Modified Alcubierre Warp Drive I

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Abstract

A solution of general relativity is presented that describes an Alcubierre [1] propulsion system in which it is possible to travel at superluminal speed while reducing the components of the energy-impulse tensor (thus reducing energy density) by an arbitrary value.

1 Introduction:

Alcubierre [1] in 1994 proposed a solution of the equations of general relativity which provides the only viable means to accelerate a spaceship up to superluminal velocities without using wormholes. A problem was soon identified: Pfenning [4] showed that the required energy is comparable to the total energy of the universe and that it is negative. Moreover he used quantum inequalities to show that this energy gets distributed at very short scale (about 100 times the Planck length) up to a multiplicative factor equal to the squared speed. This paper (part I), and the following one (part II) going to be published soon, investigate these problems, introducing a way to reduce the amount of energy involved and its spacial distribution within the warp bubble. Later Hiscock [10] proved the existence of an event horizon for superluminal travels which would imply the presence of Hawking radiation responsible for the rapid destruction of the spaceship.

Note: In the following we adopt the notation used by Landau and Lifshitz in the second volume ("The Classical Theory of Fields") of their well known Course of Theoretical Physics [12].

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We propose the use of the metric

$$ds^2=\left(1-v^2\frac{f(x,y,z-k(t))^2}{a(x,y,z-k(t))^2}\right)dt^2+2v\frac{f(x,y,z-k(t))}{a(x,y,z-k(t))}dt\,dz-dx^2-dy^2-dz^2 \quad (1)$$

or in implicit form:

$$ds^2=dt^2-\left[dz-v\frac{f(x,y,z-k(t))}{a(x,y,z-k(t))}dt\right]^2-dx^2-dy^2 \quad (2)$$

while Miguel Alcubierre solutions [1] is:

$$ds^2=dt^2-[dz-v\,f(x,y,z-k(t))dt]^2-dx^2-dy^2 \quad (3)$$

- 1)-The Pfenning zone is the zone within the interval: $R-\frac{\Delta}{2}<r<R+\frac{\Delta}{2}$ where $\Delta\ll 1$ R is the radius of the Warp bubble and Δ is the wall thickness of the Warp bubble $R\gg\Delta$.
- 2)- $r=(x^2+y^2+(z-k(t))^2)^{\frac{1}{2}}$ and $\frac{dk(t)}{dt}=v=const$
- 3)-In the Pfenning zone we let $a(r)=a(x,y,z-k(t))\gg 1$ (there is the source of esotic matter)

2 Einstein tensor in contravariant form in the Pfenning zone is G^{ik} :

where $D_i=\frac{\partial}{\partial x^i}, x^i=x,y,z(i=1,2,3)$ and $D_{i,k}=\frac{\partial^2}{\partial x^i\partial x^k}, x^i, x^k=x,y,z(i,k=1,2,3)$

$$G^{tt} = -\frac{1}{4}\frac{1}{a(x,y,z-k(t))^4}(v^2(f(x,y,z-k(t))^2 D_2(a)(x,y,z-k(t))^2 + f(x,y,z-k(t))^2 D_1(a)(x,y,z-k(t))^2$$

$$+ D_1(f)(x,y,z-k(t))^2 a(x,y,z-k(t))^2 + D_2(f)(x,y,z-k(t))^2 a(x,y,z-k(t))^2$$

$$-2 D_1(f)(x,y,z-k(t))\, a(x,y,z-k(t))\, f(x,y,z-k(t))\, D_1(a)(x,y,z-k(t))$$

$$-2 D_2(f)(x,y,z-k(t))\, a(x,y,z-k(t))\, f(x,y,z-k(t))\, D_2(a)(x,y,z-k(t))))$$

$$G^{tx} = \frac{1}{2}\frac{1}{a(x,y,z-k(t))^3}(v(-a(x,y,z-k(t))\, D_3(f)(x,y,z-k(t))\, D_1(a)(x,y,z-k(t))$$

$$- a(x,y,z-k(t))\, D_1(f)(x,y,z-k(t))\, D_3(a)(x,y,z-k(t))$$

$$- a(x,y,z-k(t))\, f(x,y,z-k(t))\, D_{1,\,3}(a)(x,y,z-k(t))$$

$$+ 2 f(x,y,z-k(t))\, D_1(a)(x,y,z-k(t))\, D_3(a)(x,y,z-k(t)) + a(x,y,z-k(t))^2 D_{1,\,3}(f)(x,y,z-k(t))))$$

$$G^{ty} = \frac{1}{2}\frac{1}{a(x,y,z-k(t))^3}(v(-a(x,y,z-k(t))\, f(x,y,z-k(t))\, D_{2,\,3}(a)(x,y,z-k(t))$$

$$- a(x,y,z-k(t))\, D_2(f)(x,y,z-k(t))\, D_3(a)(x,y,z-k(t))$$

$$- a(x,y,z-k(t))\, D_3(f)(x,y,z-k(t))\, D_2(a)(x,y,z-k(t)) + a(x,y,z-k(t))^2 D_{2,\,3}(f)(x,y,z-k(t))$$

$$+ 2 f(x,y,z-k(t))\, D_2(a)(x,y,z-k(t))\, D_3(a)(x,y,z-k(t))))$$

$$G^{tz} = -\frac{1}{4}\frac{1}{a(x,y,z-k(t))^5}(v(-2\, a(x,y,z-k(t))^3 f(x,y,z-k(t))\, D_{2,\,2}(a)(x,y,z-k(t))$$

