}\right)^2\right] \tag{18}$$

This expression might be regarded as a generalized Compton wavelength, the last term representing a small correction as one approaches the Planck point from the left.

The GUP has important implications for the black hole horizon size, as can be seen by examining what happens as one approaches the intersect point from the right Fig. 1. In this limit, it is natural to write (18) as

$$R > R_C' = \frac{\alpha G M}{c^2}\left[1 + \frac{1}{\alpha}\left(\frac{M_{\rm Planck}}{M}\right)^2\right] \tag{19}$$

and this represents a small perturbation to the Schwarzschild radius for $M \gg M_{\rm Planck}$ if one assumes $\alpha = 2$. There is no reason for anticipating $\alpha = 2$ in the heuristic derivation of the GUP. However, the factor of 2 in the expression for the Schwarzschild radius is precise, whereas the coefficient associated with the Compton term is somewhat arbitrary. This motivates an alternative approach in which the free constant in (18) is associated with the first term rather than the second. One then replaces Eqs. (18) and (19) with the expressions

$$R'_{\rm C} = \frac{\beta\hbar}{Mc}\left[1 + \frac{2}{\beta}\left(\frac{M}{M_{\rm Planck}}\right)^2\right] \tag{20}$$

and

$$R'_{\rm S} = \frac{2GM}{c^2}\left[1 + \frac{\beta}{2}\left(\frac{M_{\rm Planck}}{M}\right)^2\right] \tag{21}$$

for some constant β, with the second expression being regarded as a generalized event horizon (GEH).

An important caveat is that (17) assumes the two uncertainties add linearly. On the other hand, since they are independent, it might be more natural to assume that they add quadratically [18]:

$$\Delta x > \sqrt{\left(\frac{\hbar}{\Delta p}\right)^2 + \left(\frac{\alpha \ell_{\rm Pl}^2 \Delta p}{\hbar}\right)^2}\,. \tag{22}$$

One refers to Eqs. (17) and (22) as the linear and quadratic forms of the GUP. Adopting the β formalism, then gives a unified expression for generalized Compton wavelength and event horizon size

$$R'_{\rm C} = R'_{\rm S} = \sqrt{\left(\frac{\beta\hbar}{Mc}\right)^2 + \left(\frac{2GM}{c^2}\right)^2}\,, \tag{23}$$

leading to the approximations

$$R'_{\rm C} \approx \frac{\beta\hbar}{Mc}\left[1 + \frac{2}{\beta^2}\left(\frac{M}{M_{\rm Planck}}\right)^4\right] \tag{24}$$

and

$$R'_{\rm S} \approx \frac{2GM}{c^2}\left[1 + \frac{\beta^2}{8}\left(\frac{M_{\rm Planck}}{M}\right)^4\right] \tag{25}$$

for $M \ll M_{\rm Planck}$ and $M \gg M_{\rm Planck}$, respectively. These might be compared to the exact expressions in the linear case, given by Eqs. (20) and (21).

Regardless of the exact form of the GUP, these arguments suggest that there is a connection between the uncertainty principle on microscopic scales and black holes on macroscopic scales. This is termed the black hole uncertainty principle (BHUP) correspondence and it is manifested in a unified expression for the Compton wavelength and Schwarzschild radius [20]. It is a natural consequence of combining the notions of the GUP and the GEH. Indeed, it would be satisfied for any form of the function $R'_{\rm C} \equiv R'_{\rm S}$ which asymptotes to $R_{\rm C}$ for $M \ll M_{\rm Planck}$ and $R_{\rm S}$ for $M \gg M_{\rm Planck}$. Models in which this function is symmetric under the duality transformation $M \leftrightarrow 1/M$ (such as the linear and quadratic forms given above) are said to satisfy the strong BHUP correspondence [20].

One controversial implication of the BHUP correspondence is that it suggests there could be sub-Planckian black holes with a size of order their Compton wavelength. One can argue that there is only a low probability of sub-Planckian objects becoming black holes.

The authors explore another type of solution which involves sub-Planckian black holes but avoids some of the complications associated with the LBH solution. In particular, it implies a linear rather than quadratic form of the GUP and it does not involve another asymptotic space.

