

Wave functions for the electron and positron

© 29/5/13 Declan Traill
Updated 1/6/2015
declan@netspace.net.au

PACS [14.60.Cd, 03.70.+k, 03.50.De, 11.10.-z]

Abstract

The Wave/Particle duality of particles in Physics is well known. Particles have properties that uniquely characterize them from one another, such as mass, charge and spin. Charged particles have associated Electric and Magnetic fields. Also every moving particle has a De Broglie wavelength determined by its mass and velocity. In this paper I show that all of these properties of a particle can be derived from a single wave function equation for that particle. I present wave functions for the Electron and the Positron and provide principles that can be used to calculate the wave functions of all the fundamental particles in Physics.

1. Introduction

In order to provide the foundations of a link between Classical Physics field concepts and the wave/particle duality in Quantum Mechanics it is necessary to demonstrate how particles can be modeled both from a Classical Wave perspective while also satisfying the requirements of Quantum Mechanics, in particular the Schrödinger wave equation and the De Broglie equations.

There is already evidence of this connection in the energy sum of the Electric and Magnetic fields in the Hamiltonian function that expresses the total energy of an atomic system:

[Ref. 1] “In 1926 Schrödinger used energy conservation to obtain a quantum mechanical equation in a variable called the wave function that accurately described single-electron states such as the hydrogen atom. The wave function depended on a Hamiltonian function and the total energy of an atomic system, and was compatible with Hertz's potential formulation. The wave function depends on the sum of the squares of E- and H-fields as is seen by examining the energy density function of the electromagnetic field.”

In order to satisfy both the wave and particle natures of particles in a model of a particle, the particle's wave function must satisfy both the Classical wave equation (which ensures that the wave function can represent a vibration of the space-time continuum) and the Schrödinger wave equation (which ensures that the wave function can represent a quantum of energy – thus a particle) (Ref. [3]).

A wave function solution to the Classical wave equation describes the motion of all points on the wave at any location in space and time. The position of a test point in space as it is affected by the wave motion can be represented as a displacement vector drawn from the starting location of the point to its current location.

In the case of Electromagnetism there is a single vector field that describes the motion of an Electromagnetic wave in this way, it is known as the Hertzian vector field [Ref. 2, 6]. The Electric and Magnetic fields can both be derived from the Hertzian vector field by differentiation with respect to space and time.

[Ref. 2] “in a vacuum a single Hertz vector written as the product of a scalar potential and a constant vector, naturally arises as consequence of the transversality of the electromagnetic fields”

Therefore, a wave function that describes a field of vectors representing Hertzian vectors can also represent a wave function describing Electric and Magnetic field vectors. If the wave function satisfies both the Classical wave equation and the Schrödinger wave equation then it can also represent a vibration of space-time and a potential solution for a Quantum particle.

This paper presents two such solutions, one representing an electron and one representing a positron. In addition I show that the correct Classical fields are produced by them and that the Quantum Mechanical requirements of the De Broglie equations are also met by them.

2. The solutions

These are the suggested wave function equations for the Electron and the Positron. The images located in [Fig. 2-10] show graphical representations of the fields derived from these wave functions using a 3D vector modelling program I wrote to aid in the visualization and testing of proposed wave function solutions.

For the Electron:

$$\Psi_e = \frac{\sqrt{2}Q_e\hbar}{8\pi iM_e c\epsilon_0} \exp\left[\frac{-iM_e c^2}{\hbar}\left(t - \frac{r}{c}\right)\right] \quad (1)$$

For the Positron:

$$\Psi_p = \frac{\sqrt{2}Q_p\hbar}{8\pi iM_p c\epsilon_0} \exp\left[\frac{-iM_p c^2}{\hbar}\left(t + \frac{r}{c}\right)\right] \quad (2)$$

Where:

Ψ_e = Electron wave function Ψ_p = Positron wave function

Q_e = Electron Charge (-) Q_p = Positron Charge (+)

M_e = Mass of an Electron M_p = Mass of a Positron

ϵ_0 = Permittivity of free space

t = Time

r = Distance from particle's centre

c = The speed of light

\hbar = The Reduced Plank's Constant

3. The solutions satisfy the wave equations

The wave nature of particles is being modelled here as a vibration of the space-time continuum and the particle nature is modelled as localized quanta of this wave energy. In order to satisfy both the wave and particle natures of particles in the model, the wave function must satisfy both the Classical wave equation (which ensures that the wave function can represent a vibration of the space-time continuum) and the Schrödinger wave equation (which ensures that the wave function can represent a quantum of energy – thus a particle) (Ref.[3]).

$$\text{Classical wave equation: } \nabla^2\psi = \frac{1}{c^2} \frac{\partial^2\psi}{\partial t^2} \quad (3)$$

$$\text{Schrödinger wave equation: } i\hbar \frac{\partial\psi}{\partial t} = \hat{H}\psi$$

$$\text{Where: } \hat{H}\psi = \text{Total Energy}$$

$$\hat{H}\psi = \text{Kinetic Energy}(KE) + \text{Potential Energy}(PE)$$

Due to Equipartition of energy in a Classical wave(Ref. [4]):

$$KE = PE = -\frac{\hbar^2}{2m} \nabla^2\psi$$

$$\text{So: } i\hbar \frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{m} \nabla^2\psi \quad (4)$$

4. The wave function and electromagnetism

Each of the measurable fields in Electromagnetic Theory (Ref. [5, 6]), and their connection back to the wave function, can be expressed quite simply by the following set of equations and illustrated by Fig (1).

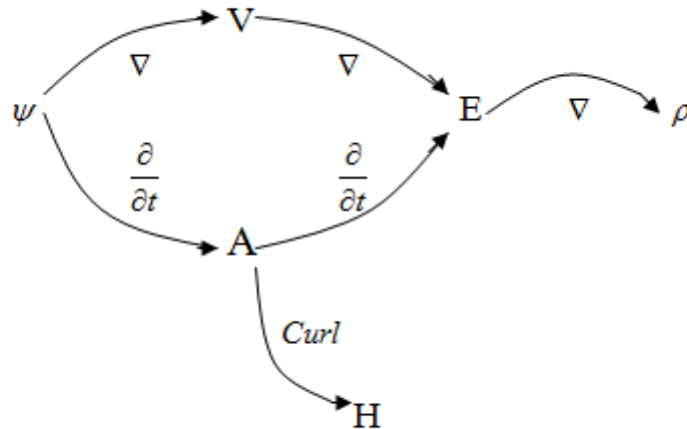


Fig. 1. The mathematical connections between the fields.

$$A = -\frac{1}{c} \frac{\partial \psi}{\partial t} \quad (5)$$

$$V = \nabla \cdot \psi \quad (6)$$

$$E = -\nabla V - \frac{1}{c} \frac{\partial A}{\partial t} \quad (7)$$

$$H = \nabla \times A \quad (8)$$

$$\rho = -\frac{1}{4\pi} \nabla \cdot E \quad (9)$$

Where:

ψ = Wave function

V = Voltage (electric potential)

E = Electric field vector

A = Vector potential

H = Magnetic field vector

ρ = Charge density

5. Analysis of the wave functions

Both wave functions represent a field of rotating vectors. The pattern described by the phases of the field of rotating vectors is that of a spinning spiral wave. The phase wave flows either away from or towards the centre of the particle (Fig.[2, 3a, 3b]). The Electron spins with the phase wave flowing outward and the Positron with the phase wave flowing inwards (Ref[5]).