$$-2\, a(x,y,z-k(t))^3 f(x,y,z-k(t))\, D_{1,\,1}(a)(x,y,z-k(t))$$

$$+ 4 D_2(a)(x,y,z-k(t))^2 a(x,y,z-k(t))^2 f(x,y,z-k(t))$$

$$- 4 D_2(a)(x,y,z-k(t))\, a(x,y,z-k(t))^3 D_2(f)(x,y,z-k(t))$$

$$+ 4 D_1(a)(x,y,z-k(t))^2 a(x,y,z-k(t))^2 f(x,y,z-k(t))$$

$$- 4\, D_1(a)(x, y, z - k(t))\, a(x, y, z - k(t))^3\, D_1(f)(x, y, z - k(t))$$

$$+ v^2 f(x, y, z - k(t))\, a(x, y, z - k(t))^2\, D_2(f)(x, y, z - k(t))^2$$

$$+ v^2 f(x, y, z - k(t))\, a(x, y, z - k(t))^2\, D_1(f)(x, y, z - k(t))^2 + v^2 f(x, y, z - k(t))^3\, D_1(a)(x, y, z - k(t))^2$$

$$+ v^2 f(x, y, z - k(t))^3\, D_2(a)(x, y, z - k(t))^2 + 2\, a(x, y, z - k(t))^4\, D_{2,\,2}(f)(x, y, z - k(t))$$

$$+ 2\, a(x, y, z - k(t))^4\, D_{1,\,1}(f)(x, y, z - k(t))$$

$$- 2\, v^2 f(x, y, z - k(t))^2\, a(x, y, z - k(t))\, D_1(f)(x, y, z - k(t))\, D_1(a)(x, y, z - k(t))$$

$$- 2\, v^2 f(x, y, z - k(t))^2\, a(x, y, z - k(t))\, D_2(f)(x, y, z - k(t))\, D_2(a)(x, y, z - k(t))))$$

$$G^{x\,x} = - \frac{1}{4} \frac{1}{a(x, y, z - k(t))^4} \left(v \left(-v\, D_1(f)(x, y, z - k(t))^2\, a(x, y, z - k(t))^2 \right.\right.$$

$$+ 2\, v\, D_1(f)(x, y, z - k(t))\, a(x, y, z - k(t))\, f(x, y, z - k(t))\, D_1(a)(x, y, z - k(t))$$

$$- v\, f(x, y, z - k(t))^2\, D_1(a)(x, y, z - k(t))^2 + 4\, v\, D_3(f)(x, y, z - k(t))^2\, a(x, y, z - k(t))^2$$

$$- 4 \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))^3\, D_{3,\,3}(f)(x, y, z - k(t)) + v\, D_2(f)(x, y, z - k(t))^2\, a(x, y, z - k(t))^2$$

$$+ v\, f(x, y, z - k(t))^2\, D_2(a)(x, y, z - k(t))^2 + 4\, v\, f(x, y, z - k(t))\, a(x, y, z - k(t))^2\, D_{3,\,3}(f)(x, y, z - k(t))$$

$$+ 4 \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))^2\, f(x, y, z - k(t))\, D_{3,\,3}(a)(x, y, z - k(t))$$

$$- 4\, v\, f(x, y, z - k(t))^2\, a(x, y, z - k(t))\, D_{3,\,3}(a)(x, y, z - k(t))$$

$$+ 8\, D_3(a)(x, y, z - k(t)) \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))^2\, D_3(f)(x, y, z - k(t))$$

$$- 8\, D_3(a)(x, y, z - k(t))^2 \left(\frac{d}{dt} k(t)\right) a(x, y, z - k(t))\, f(x, y, z - k(t))$$

$$+ 12\, v\, f(x, y, z - k(t))^2\, D_3(a)(x, y, z - k(t))^2$$

$$- 16\, v\, D_3(f)(x, y, z - k(t))\, a(x, y, z - k(t))\, f(x, y, z - k(t))\, D_3(a)(x, y, z - k(t))$$

$$- 2\, v\, D_2(f)(x, y, z - k(t))\, a(x, y, z - k(t))\, f(x, y, z - k(t))\, D_2(a)(x, y, z - k(t)) \Big) \Big)$$

$$G^{x\,y} = \frac{1}{2} \frac{1}{a(x, y, z - k(t))^4} (v^2\, (D_1(f)(x, y, z - k(t))\, a(x, y, z - k(t)) - f(x, y, z - k(t))\, D_1(a)(x, y, z - k(t)))\, (D_2(f)(x, y,$$

$$z - k(t))\, a(x, y, z - k(t)) - f(x, y, z - k(t))\, D_2(a)(x, y, z - k(t))))$$

$$G^{x\,z} = -\frac{1}{2} \frac{1}{a(x, y, z - k(t))^4} \left(v \left(\left(\frac{d}{dt} k(t)\right) a(x, y, z - k(t))^3\, D_{1,\,3}(f)(x, y, z - k(t)) \right.\right.$$

$$- 2\, v\, f(x, y, z - k(t))\, a(x, y, z - k(t))^2\, D_{1,\,3}(f)(x, y, z - k(t))$$

$$+ 2\, v\, f(x, y, z - k(t))^2\, a(x, y, z - k(t))\, D_{1,\,3}(a)(x, y, z - k(t))$$