The continuity between the Compton and Schwarzschild lines suggests some link between elementary particles and sub-Planckian black holes. However, one might prefer to maintain a distinction between these objects. For example, the function $|\Delta x|$ has a minimum at 0 for models with $\alpha < 0$ but with a discontinuity in the gradient. Since $R'_C = R'_S = 0$ at this point, one effectively has $G \to 0$ (no gravity) and $\hbar \to 0$ (no quantum discreteness) The distinction between particles and black holes could also be maintained with more general forms of the GUP and GEH.

In the standard picture, the Schwarzschild solution is obtained by solving Einstein's equations in vacuum and matching the metric coefficients with the Newtonian potential as a boundary condition to fix the integration constant. This constant relates to the mass specified by the Komar integral [21, p. 251]:

$$M \equiv \frac{1}{4\pi G} \int_{\partial\Sigma} d^2x \sqrt{\gamma^{(2)}}\, n_\mu \sigma_\nu \nabla^\mu K^\nu \tag{26}$$

where K^ν is a timelike vector, Σ is a spacelike surface with unit normal n^μ, and $\partial\Sigma$ is the boundary of Σ (typically a 2-sphere at spatial infinity) with metric $\gamma^{(2)ij}$ and outward normal σ^μ.

The authors consider only the particle case in the sub-Planckian regime and write (26) as

$$M \equiv \int_\Sigma d^3x \sqrt{\gamma}\, n_\mu K_\nu T^{\mu\nu} \simeq -4\pi \int_0^{R_\mathrm{C}} dr\, r^2 {T_0}^0 \tag{27}$$

where γ is the determinant of the spatially induced metric γ^{ij}, $T^{\mu\nu}$ is the stress-energy tensor and ${T_0}^0$ accounts for the particle distribution on a scale of order R_C. This corresponds to the rest mass appearing in the expression for the Compton wavelength, $R_\mathrm{C} = \hbar/(Mc)$.

Consider now a decaying black hole with mass $M \gtrsim M_\mathrm{Planck}$. The fate of such an object is an open problem in quantum gravity with at least three possible scenarios for the end-point of evaporation.

(**1**) The black hole keeps decaying semi-classically with a runaway increase of the temperature and a final explosion involving non-thermal emission of hard quanta. In this case, the energy momentum tensor exhibits an integrable singularity, ${T_0}^0 = -M\delta(\vec{x})$, and the Komar energy has a standard profile. However, this scenario may be criticized since it relies on classical and semi-classical arguments applied to a quantum gravity dominated regime.

(**2**) Quantum gravity effects modify the classical profile of the mass-energy distribution, so that ${T_0}^0 \neq -M\delta(\vec{x})$. This happens in a variety of proposals, including asymptotically safe gravity, non-commutative geometry, non-local gravity and gravitational self-completeness, for refs. see [17]. In all these cases, the end-point of evaporation turns out to be a stable zero-temperature extremal black hole configuration, preceded by a positive heat capacity cooling phase. The Komar energy would again be defined by (26), while the size of the black hole remnant would correspond to the natural ultraviolet cut-off of quantum gravity. This means that the endpoint of evaporation separates the two phases, i.e. particles and black holes. Such a scenario has the following three properties:

- singularity avoidance or inaccessibility
- non-singular final stage of evaporation
- consistent definition of black hole size with $R_{\mathrm{S}} > \ell_{\mathrm{Pl}}$ for all masses.

Only a self-consistent theory of quantum gravity can confirm this possibility.
(**3**) In the absence of further theoretical indications or experimental evidence, the authors explore a third scenario, which reverses the usual logic but still assumes the above three properties. In so far as the black hole undergoes a final stage of evaporation, the major contribution to integral (26) will be

$$M = -4\pi \int_0^{\ell_{\mathrm{Pl}}} dr\, r^2 T_0{}^0 \tag{28}$$

where $T_0{}^0$ accounts for an unspecified quantum-mechanical distribution of matter and energy. One still has $M \neq -M\delta(\vec{x})$ but the profile differs from the second scenario. Integral (28) is generally not known and might lead to a completely different definition of the Komar energy. Some anomalies are expected to emerge at the Planck scale since they already emerge at the GUP level.

3 Inflation

It is assumed that the universe originated from a primordial quantum fluctuation in vacuum, creation of a gravon field in a false vacuum. That lead in the next phase to inflation where gravity and the Higgs play major roles.