The angular frequency in the wave function is derived from the following three known equations.

$$E = h\nu \quad (10)$$

$$E = mc^2 \quad (11)$$

$$h = 2\pi\hbar \quad (12)$$

Substituting Eq. (11) and Eq. (12) into Eq. (10) and solving for ν we have:

$$\nu = \frac{M_e c^2}{2\pi\hbar} \quad (13)$$

Then to convert to angular frequency:

$$\omega = 2\pi \times \nu \quad (14)$$

Substituting Eq.(13) into Eq.(14) gives:

$$\omega = \frac{M_e c^2}{\hbar} \quad \text{Radians per Second} \quad (15)$$

This describes the rate of rotation of the vectors in the vector field that describes the Electron/Positron wave function.

The centre of the Electron comprises a vector that rotates around a fixed position at the particle's centre over time. As time progresses this vector propagates radially outwards away from the centre thus forming the phase wave spiral. Every point on this spiral comprises a vector that rotates on the spot as the phase waves pass through each point.

In a classical wave, each point in the medium supporting that wave (such as the water molecules in a water wave) moves in a circular motion as the

wavepasses. The frequency of this circular motion is the same as that of the wave. However, when two waves of equal frequency (but travelling in opposite directions) combine to form a standing wave, each point in the medium rotates at twice the angular frequency of each of the two component waves.

The spinning spiral of rotating vectors that the wave function describes can be modelled as a standing wave comprised from two interfering waves: a spherical IN wave and a spherical OUT wave. Thus each point in the medium supporting this standing wave is rotating at twice the frequency of either the IN or OUT wave alone.

The spherical IN and OUT waves work together, by means of constructive and destructive interference due to a slight frequency difference between the IN and OUT waves, forming the spinning spiral structure of the particle. As each point in this spatial structure is being influenced by both IN and OUT waves (one wave from each side), a vector at that point spins around at a rate which is the sum of the IN and OUT wave frequencies, as it receives wave crests from both sides simultaneously. The vector rotation period at each point is completed in the time it takes only half of the wavelength of an IN or OUT wave to pass each point (π Radians rather than 2π Radians). The frequency of the IN and OUT waves is the same except for a slight difference that modulates this fundamental frequency and thereby forms the spiral pattern, therefore the rotation rate of each vector is two times the fundamental frequency of an IN or OUT wave.

The frequency of the vector rotation for any point in the wave function is given by Eq.(15). Thus the angular wave frequency of each IN/OUT wave is given by:

$$\frac{\omega}{2} \quad \text{Radians per Second} \quad (16)$$

From Eq. (14) and Eq.(16), and the IN/OUT wave's propagation speed (the speed of light c), the travelling wave fundamental frequency of an IN or OUT wave is:

$$\nu = \frac{\omega}{4\pi c} \quad \text{Hertz} \quad (17)$$

So, from Eq. (15) and Eq.(17), the Electron's IN/OUT wave frequency is:

$$f_{electron} = 2.06074224119 \times 10^{11} \quad \text{Hertz} \quad (18)$$

6. Verification using the De Broglie equations

The De Broglie wave frequency for a moving particle is $\frac{mv}{h}$ (19)

This is 13747.792 Hertz for an Electron travelling at $10msec^{-1}$ (20)

The Classical interpretation of the De Broglie wave is that of a beat frequency of the upstream and downstream components (with respect to the particle's direction of motion) of the Electron's IN/OUT wave, so:

$$f_{up} = f_{electron} \times \frac{c}{c - v} \quad (21)$$

$$f_{down} = f_{electron} \times \frac{c}{c + v} \quad (22)$$

Again, the speed of the Electron $v = 10msec^{-1}$

Thus from Eq. (21) and Eq.(22)the beat frequency is:

$$f_{up} - f_{down} = 13747.792 \text{ Hertz} \quad (23)$$

So we can see that the De Broglie wave matches the beat frequency of the calculated Electron IN/OUT waves for an Electron travelling at $10msec^{-1}$ (Eq.(20) equals Eq.(23)).

The Energy of the Electron can be checked too, using the De Broglie relation:

$$E = \hbar\omega \quad (24)$$

Using Eq.(15):

$$E = \hbar \frac{M_e c^2}{\hbar} = M_e c^2 \quad (25)$$

Which is the Energy/Mass relationship as it should be.

7. Derivation of the classical electric potential

For the Electron wave function, the Electric Potential (V) is $div(\psi_e)$, which in polar coordinates is (Ref.8)].

$$\frac{1}{r^2} \frac{\partial(r^2 \Psi_e)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\Psi_e \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial \Psi_e}{\partial \phi} \quad (26)$$

As all the vectors described by the wave function at the same distance from the particle's centre in a plane along the spin axis are of the same direction and magnitude, the last term in Eq.(26) is zero. The second last term represents a periodic 'wobble' in the divergence due to the spiral arms as they spin.