$$- \left(\frac{d}{dt} k(t)\right) a(x, y, z - k(t))^2\, D_1(f)(x, y, z - k(t))\, D_3(a)(x, y, z - k(t))$$

$$- 2\, v\, D_3(f)(x, y, z - k(t))\, a(x, y, z - k(t))^2\, D_1(f)(x, y, z - k(t))$$

$$- \left(\frac{d}{dt} k(t)\right) a(x, y, z - k(t))^2\, f(x, y, z - k(t))\, D_{1,\,3}(a)(x, y, z - k(t))$$

$$- \left(\frac{d}{dt} k(t)\right) a(x, y, z - k(t))^2\, D_3(f)(x, y, z - k(t))\, D_1(a)(x, y, z - k(t))$$

$$+ 4\, v\, D_3(f)(x, y, z - k(t))\, a(x, y, z - k(t))\, f(x, y, z - k(t))\, D_1(a)(x, y, z - k(t))$$

$$+ 4\, v\, f(x, y, z - k(t))\, a(x, y, z - k(t))\, D_1(f)(x, y, z - k(t))\, D_3(a)(x, y, z - k(t))$$

$$- 6\, v\, f(x, y, z - k(t))^2\, D_3(a)(x, y, z - k(t))\, D_1(a)(x, y, z - k(t))$$

$$+ 2\, D_3(a)(x, y, z - k(t)) \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))\, f(x, y, z - k(t))\, D_1(a)(x, y, z - k(t)) \Big) \Big)$$

$$G^{yy} = -\frac{1}{4} \frac{1}{a(x, y, z - k(t))^4} \Big(v \Big(-v\, D_2(f)(x, y, z - k(t))^2\, a(x, y, z - k(t))^2$$

$$+ 2\, v\, D_2(f)(x, y, z - k(t))\, a(x, y, z - k(t))\, f(x, y, z - k(t))\, D_2(a)(x, y, z - k(t))$$

$$- v\, f(x, y, z - k(t))^2\, D_2(a)(x, y, z - k(t))^2 + 4\, v\, D_3(f)(x, y, z - k(t))^2\, a(x, y, z - k(t))^2$$

$$- 4 \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))^3\, D_{3,\,3}(f)(x, y, z - k(t)) + v\, D_1(f)(x, y, z - k(t))^2\, a(x, y, z - k(t))^2$$

$$+ v\, f(x, y, z - k(t))^2\, D_1(a)(x, y, z - k(t))^2 + 4\, v\, f(x, y, z - k(t))\, a(x, y, z - k(t))^2\, D_{3,\,3}(f)(x, y, z - k(t))$$

$$+ 4 \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))^2\, f(x, y, z - k(t))\, D_{3,\,3}(a)(x, y, z - k(t))$$

$$- 4\, v\, f(x, y, z - k(t))^2\, a(x, y, z - k(t))\, D_{3,\,3}(a)(x, y, z - k(t))$$

$$+ 8\, D_3(a)(x, y, z - k(t)) \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))^2\, D_3(f)(x, y, z - k(t))$$

$$- 8\, D_3(a)(x, y, z - k(t))^2 \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))\, f(x, y, z - k(t))$$

$$+ 12\, v\, f(x, y, z - k(t))^2\, D_3(a)(x, y, z - k(t))^2$$

$$- 16\, v\, D_3(f)(x, y, z - k(t))\, a(x, y, z - k(t))\, f(x, y, z - k(t))\, D_3(a)(x, y, z - k(t))$$

$$- 2\, v\, D_1(f)(x, y, z - k(t))\, a(x, y, z - k(t))\, f(x, y, z - k(t))\, D_1(a)(x, y, z - k(t)) \Big) \Big)$$

$$G^{yz} = -\frac{1}{2}\,\frac{1}{a(x, y, z - k(t))^4}\left(v\left(\left(\frac{d}{dt}k(t)\right) a(x, y, z - k(t))^3\,\mathrm{D}_{2,\,3}(f)(x, y, z - k(t))\right.\right.$$

$$+ 2\,v\,f(x, y, z - k(t))^2\,a(x, y, z - k(t))\,\mathrm{D}_{2,\,3}(a)(x, y, z - k(t))$$

$$- 2\,v\,f(x, y, z - k(t))\,a(x, y, z - k(t))^2\,\mathrm{D}_{2,\,3}(f)(x, y, z - k(t))$$

$$-\left(\frac{d}{dt}k(t)\right) a(x, y, z - k(t))^2\,f(x, y, z - k(t))\,\mathrm{D}_{2,\,3}(a)(x, y, z - k(t))$$

$$-\left(\frac{d}{dt}k(t)\right) a(x, y, z - k(t))^2\,\mathrm{D}_3(f)(x, y, z - k(t))\,\mathrm{D}_2(a)(x, y, z - k(t))$$

$$-\left(\frac{d}{dt}k(t)\right) a(x, y, z - k(t))^2\,\mathrm{D}_2(f)(x, y, z - k(t))\,\mathrm{D}_3(a)(x, y, z - k(t))$$

$$- 2\,v\,\mathrm{D}_3(f)(x, y, z - k(t))\,a(x, y, z - k(t))^2\,\mathrm{D}_2(f)(x, y, z - k(t))$$

$$+ 4\,v\,\mathrm{D}_3(f)(x, y, z - k(t))\,a(x, y, z - k(t))\,f(x, y, z - k(t))\,\mathrm{D}_2(a)(x, y, z - k(t))$$