3.1 False Vacuum and Higgs Inflation

Inflation [22, 48, 24] stretches the initial quantum vacuum fluctuations to the size of the present Hubble patch, seeding the initial perturbations for the cosmic microwave background radiation and large scale structure in the universe [25]. For a theoretical review, see [26]. Since inflation dilutes all matter it is pertinent that after the end of inflation the universe is filled with the right thermal degrees of freedom: the standard model particles together with dark matter. For a review on pre- and post-inflationary dynamics, see [27].

The decay of the initial false vacuum is a nucleation process in a first order phase transitions [28]. It is initiated by the materialization of a bubble of true vacuum within the false vacuum by quantum tunneling causing a change in the cosmological constant [29].

I assume the tunneling of the scalar gravon takes place from a de Sitter vacuum to a lower energy vacuum, de Sitter or flat, by the one bubble inflationary scenario [30, 31]. Slow roll inflation, by the scalar field potential, follows after the gravon tunneling to the true vacuum in the standard inflationary way. The gravon decay produces primordial black holes which slow down inflation towards exit.

The Higgs scalar field inflation action is [32, 33]

$$S = \int d^4x\sqrt{-g}\left[\mathcal{L}_{\rm SM} - \left(\frac{\bar{M}^2_{\rm Planck}}{2} + \xi|\mathcal{H}|^2\right)R\right] \tag{29}$$

where $\mathcal{L}_{\rm SM}$ is the SM Lagrangian minimally coupled to gravity, ξ is the parameter that determines the non-minimal coupling between the Higgs and the Ricci scalar R, $\mathcal{H}$ is the Higgs doublet and, as a consequence of such large non-minimal coupling, there is a new scale in the theory, $\bar{M}_{\rm Pl}/\sqrt{\xi}$, lower than the standard reduced Planck mass, $\bar{M}_{\rm Pl} \approx 2.43 \times 10^{18}$ GeV. The part of the action that depends on the metric and the Higgs field only (the scalar-tensor part) is

$$S_{\rm st} = \int d^4x\sqrt{-g}\left[|\partial\mathcal{H}|^2 - V - \left(\frac{\bar{M}^2_{\rm Planck}}{2} + \xi|\mathcal{H}|^2\right)R\right], \tag{30}$$

where $V = \lambda(|\mathcal{H}|^2 - v^2/2)^2$ is the Higgs potential and v is the electroweak Higgs vacuum expectation value. In [33] a sizable non-minimal coupling is taken, $\xi > 1$, because it is required by inflation.

3.2 Starobinsky Model

Starobinsky has pointed out that quantum corrections to general relativity should be important in the early universe. The Starobinsky model action is [34]

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G}R + \frac{1}{b}R^2\right) \tag{31}$$

with the dimensionless coupling $b = 6M^2/M^2_{\rm Planck}$, where M is a constant of mass dimension one, $M_{\rm Planck} = G^{-1/2}$, G is the Newton's constant with scale dependence and g is the determinant of the metric. This action creates de Sitter expansion phase in the early universe and removes the early singularity.

Usually the Einstein term is regarded as the fundamental term, and the other terms (higher powers in R) are secondary in the sense that they are originate from quantum corrections. But one can take the view that the fundamental term is the one-loop second term R^2 rather than the linear term R.

A non-perturbative renormalization group (RG) analysis the Starobinsky action leads to asymptotically safe (AS) gravity [12]. There exists a non-trivial, or non-Gaussian, UV fixed point, where G is asymptotically safe and the R^2 coupling vanishes. The starting point for RG calculations is an exact renormalization group equation (ERGE) in Wilsonian context, for details see [35]. The aim of [33] is to address both the classical and quantum issues. The latter issue is more of a challenge, but the authors have performed both of them carefully.

4 Theoretical Directions

When any simple model turns out inadequate one has to turn to an analysis with more mathematical machinery. At present there are several possibilities to follow. I mention below just a few.

4.1 Dynamic Planck Scale

Assuming scalar and fermion fields a scheme for dynamic generation of the Planck scale with inflation seems possible as discussed in [36]. The authors aim at a model independent analysis and make the interesting proposal that a complete theory of quantum gravity may not even be needed because inflation is described by Einstein gravity at energies below the Planck scale. This is supported by the model of the present note, the quantum era of gravity occurs only extremely briefly before the big bang.