Its value is
$$\frac{A}{r} \left(\frac{\cos^2 \theta}{\sin \theta} - i \cos \theta \right),$$

where
$$\theta = \frac{-M_e c^2}{\hbar} \left(t - \frac{r}{c} \right) \quad \text{and} \quad A = \frac{\sqrt{2} Q_e \hbar}{8 \pi i M_e c \epsilon_0}$$

Due to the high speed of rotation of the spiral, this term's effect over time averages to zero too. Given these considerations, Eq.(26) effectively becomes:

$$\frac{1}{r^2} \frac{\partial(r^2 \Psi_e)}{\partial r} \quad (27)$$

This is the divergence of the vector field lines with increasing distance (r) from the centre of the Electron. So substituting Eq.(1) into Eq.(27) and simplifying gives:

$$V = div(\Psi_e) = \frac{1}{r^2} \frac{\partial(r^2 \Psi_e)}{\partial r} \Psi_e$$

So

$$V = \frac{\sqrt{2} Q_e}{4 \pi r \epsilon_0} \exp \left[\frac{-i M_e c^2}{\hbar} \left(t - \frac{r}{c} \right) \right] \quad (28)$$

Similarly for the Positron:

$$V = \text{div}(\Psi_p) = \frac{1}{r^2} \frac{\partial(r^2 \Psi_p)}{\partial r} \Psi_p$$

So

$$V = \frac{\sqrt{2}Q_p}{4\pi r \epsilon_0} \exp\left[\frac{-iM_p c^2}{\hbar} \left(t + \frac{r}{c}\right)\right] \quad (29)$$

When viewed close up, the spinning spiral and charge layers that comprise the electron/positron are clearly visible. Due to the fast spinning of the spiral (and outward or inward phase flow), or at large distance scales where the undulations of the spinning charge layers are small by comparison, the fields appear to become smooth and be of a continuous nature [Fig. 5-10]. So, for example, the Electric Potential for the Electron in this case appears to be the RMS (Root-Mean-Squared Ref. [9]) of Eq.(28), which is equal to the classical equation:

$$V = \frac{Q_e}{4\pi r \epsilon_0} \quad (30)$$

8. Conclusion

The wave functions presented here describe particles with all the correct properties for an Electron and a Positron and satisfy the requirements of both the Classical and Quantum Mechanical interpretations.

The wave function represents a field of rotating vectors. The spinning vectors form a phase wave that describes a spinning spiral. The phase wave flows either away from or towards the centre of the particle. Interactions between the phase waves of two or more particles could be the cause of the Electrical & Magnetic attraction/repulsion between charged particles due to momentum exchanges between the wave structures (Ref. [7]).

In general, the concepts use to build these two wave equations could be applied to all particles in Physics. The key principles are:

1. The frequency of the waves that comprise the three dimensional wave structure of the particle is based on the particle's mass (via the calculation shown above).
2. A particle's charge is defined by either an outward or inward flowing phase wave. A neutral particle would have no net phase flow inward or outward, but may contain regions of either inward or outward flow, which cancel out in the region surrounding the particle.
3. The completed wave function must satisfy both the Classical and Schrödinger wave equation.
4. Particles such as Protons (or other particles containing Quarks) would contain several components to the overall wave function, which work together to form a stable particle (i.e. together they satisfy the other three principles stated here).

9. Supplementary material

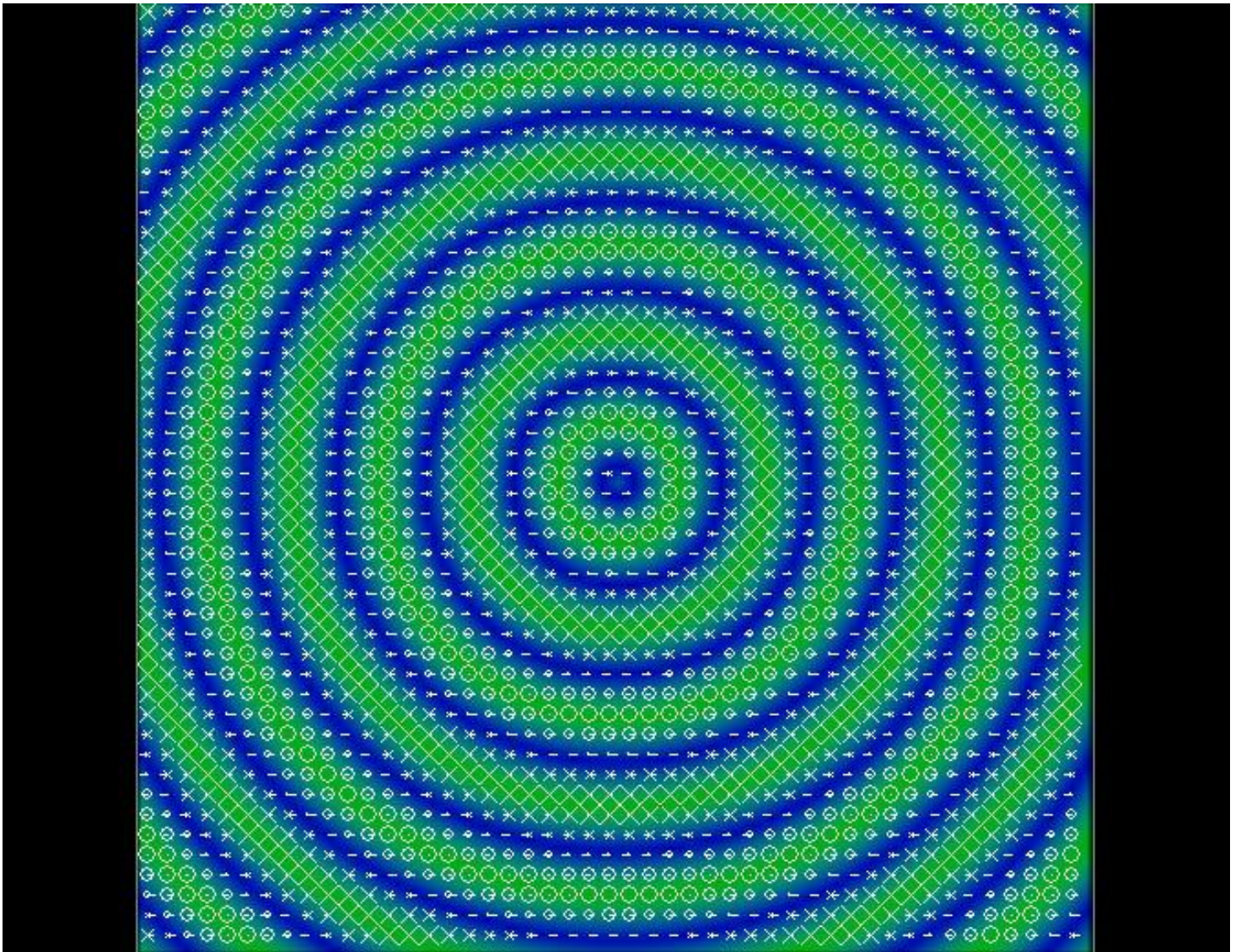


Fig. 2. The electron wave function from the side (the spin axis is vertical).