$$+ 4\,v\,f(x, y, z - k(t))\,a(x, y, z - k(t))\,\mathrm{D}_2(f)(x, y, z - k(t))\,\mathrm{D}_3(a)(x, y, z - k(t))$$

$$- 6\,v\,f(x, y, z - k(t))^2\,\mathrm{D}_3(a)(x, y, z - k(t))\,\mathrm{D}_2(a)(x, y, z - k(t))$$

$$\left.\left. + 2\,\mathrm{D}_3(a)(x, y, z - k(t))\left(\frac{d}{dt}k(t)\right) a(x, y, z - k(t))\,f(x, y, z - k(t))\,\mathrm{D}_2(a)(x, y, z - k(t))\right)\right)$$

$$G^{zz} = -\frac{1}{4}\,\frac{1}{a(x, y, z - k(t))^6}\,(v^2\,(4\,f(x, y, z - k(t))\,a(x, y, z - k(t))^4\,\mathrm{D}_{1,\,1}(f)(x, y, z - k(t))$$

$$+ 4\,f(x, y, z - k(t))\,a(x, y, z - k(t))^4\,\mathrm{D}_{2,\,2}(f)(x, y, z - k(t))$$

$$+ 11\,f(x, y, z - k(t))^2\,\mathrm{D}_1(a)(x, y, z - k(t))^2\,a(x, y, z - k(t))^2$$

$$+ 11\,f(x, y, z - k(t))^2\,\mathrm{D}_2(a)(x, y, z - k(t))^2\,a(x, y, z - k(t))^2$$

$$- 4\, f(x, y, z - k(t))^2\, a(x, y, z - k(t))^3\, D_{1,\,1}(a)(x, y, z - k(t))$$

$$- 4\, f(x, y, z - k(t))^2\, a(x, y, z - k(t))^3\, D_{2,\,2}(a)(x, y, z - k(t)) + 3\, D_1(f)(x, y, z - k(t))^2\, a(x, y, z - k(t))^4$$

$$+ 3\, D_2(f)(x, y, z - k(t))^2\, a(x, y, z - k(t))^4$$

$$- 2\, v^2\, f(x, y, z - k(t))^3\, a(x, y, z - k(t))\, D_1(f)(x, y, z - k(t))\, D_1(a)(x, y, z - k(t))$$

$$- 2\, v^2\, f(x, y, z - k(t))^3\, a(x, y, z - k(t))\, D_2(f)(x, y, z - k(t))\, D_2(a)(x, y, z - k(t))$$

$$+ v^2\, f(x, y, z - k(t))^2\, a(x, y, z - k(t))^2\, D_2(f)(x, y, z - k(t))^2$$

$$+ v^2\, f(x, y, z - k(t))^2\, a(x, y, z - k(t))^2\, D_1(f)(x, y, z - k(t))^2 + v^2\, f(x, y, z - k(t))^4\, D_1(a)(x, y, z - k(t))^2$$

$$+ v^2\, f(x, y, z - k(t))^4\, D_2(a)(x, y, z - k(t))^2$$

$$- 14\, D_2(f)(x, y, z - k(t))\, a(x, y, z - k(t))^3\, f(x, y, z - k(t))\, D_2(a)(x, y, z - k(t))$$

$$- 14\, D_1(f)(x, y, z - k(t))\, a(x, y, z - k(t))^3\, f(x, y, z - k(t))\, D_1(a)(x, y, z - k(t))))$$

The functions $f(r)=f(x,y,z-k(t))$ and $a(r)=a(x,y,z-k(t))$ assume the following values:

- 1)-inside the warp bubble $(0<r<R-\frac{\Delta}{2})$ $f(r)=1$ and $a(r)=1$
- 2)-outside the warp bubble $(r>R+\frac{\Delta}{2})$ $f(r)=0$ and $a(r)=1$
- 3)-in the Alcubierre warped region $(R-\frac{\Delta}{2}<r<R+\frac{\Delta}{2})$ $0<f(r)<1$ (Pfenning zone [4]) $a(r)=a(x,y,z-k(t))\gg 1$ possessing extremely large values

Because of the value $a(r)=a(x,y,z-k(t))\gg 1$ (extremely large) the components of T^{ik} can be reduced by an arbitrary value in the Pfenning zone.

Einstein Equation:

$$G^{ik}=\frac{8\pi G}{c^4}T^{ik} \qquad T^{ik} \text{ (energy-impulse tensor) [12]}$$

- 1)-Internal metric of the Warp bubble $\left(0<r<R-\frac{\Delta}{2}\right)$ is:

$$ds^2=dt^2-(dz-vdt)^2-dx^2-dy^2 \tag{4}$$

moving with velocity v (multiple of the speed of light c) along the z-axis.

- 2)-Metric outside of the bubble beyond the Pfenning zone $\left(r>R+\frac{\Delta}{2}\right)$ is:

$$ds^2=dt^2-dx^2-dy^2-dz^2 \tag{5}$$

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3 Conclusions

These calculations show that the modified Alcubierre propulsion system can achieve superluminal speeds without giving rise to problems in the energy density and in the components of the energy-impulse tensor.