4.2 Higher Derivative Gravity

There is ample literature of higher derivative gravity but I mention only a recent paper [37] which gives an up-to-date view to the field (authors include the originator of the idea). The authors start with a general second-plus-fourth-order action

$$I = \int d^4x\sqrt{-g}\left(\gamma R - \alpha C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + \beta R^2\right) \tag{32}$$

where $C_{\mu\nu\rho\sigma}$ is the Weyl tensor, the traceless part of the curvature tensor $R_{\mu\nu\rho\sigma}$, one obtains a renormalisable system [38]. The spectrum of this theory contains [39] a massless graviton, a massive spin-two ghost excitation with $(m_2)^2 = \frac{\gamma}{2\alpha}$, and a massive non-ghost spin-zero excitation with $(m_0)^2 = \frac{\gamma}{6\beta}$. The canonical value of γ is $\frac{1}{16\pi G} = \frac{2}{\kappa^2}$, where G is the 4D Newton constant. Quadratic curvature terms in the action arise in most effective theories of quantized gravity, including string theory.

This article explores the set of static, spherically symmetric and asymptotically flat solutions of this class of theories. From a Frobenius analysis of the asymptotic small-radius behavior, the solution space is found to split into three asymptotic families, one of which contains the classic Schwarzschild solution. These three families are carefully analyzed to determine the corresponding numbers of free parameters in each. One solution family is capable of arising from coupling to a distributional shell of matter near the origin; this family can then match on to an asymptotically flat solution at spatial infinity without encountering a horizon. Another family, with horizons, contains the Schwarzschild solution but includes also non-Schwarzschild black holes. The third family of solutions obtained from the Frobenius analysis is nonsingular and corresponds to ‘vacuum’ solutions. In addition to the three families identified from near-origin behavior, there are solutions that may be identified as ‘wormholes’, which can match symmetrically on to another sheet of spacetime at finite radius.

Without a full stability analysis of the various phases of the static solution space one can extract some partial stability information from various quasinormal mode studies of the stability of the Schwarzschild solution itself, considered as a solution of the higher-derivative (32) theory. This has been studied in ref. [40]. It was, firstly, found there that the Schwarzschild solution is stable in the $\gamma R + \beta R^2$ theory with $\alpha = 0$. This is not surprising, because that theory is equivalent to ordinary general relativity coupled to a positive-energy massive scalar field.

In ref. [40] it was also suggested that the Schwarzschild solution could become unstable, for nontachyonic values of $(m_2)^2 = \frac{\gamma}{2\alpha}$, for sufficiently small values of

$$\mu_W = \frac{M m_2}{M_{\rm Pl}^2}\,, \tag{33}$$

where $M_{\rm Pl}$ is the Planck mass. ref. [40] then went on to claim, nonetheless, that detailed analysis of the quasinormal modes of the theory (32) showed no such instability. This conclusion has, however, been challenged more recently in ref. [41], where it is claimed that ref. [40] erred in considering only a static S-wave potential instability. Instead, the analysis of ref. [41] does find Schwarzschild S-wave instabilities for $\mu_W \lesssim 1$ by treating the Ricci tensor $R_{\mu\nu}$ as an effective massive field. This instability is compared to Schwarzschild instabilities found in massive theories of gravity [42].

Instability of the Schwarzschild solution for small black holes (i.e. small μ_W) raises the question whether a stable sector of the static solution space exists, and whether one or another of the non-Schwarzschild solutions the authors have discussed could then represent a stable final phase. Clearly, the relation between μ_W and the branch point in the black-hole solution space could be an important issue in this regard.

4.3 Extra Dimensions

In [43] the Starobinsky model is studied from the point of view of extra dimensions, usually taking six extra dimensions. The authors take the view that the main term is the R^2 term. The pure R^2 theory does not contain any dimensional constant and is therefore scale invariant. Scale symmetry may be spontaneously broken, eg. by coupling to matter sector, leading to a scale Λ. The authors give an estimate of the lower limit of scale $\Lambda \sim 5 \times 10^{15}$ GeV. This is very close to the grand unified theory (GUT) value and the authors suggest associating higher dimensional theory with GUT.