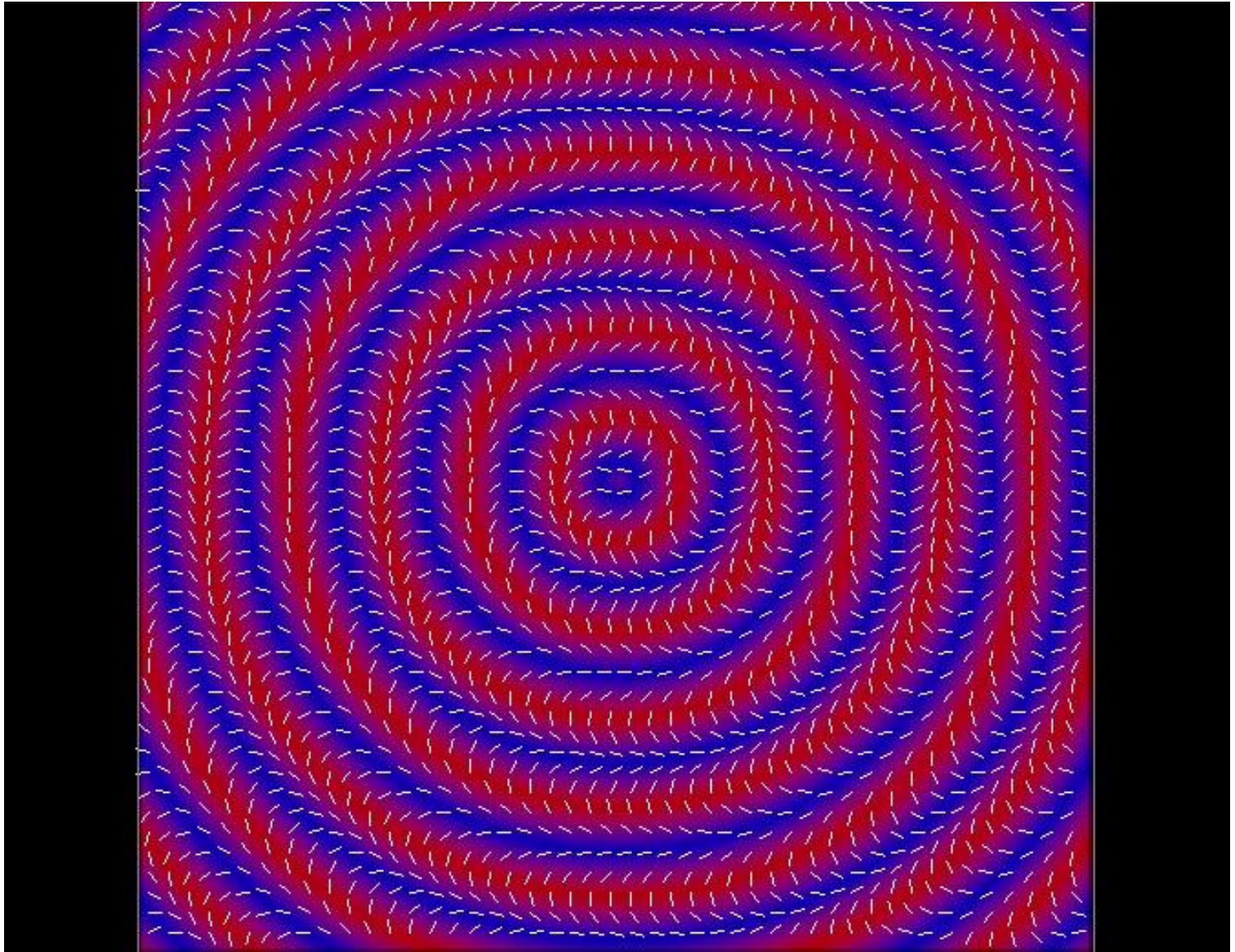


Fig. 3(a). The electron wave function viewed from the top (looking down the spin axis).

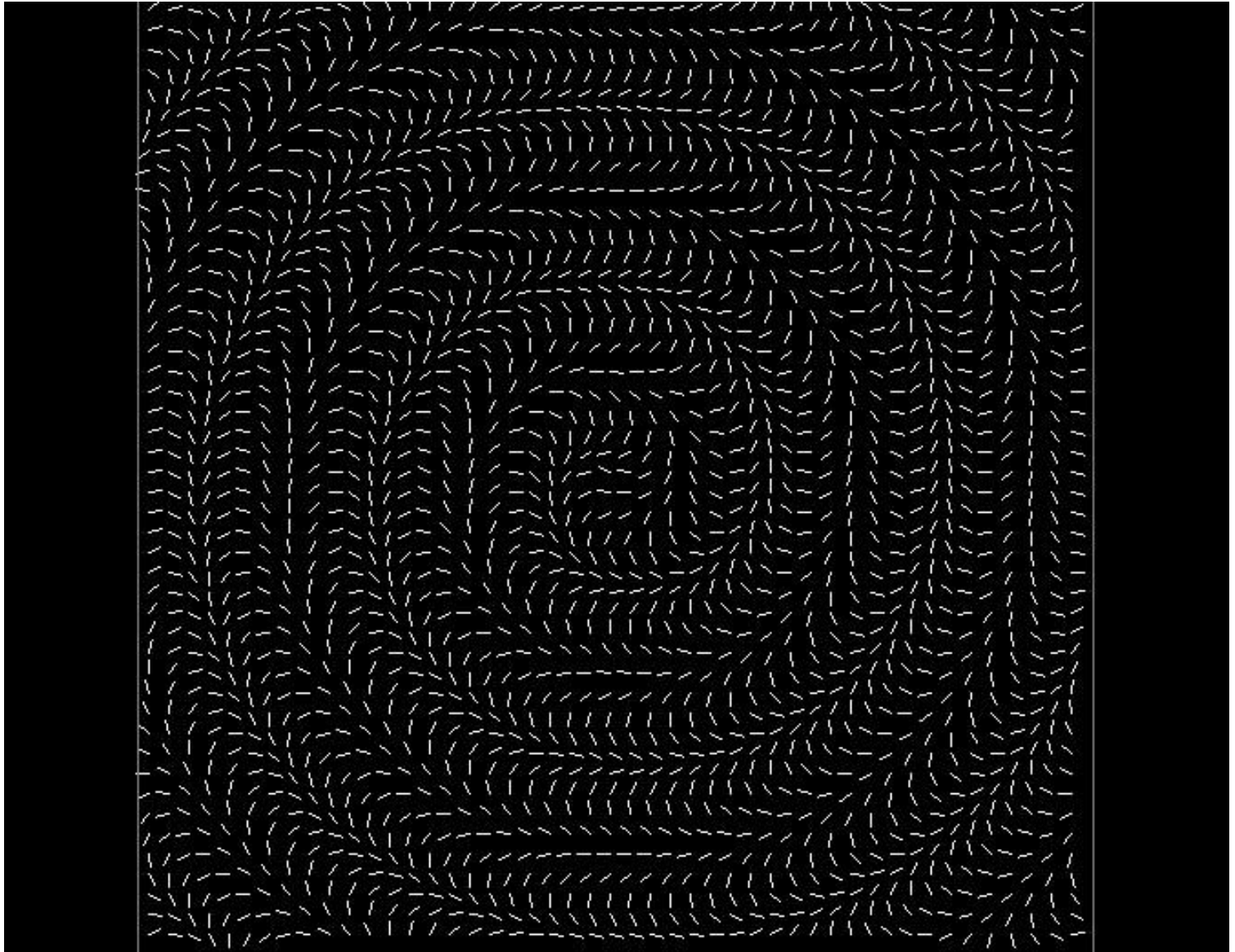


Fig. 3(b).The electron wave function (vector arrows only) viewed from the top (looking down the spin axis).