4 Appendix I:

The components of the Einstein tensor proportional to the energy-impulse tensor have been calculated with reference to an observer whose gravitational field is very weak and whose speeds are far lower than the speed of light, observing the spaceship and the warp bubble moving at speed v, i.e., an inertial reference frame in which the spaceship is moving at speed v . If we want to calculate in the Eulerian reference frame, that is moving with the spaceship, we get for each component of the energy-impulse tensor in implicit form the following:

$$(energy\ density) = k\, G^{tt} \qquad (6)$$

$$(impulse\ density\ x) = -k\, G^{tx} \qquad (7)$$

$$(impulse\ density\ y) = -k\, G^{ty} \qquad (8)$$

$$(impulse\ density\ z) = k\left[G^{tt}\, v\,(\frac{f}{a}) - G^{tz}\right] \qquad (9)$$

$$(stress\ xx) = k\, G^{xx} \qquad (10)$$

$$(stress\ yy) = k\, G^{yy} \qquad (11)$$

$$(stress\ zz) = k\left[G^{tt}\, v^2 (\frac{f}{a})^2 - 2\mathrm{v}\,(\frac{f}{a}) G^{tz} + G^{zz}\right] \qquad (12)$$

$$(stress\ xy) = k\, G^{xy} \qquad (13)$$

$$(stress\ xz) = k\left[-v\,(\frac{f}{a}) G^{xt} + G^{xz}\right] \qquad (14)$$

$$(stress\ yz) = k\left[-v\,(\frac{f}{a}) G^{yt} + G^{yz}\right] \qquad (15)$$

The various stress xx, stress yy, stress zz, stress xy, stress xz, stress yz and their symmetric counterparts are the components of the stress tensor [12], and $k = \frac{c^4}{8\pi G}$; c is the speed of light and G is Newton's gravitational constant; $a(r) = a(x, y, z - k(t)) \gg 1$ in Pfenning zone where the energy-impulse tensor is not zero.

5 Appendix II: In the non-constant case $v=v(t)$ the metric becomes:

$$ds^2=\left(1-v(t)^2\frac{f(x,y,z-k(t))^2}{a(x,y,z-k(t))^2}\right)dt^2+2\mathrm{v}(t)\frac{f(x,y,z-k(t))}{a(x,y,z-k(t))}dtdz-dx^2-dy^2-dz^2 \quad (16)$$

which can be recast in implicit form as:

$$ds^2=dt^2-\left[dz-v(t)\frac{f(x,y,z-k(t))}{a(x,y,z-k(t))}dt\right]^2-dx^2-dy^2 \quad (17)$$

Using this metric we can calculate the Einstein tensor as follows:

$$G^{tt}=-\frac{1}{4}\frac{1}{a(x,y,z-k(t))^4}(v(t)^2(f(x,y,z-k(t))^2 \mathrm{D}_2(a)(x,y,z-k(t))^2+f(x,y,z-k(t))^2\mathrm{D}_1(a)(x,y,z-k(t))^2$$

$$+\mathrm{D}_2(f)(x,y,z-k(t))^2 a(x,y,z-k(t))^2+\mathrm{D}_1(f)(x,y,z-k(t))^2 a(x,y,z-k(t))^2$$

$$-2\,\mathrm{D}_2(f)(x,y,z-k(t))\,a(x,y,z-k(t))\,f(x,y,z-k(t))\,\mathrm{D}_2(a)(x,y,z-k(t))$$

$$-2\,\mathrm{D}_1(f)(x,y,z-k(t))\,a(x,y,z-k(t))\,f(x,y,z-k(t))\,\mathrm{D}_1(a)(x,y,z-k(t))))$$

$$G^{tx}=-\frac{1}{2}\frac{1}{a(x,y,z-k(t))^3}(v(t)\,(a(x,y,z-k(t))\,f(x,y,z-k(t))\,\mathrm{D}_{1,\,3}(a)(x,y,z-k(t))$$

$$+a(x,y,z-k(t))\,\mathrm{D}_1(f)(x,y,z-k(t))\,\mathrm{D}_3(a)(x,y,z-k(t))$$

$$+a(x,y,z-k(t))\,\mathrm{D}_3(f)(x,y,z-k(t))\,\mathrm{D}_1(a)(x,y,z-k(t))-a(x,y,z-k(t))^2\,\mathrm{D}_{1,\,3}(f)(x,y,z-k(t))$$

$$-2\,f(x,y,z-k(t))\,\mathrm{D}_1(a)(x,y,z-k(t))\,\mathrm{D}_3(a)(x,y,z-k(t))))$$

$$G^{ty}=\frac{1}{2}\frac{1}{a(x,y,z-k(t))^3}(v(t)\,(-a(x,y,z-k(t))\,\mathrm{D}_3(f)(x,y,z-k(t))\,\mathrm{D}_2(a)(x,y,z-k(t))$$

$$-a(x,y,z-k(t))\,\mathrm{D}_2(f)(x,y,z-k(t))\,\mathrm{D}_3(a)(x,y,z-k(t))$$

$$-a(x,y,z-k(t))\,f(x,y,z-k(t))\,\mathrm{D}_{2,\,3}(a)(x,y,z-k(t))$$

$$+2\,f(x,y,z-k(t))\,\mathrm{D}_2(a)(x,y,z-k(t))\,\mathrm{D}_3(a)(x,y,z-k(t))+a(x,y,z-k(t))^2\,\mathrm{D}_{2,\,3}(f)(x,y,z-k(t))))$$