In ten dimensional theories, originally in 5D Kaluza-Klein theory, a dilaton comes always with gravity. If the Newton's constant, or Planck mass, is promoted to a dynamical field the result is the dilaton. The dilaton field has been considered as a model of dark energy in [44].

4.4 Non-Supersymmetric Strings

In the light of present LHC results research based on non-supersymmetric vacua is becoming more important. In non-supersymmetric vacua almost all the moduli are lifted up perturbatively, contrary to the supersymmetric ones which typically possess tens or even hundreds of flat directions that cannot be raised perturbatively. An interesting analysis of non-supersymmetric SO(16) $\times$ SO(16) heterotic string theory is presented in [45]. It is based on the observation that there is a triple coincidence with the Higgs potential

$$V = m^2|\mathcal{H}|^2 + \lambda|\mathcal{H}|^4 \tag{34}$$

(with $m^2 \sim -(90GeV)^2$ and $\lambda \simeq 0.13$) namely: quartic coupling λ, its running, and the bare Higgs mass can all be accidentally small at around the Planck scale. This is a direct hint for Planck scale physics in the context of superstring theory. The vanishing bare Higgs mass implies that the supersymmetry is restored at the Planck scale and that the Higgs field resides in a massless string state. The smallness of both λ and its beta function is consistent with the Higgs potential being very flat around the string scale. Such a flat potential opens up the possibility that the Higgs field plays the role of inflaton in the early universe.

In [45] the authors study the concrete model: the $SO(16) \times SO(16)$ heterotic string theory [46, 47]. This model breaks supersymmetry at the string scale but, unlike the bosonic string theory in 26 dimensions, the tachyonic modes are projected out as in the ordinary heterotic superstring theories. In the fermionic construction, the modular invariance of the partition function restricts the allowed set of the fermion numbers in Neveu-Schwarz (NS) and Ramond (R) sectors. The $SO(16) \times SO(16)$ model is the only one that has neither a tachyon nor supersymmetry in ten dimensions.

There are two possibilities for the potential beyond the maximum: (i) the potential smoothly becomes runaway (ii) the potential has another local minimum

In the latter case, the false vacuum gives a mechanism of eternal inflation. This situation is similar to the idea of the inflation being a first order phase transition. In the medium of the false vacuum, there appears a bubble of the electroweak vacuum due to the tunneling. This eternal inflation in the false vacuum has caused the so-called the graceful exit problem in the old inflation scenario [48]. However in the case (ii) there is a down hill, slow roll and a down hill structure. The space inside the bubble experiences the second stage of inflation hence this problem is ameliorated as one does not need let bubbles collide.

The above described inflation scheme is close to one considered in sec. 3.1, for both the Higgs and the gravon. May be the gravon and gravion are superpartners. Further details should be checked out and there are a lot of subtleties to be resolved.

5 Conclusions

The present note contains a proposal of a schematic model, and references to literature for more details with calculations and a hint of tests. It takes a step beyond the standard model of particles towards a model of Planck scale phenomena, assuming the standard model is valid up to that scale. At the Planck scale black holes are the key objects to study. Unfortunately not all existing calculational results concerning Planck mass region black holes are in consensus. ERGE based calculations provide rather solid results for $f(R)$ type gravity [49].

The scheme I propose here can be summarized as having the gravon a fundamental elementary particle of quantum gravity, which should be included in the standard model and the modified theory of Einstein-Hilbert gravity. But the main conclusion is, unfortunately, that no detailed action could be written for the present model. I only present the gravon as a candidate for non-singular blacks hole and dark matter. In [45] the Higgs has been considered to be a string state. The gravon in turn could be constructed as a massless black string, it remains open at this stage whether it is stable enough or suitably unstable.

One might classify the gravon and the Higgs as the "arsenal" sector and the traditional SM as the "customer" sector of the standard model at Planck scale (SM@P). The details of the present scheme remain to be fixed to a mathematical structure, a task I wish to return in the future. One of the most interesting questions is what happens to a black hole below the Planck mass value. Here it is tentatively supposed that the hole becomes particle-like and "explodes" to known particles. A simple toy model is sometimes a useful tool until experimental evidence is found for theories of more mathematical structure.

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