Fig. 4. The electric potential of the electron showing the double spiral of charge layers.

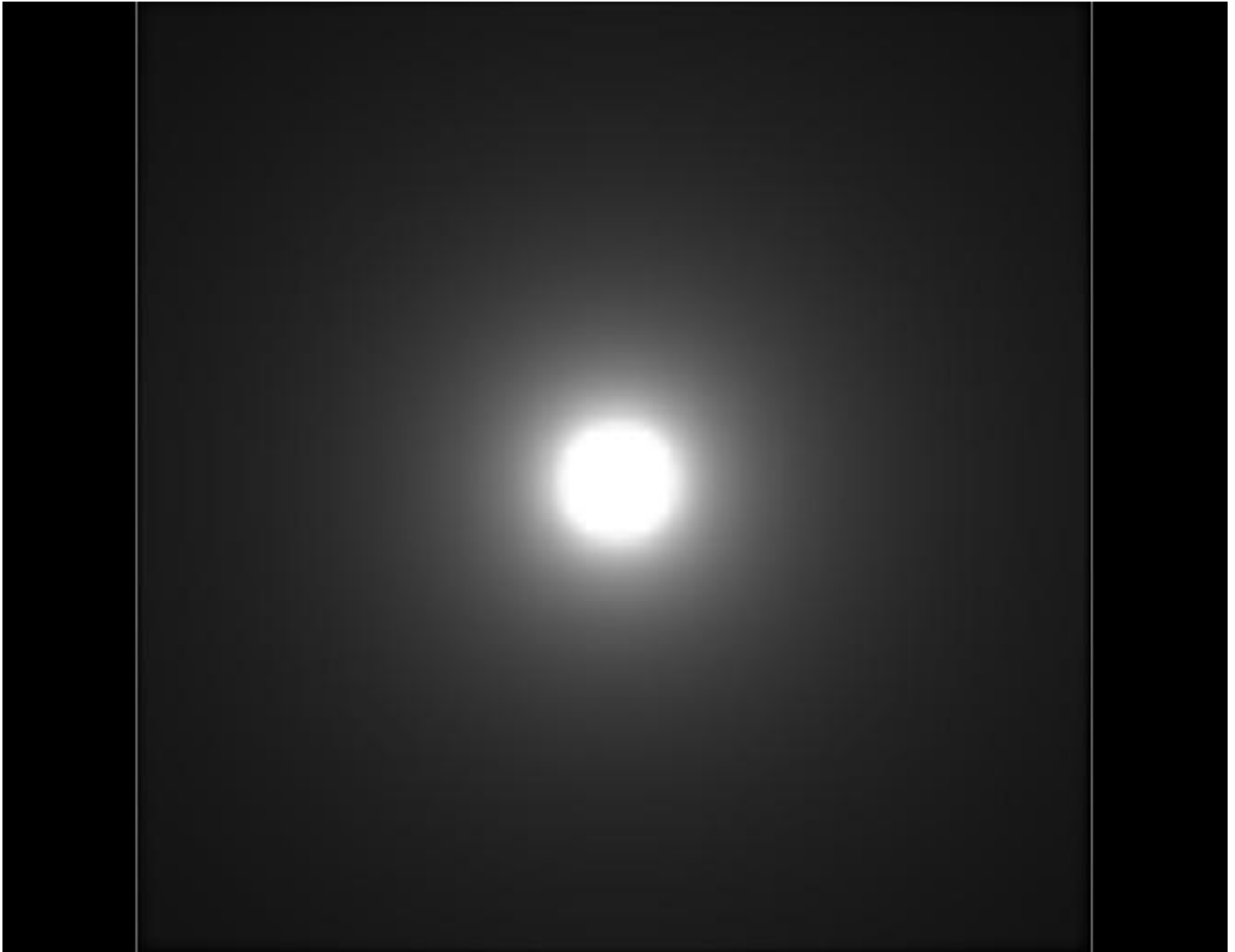


Fig.5. The electric potential of the electron with the small scale wave function undulations smoothed out (the individual charge layers are not visible).