$$G^{tz}=-\frac{1}{4}\frac{1}{a(x,y,z-k(t))^5}(v(t)\,(4\,\mathrm{D}_2(a)(x,y,z-k(t))^2\,a(x,y,z-k(t))^2\,f(x,y,z-k(t))$$

$$-4\,\mathrm{D}_2(a)(x,y,z-k(t))\,a(x,y,z-k(t))^3\,\mathrm{D}_2(f)(x,y,z-k(t))$$

$$- 2\, a(x, y, z - k(t))^3 f(x, y, z - k(t))\, D_{2,\,2}(a)(x, y, z - k(t))$$

$$- 4\, D_1(a)(x, y, z - k(t))\, a(x, y, z - k(t))^3\, D_1(f)(x, y, z - k(t))$$

$$+ 4\, D_1(a)(x, y, z - k(t))^2\, a(x, y, z - k(t))^2 f(x, y, z - k(t))$$

$$- 2\, a(x, y, z - k(t))^3 f(x, y, z - k(t))\, D_{1,\,1}(a)(x, y, z - k(t))$$

$$- 2\, v(t)^2 f(x, y, z - k(t))^2\, a(x, y, z - k(t))\, D_2(f)(x, y, z - k(t))\, D_2(a)(x, y, z - k(t))$$

$$- 2\, v(t)^2 f(x, y, z - k(t))^2\, a(x, y, z - k(t))\, D_1(f)(x, y, z - k(t))\, D_1(a)(x, y, z - k(t))$$

$$+ v(t)^2 f(x, y, z - k(t))\, a(x, y, z - k(t))^2\, D_1(f)(x, y, z - k(t))^2 + v(t)^2 f(x, y, z - k(t))^3\, D_1(a)(x, y, z - k(t))^2$$

$$+ v(t)^2 f(x, y, z - k(t))^3\, D_2(a)(x, y, z - k(t))^2 + 2\, a(x, y, z - k(t))^4\, D_{2,\,2}(f)(x, y, z - k(t))$$

$$+ 2\, a(x, y, z - k(t))^4\, D_{1,\,1}(f)(x, y, z - k(t)) + v(t)^2 f(x, y, z - k(t))\, a(x, y, z - k(t))^2\, D_2(f)(x, y, z - k(t))^2))$$

$$G^{x\,x} = \frac{1}{4}\, \frac{1}{a(x, y, z - k(t))^4} \Big(v(t)^2\, D_1(f)(x, y, z - k(t))^2\, a(x, y, z - k(t))^2$$

$$- 2\, v(t)^2\, D_1(f)(x, y, z - k(t))\, a(x, y, z - k(t))\, f(x, y, z - k(t))\, D_1(a)(x, y, z - k(t))$$

$$+ v(t)^2 f(x, y, z - k(t))^2\, D_1(a)(x, y, z - k(t))^2 - 4 \left(\frac{d}{dt} v(t) \right) a(x, y, z - k(t))^3\, D_3(f)(x, y, z - k(t))$$

$$- 4\, v(t)^2\, D_3(f)(x, y, z - k(t))^2\, a(x, y, z - k(t))^2 - v(t)^2\, D_2(f)(x, y, z - k(t))^2\, a(x, y, z - k(t))^2$$

$$- v(t)^2 f(x, y, z - k(t))^2\, D_2(a)(x, y, z - k(t))^2$$

$$+ 4 \left(\frac{d}{dt} v(t) \right) a(x, y, z - k(t))^2 f(x, y, z - k(t))\, D_3(a)(x, y, z - k(t))$$

$$+ 4\, v(t) \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))^3\, D_{3,\,3}(f)(x, y, z - k(t))$$

$$- 4\, v(t)^2 f(x, y, z - k(t))\, a(x, y, z - k(t))^2\, D_{3,\,3}(f)(x, y, z - k(t)) - 12\, v(t)^2 f(x, y, z - k(t))^2\, D_3(a)(x, y, z - k(t))^2$$

$$+ 4\, v(t)^2 f(x, y, z - k(t))^2\, a(x, y, z - k(t))\, D_{3,\,3}(a)(x, y, z - k(t))$$

$$- 4\, v(t) \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))^2 f(x, y, z - k(t))\, D_{3,\,3}(a)(x, y, z - k(t))$$

$$- 8\, v(t)\, D_3(a)(x, y, z - k(t)) \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))^2\, D_3(f)(x, y, z - k(t))$$

$$+ 2\, v(t)^2\, D_2(f)(x, y, z - k(t))\, a(x, y, z - k(t))\, f(x, y, z - k(t))\, D_2(a)(x, y, z - k(t))$$

$$+ 16\, v(t)^2\, D_3(f)(x, y, z - k(t))\, a(x, y, z - k(t))\, f(x, y, z - k(t))\, D_3(a)(x, y, z - k(t))$$

$$+ 8\, v(t)\, D_3(a)(x, y, z - k(t))^2 \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))\, f(x, y, z - k(t)) \Bigg)$$

$$G^{xy} = -\frac{1}{2} \frac{1}{a(x, y, z - k(t))^4} (v(t)^2 (D_1(f)(x, y, z - k(t))\, a(x, y, z - k(t)) - f(x, y, z - k(t))\, D_1(a)(x, y, z - k(t)))\, ($$

$$-D_2(f)(x, y, z - k(t))\, a(x, y, z - k(t)) + f(x, y, z - k(t))\, D_2(a)(x, y, z - k(t))))$$

$$G^{xz} = -\frac{1}{2} \frac{1}{a(x, y, z - k(t))^4} \Bigg(-\left(\frac{d}{dt} v(t) \right) a(x, y, z - k(t))^3\, D_1(f)(x, y, z - k(t))$$

$$+ 2\, v(t)^2 f(x, y, z - k(t))^2\, a(x, y, z - k(t))\, D_{1,\,3}(a)(x, y, z - k(t))$$

$$- 2\, v(t)^2 f(x, y, z - k(t))\, a(x, y, z - k(t))^2\, D_{1,\,3}(f)(x, y, z - k(t))$$

$$+ 4\, v(t)^2\, D_3(f)(x, y, z - k(t))\, a(x, y, z - k(t))\, f(x, y, z - k(t))\, D_1(a)(x, y, z - k(t))$$

$$+ 4\, v(t)^2 f(x, y, z - k(t))\, a(x, y, z - k(t))\, D_1(f)(x, y, z - k(t))\, D_3(a)(x, y, z - k(t))$$

$$- 6\, v(t)^2 f(x, y, z - k(t))^2\, D_3(a)(x, y, z - k(t))\, D_1(a)(x, y, z - k(t))$$

$$- 2\, v(t)^2\, D_3(f)(x, y, z - k(t))\, a(x, y, z - k(t))^2\, D_1(f)(x, y, z - k(t))$$

$$- v(t) \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))^2\, D_1(f)(x, y, z - k(t))\, D_3(a)(x, y, z - k(t))$$

$$- v(t) \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))^2\, D_3(f)(x, y, z - k(t))\, D_1(a)(x, y, z - k(t))$$

$$- v(t) \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))^2 f(x, y, z - k(t))\, D_{1,\,3}(a)(x, y, z - k(t))$$

$$+ 2\, v(t)\, D_3(a)(x, y, z - k(t)) \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))\, f(x, y, z - k(t))\, D_1(a)(x, y, z - k(t))$$

$$+ \left(\frac{d}{dt} v(t) \right) a(x, y, z - k(t))^2 f(x, y, z - k(t))\, D_1(a)(x, y, z - k(t))$$

$$+ v(t) \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))^3\, D_{1,\,3}(f)(x, y, z - k(t)) \Bigg)$$

$$G^{yy} = \frac{1}{4} \frac{1}{a(x, y, z - k(t))^4} \Bigg(v(t)^2\, D_2(f)(x, y, z - k(t))^2\, a(x, y, z - k(t))^2$$

$$- 2\, v(t)^2\, D_2(f)(x, y, z - k(t))\, a(x, y, z - k(t))\, f(x, y, z - k(t))\, D_2(a)(x, y, z - k(t))$$

$$+ v(t)^2 f(x, y, z - k(t))^2\, D_2(a)(x, y, z - k(t))^2 - v(t)^2\, D_1(f)(x, y, z - k(t))^2\, a(x, y, z - k(t))^2$$

$$- 4 \left(\frac{d}{dt} v(t) \right) a(x, y, z - k(t))^3 \mathrm{D}_3(f)(x, y, z - k(t)) - 4\, v(t)^2 \mathrm{D}_3(f)(x, y, z - k(t))^2 a(x, y, z - k(t))^2$$

$$- v(t)^2 f(x, y, z - k(t))^2 \mathrm{D}_1(a)(x, y, z - k(t))^2$$

$$+ 4 \left(\frac{d}{dt} v(t) \right) a(x, y, z - k(t))^2 f(x, y, z - k(t)) \mathrm{D}_3(a)(x, y, z - k(t))$$

$$+ 4\, v(t) \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))^3 \mathrm{D}_{3,\ 3}(f)(x, y, z - k(t))$$

$$- 4\, v(t)^2 f(x, y, z - k(t))\, a(x, y, z - k(t))^2 \mathrm{D}_{3,\ 3}(f)(x, y, z - k(t)) - 12\, v(t)^2 f(x, y, z - k(t))^2 \mathrm{D}_3(a)(x, y, z - k(t))^2$$

$$+ 4\, v(t)^2 f(x, y, z - k(t))^2 a(x, y, z - k(t)) \mathrm{D}_{3,\ 3}(a)(x, y, z - k(t))$$

$$- 4\, v(t) \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))^2 f(x, y, z - k(t)) \mathrm{D}_{3,\ 3}(a)(x, y, z - k(t))$$

$$- 8\, v(t) \mathrm{D}_3(a)(x, y, z - k(t)) \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))^2 \mathrm{D}_3(f)(x, y, z - k(t))$$

$$+ 2\, v(t)^2 \mathrm{D}_1(f)(x, y, z - k(t))\, a(x, y, z - k(t))\, f(x, y, z - k(t)) \mathrm{D}_1(a)(x, y, z - k(t))$$

$$+ 16\, v(t)^2 \mathrm{D}_3(f)(x, y, z - k(t))\, a(x, y, z - k(t))\, f(x, y, z - k(t)) \mathrm{D}_3(a)(x, y, z - k(t))$$

$$+ 8\, v(t) \mathrm{D}_3(a)(x, y, z - k(t))^2 \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))\, f(x, y, z - k(t)) \Bigg)$$

$$G^{\,y\,z} = -\frac{1}{2} \frac{1}{a(x, y, z - k(t))^4} \Bigg(- \left(\frac{d}{dt} v(t) \right) a(x, y, z - k(t))^3 \mathrm{D}_2(f)(x, y, z - k(t))$$