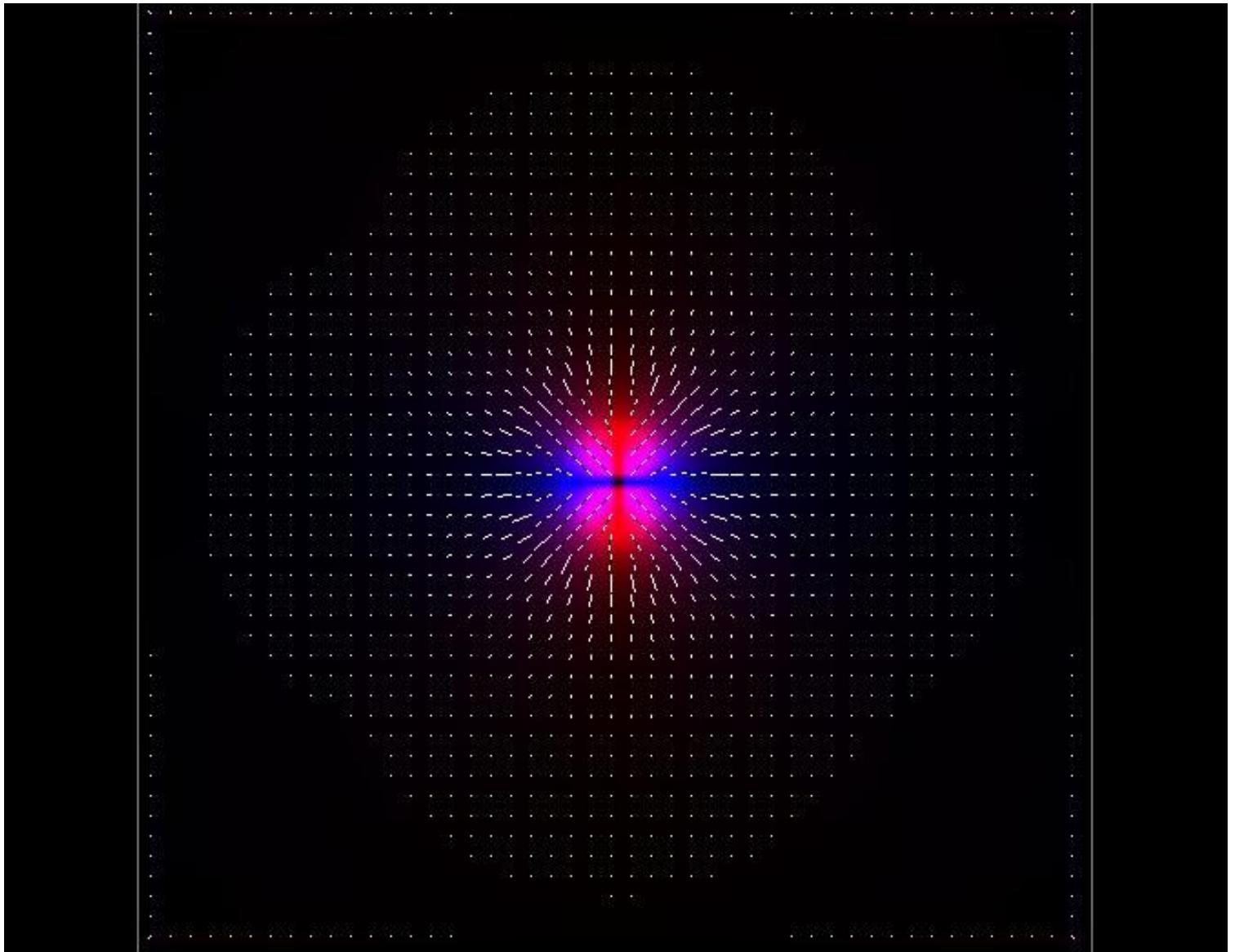


Fig.6. The electric field of the electron with the small scale wave function undulations smoothed out.

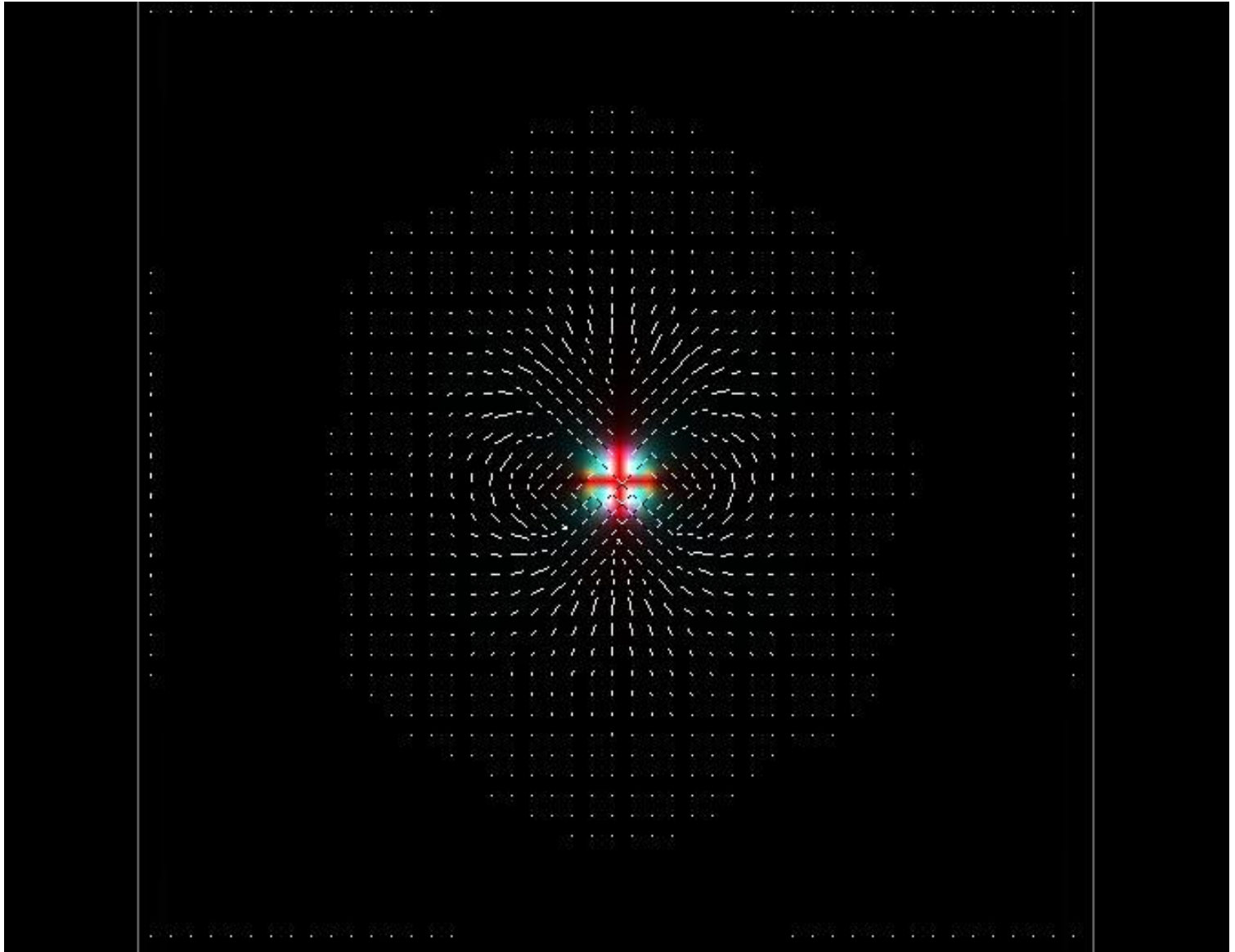


Fig.7. The magnetic field of the electron viewed from the side (spin axis is vertical) with the small scale wave function undulations smoothed out. The vectors into/out of the page are not shown in order to reveal the nice magnetic field lines.