$$+ 2\, v(t)^2 f(x, y, z - k(t))^2 a(x, y, z - k(t)) \mathrm{D}_{2,\ 3}(a)(x, y, z - k(t))$$

$$- 2\, v(t)^2 f(x, y, z - k(t))\, a(x, y, z - k(t))^2 \mathrm{D}_{2,\ 3}(f)(x, y, z - k(t))$$

$$+ \left(\frac{d}{dt} v(t) \right) a(x, y, z - k(t))^2 f(x, y, z - k(t)) \mathrm{D}_2(a)(x, y, z - k(t))$$

$$+ v(t) \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))^3 \mathrm{D}_{2,\ 3}(f)(x, y, z - k(t))$$

$$- 2\, v(t)^2 \mathrm{D}_3(f)(x, y, z - k(t))\, a(x, y, z - k(t))^2 \mathrm{D}_2(f)(x, y, z - k(t))$$

$$+ 4\, v(t)^2 f(x, y, z - k(t))\, a(x, y, z - k(t)) \mathrm{D}_2(f)(x, y, z - k(t)) \mathrm{D}_3(a)(x, y, z - k(t))$$

$$+ 4\, v(t)^2 \mathrm{D}_3(f)(x, y, z - k(t))\, a(x, y, z - k(t))\, f(x, y, z - k(t)) \mathrm{D}_2(a)(x, y, z - k(t))$$

$$- 6\, v(t)^2 f(x, y, z - k(t))^2 \mathrm{D}_3(a)(x, y, z - k(t)) \mathrm{D}_2(a)(x, y, z - k(t))$$

$$- v(t) \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))^2 \mathrm{D}_2(f)(x, y, z - k(t)) \mathrm{D}_3(a)(x, y, z - k(t))$$

$$- v(t) \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))^2 \, D_3(f)(x, y, z - k(t)) \, D_2(a)(x, y, z - k(t))$$

$$- v(t) \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t))^2 f(x, y, z - k(t)) \, D_{2,\,3}(a)(x, y, z - k(t))$$

$$+ 2\, v(t) \, D_3(a)(x, y, z - k(t)) \left(\frac{d}{dt} k(t) \right) a(x, y, z - k(t)) \, f(x, y, z - k(t)) \, D_2(a)(x, y, z - k(t)) \Bigg)$$

$$G^{zz} = -\frac{1}{4} \frac{1}{a(x, y, z - k(t))^6} \big(v(t)^2 \, (4 f(x, y, z - k(t)) \, a(x, y, z - k(t))^4 \, D_{1,\,1}(f)(x, y, z - k(t))$$

$$+ 4 f(x, y, z - k(t)) \, a(x, y, z - k(t))^4 \, D_{2,\,2}(f)(x, y, z - k(t))$$

$$- 4 f(x, y, z - k(t))^2 \, a(x, y, z - k(t))^3 \, D_{1,\,1}(a)(x, y, z - k(t))$$

$$+ 11 f(x, y, z - k(t))^2 \, D_1(a)(x, y, z - k(t))^2 \, a(x, y, z - k(t))^2$$

$$- 4 f(x, y, z - k(t))^2 \, a(x, y, z - k(t))^3 \, D_{2,\,2}(a)(x, y, z - k(t))$$

$$+ 11 f(x, y, z - k(t))^2 \, D_2(a)(x, y, z - k(t))^2 \, a(x, y, z - k(t))^2 + 3\, D_2(f)(x, y, z - k(t))^2 \, a(x, y, z - k(t))^4$$

$$+ 3\, D_1(f)(x, y, z - k(t))^2 \, a(x, y, z - k(t))^4 + v(t)^2 f(x, y, z - k(t))^2 \, a(x, y, z - k(t))^2 \, D_2(f)(x, y, z - k(t))^2$$

$$+ v(t)^2 f(x, y, z - k(t))^2 \, a(x, y, z - k(t))^2 \, D_1(f)(x, y, z - k(t))^2 + v(t)^2 f(x, y, z - k(t))^4 \, D_1(a)(x, y, z - k(t))^2$$

$$+ v(t)^2 f(x, y, z - k(t))^4 \, D_2(a)(x, y, z - k(t))^2$$

$$- 14\, D_2(f)(x, y, z - k(t)) \, a(x, y, z - k(t))^3 f(x, y, z - k(t)) \, D_2(a)(x, y, z - k(t))$$

$$- 14\, D_1(f)(x, y, z - k(t)) \, a(x, y, z - k(t))^3 f(x, y, z - k(t)) \, D_1(a)(x, y, z - k(t))$$

$$- 2\, v(t)^2 f(x, y, z - k(t))^3 \, a(x, y, z - k(t)) \, D_2(f)(x, y, z - k(t)) \, D_2(a)(x, y, z - k(t))$$

$$- 2\, v(t)^2 f(x, y, z - k(t))^3 \, a(x, y, z - k(t)) \, D_1(f)(x, y, z - k(t)) \, D_1(a)(x, y, z - k(t)))\big)$$

with $a(r) = a(x, y, z - k(t)) \gg 1$ in the Pfenning zone there is again an arbitrary reduction of the components of the energy-impulse tensor and consequently of the energy density. The equations in the Eulerian system are presented in implicit form in appendix I, we only need to replace the equations in appendix II to find the same results. The same boundary conditions that apply when $v = cost$ apply also in this case.

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