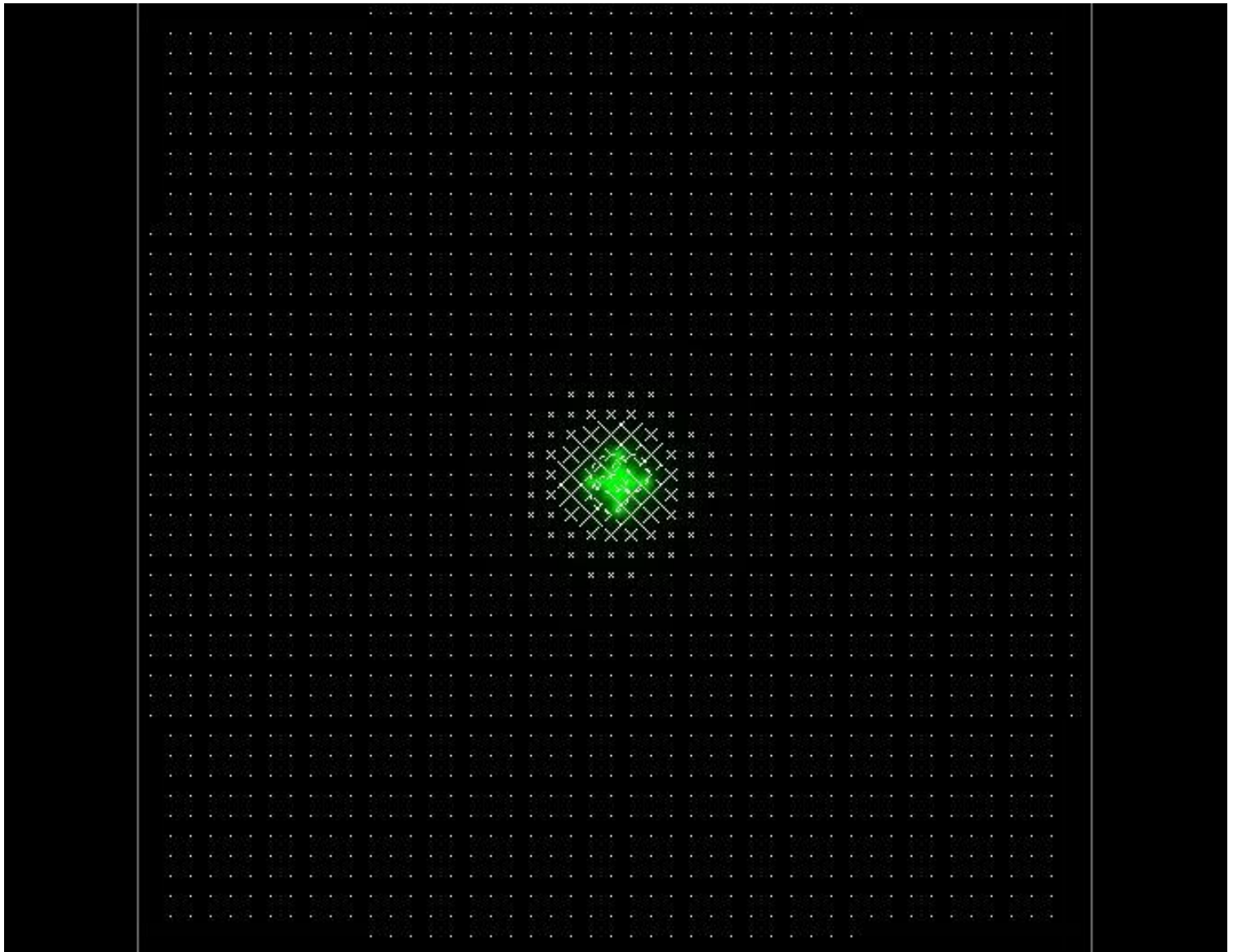


Fig.8. The magnetic field of the electron viewed from the top (looking down the spin axis) with the small scale wave function undulations smoothed out.

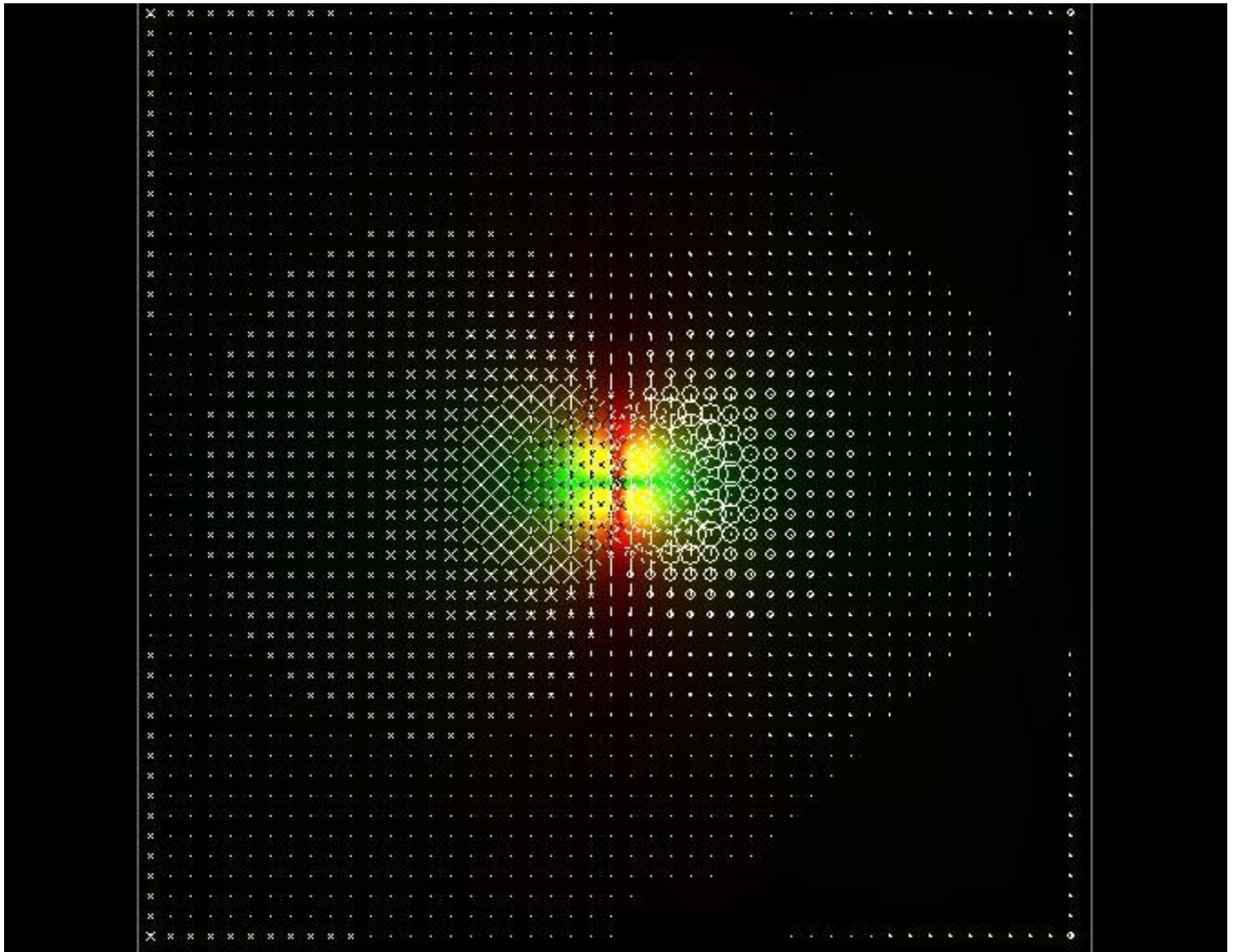


Fig.9. The vector potential field of the electron viewed from the side (the spin axis is vertical) with the small scale wave function undulations smoothed out.

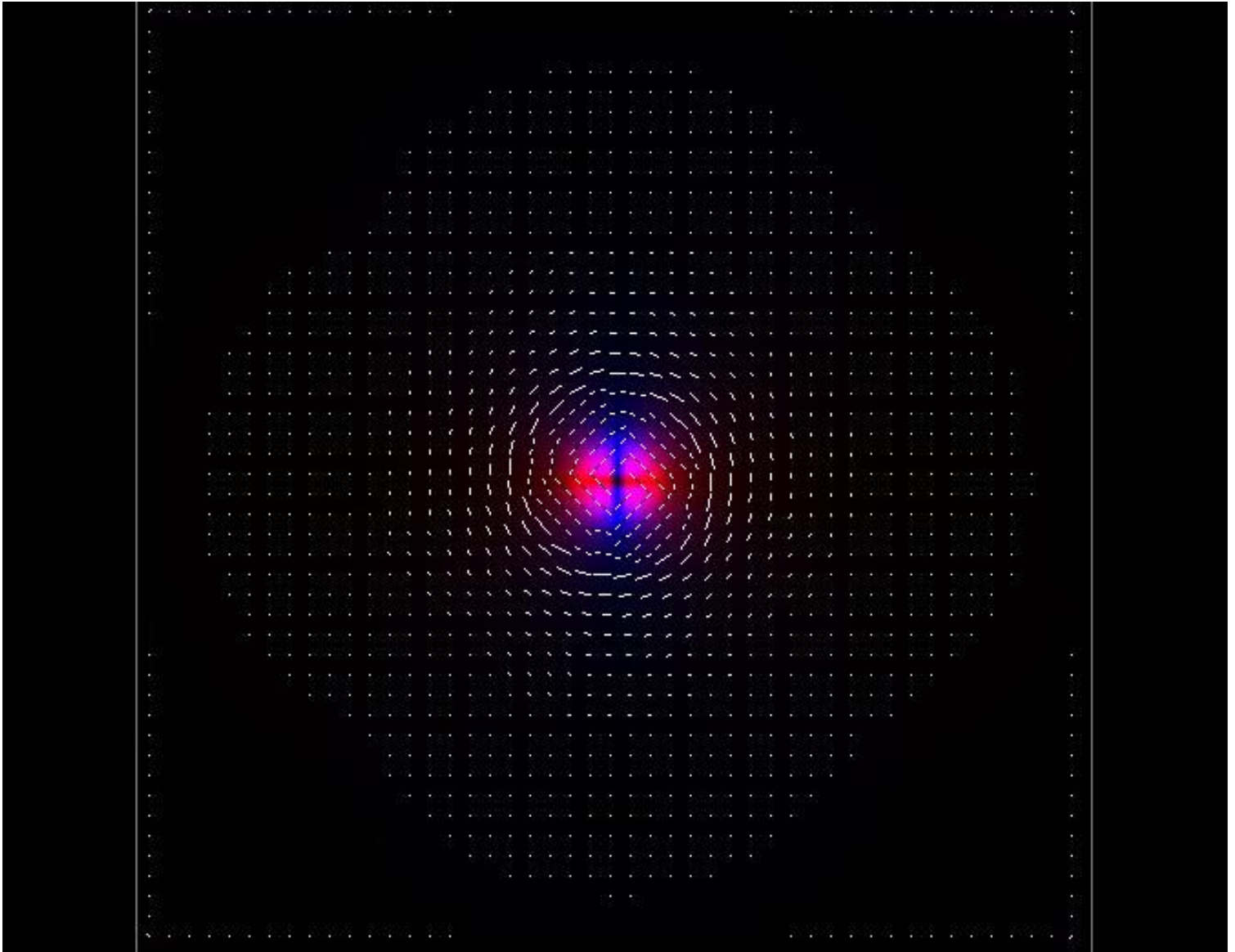


Fig.10. The vector potential field of the electron viewed from the top (looking down the spin axis) with the small scale wave function undulations smoothed out. Note how the energy of the particle flows around the spin axis in closed loops.

References

1. *A brief survey of the various field equations since 1873*, (available <http://www.unifiedphysics.com/mathematics/>, last accessed 1/6/2015)
2. Ornigotti. A and Aiello. A. 2014 *J. The Hertz vector revisited: a simple physical picture*, **16**, Journal of Optics (2014). (available at <http://iopscience.iop.org/2040-8986/16/10/105705>, last accessed 1/6/2015)
3. Simpson. G.D. *Wave Equation and Rotation*, **6**, (viXra, 2012) (available at <http://vixra.org/abs/1208.0064>, last accessed 1/6/2015)
4. Duffin. R.J. *The Equipartition of Energy in Wave Motion*, 1969 Carnegie-Mellon University (available at <http://repository.cmu.edu/cgi/viewcontent.cgi?article=1146&context=math>, last accessed 1/6/2015)
5. Traill, D.A. *Towards A Unified Field Theory*, (GSJ Journal, 2002) (available at <http://gsjournal.net/Science-Journals/Research%20Papers-Mechanics%20/%20Electrodynamics/Download/1091>, last accessed 1/6/2015)
6. Skilling. H.H. *Fundamentals Of Electric Waves*, Wiley. NY, **127-140** (1942)
7. Traill, D.A. *The Mechanics Of Charged Particles*, (GSJ Journal, 2005) (available at <http://gsjournal.net/Science-Journals/Research%20Papers-Mechanics%20/%20Electrodynamics/Download/1108>, last accessed 1/6/2015)
8. *Del in cylindrical and spherical coordinates* (available at http://en.wikipedia.org/wiki/Del_in_cylindrical_and_spherical_coordinates, last accessed 1/6/2015)
9. *The RMS Voltage of an AC Waveform* (available at <http://www.electronics-tutorials.ws/accircuits/rms-voltage.html>, last accessed 1/6/2015)