# MODIFIED ALCUBIERRE WARP DRIVE II:

Gianluca Perniciano*, Bsc in Physics with a thesis in generale relativity at the Department of Physics of the University of Cagliari, Cittadella Universitaria, 09042 Monserrato, Italy.
*email:g.perniciano@gmail.com

## Abstract:

A solution of general relativity is presented that describes an Alcubierre [1] propulsion system in which it is possible to travel at superluminal speed while reducing the components of the energy-impulse tensor (thus reducing energy density) by an arbitrary value. Here we investigate the negative energy in the Pfenning zone, and the quantum inequalities involved.

**1. Introduction:**

Alcubierre [1] in 1994 proposed a solution of the equations of general relativity which provides the only viable means to accelerate a spaceship up to superluminal velocities without using wormholes. A problem was soon identified: Pfenning [4] showed that the required energy is comparable to the total energy of the universe and that it is negative. Moreover he used quantum inequalities to show that this energy gets distributed at very short scale (about 100 times the Planck length) up to a multiplicative factor equal to the squared speed. In the previous part of this pubblication (part I), we presented a way to reduce the amount of energy involved and its spacial distribution within the warp bubble. In this second part we investigate the quantum inequalities.

Note: In the following we adopt the notation used by Landau and Lifshitz in the second volume (“The Classical Theory of Fields”) of their well known Course of Theoretical Physics [15].

We start with the metric

$$ds^2=\left(1-v^2\frac{f(x,y,z-k(t))^2}{a(x,y,z-k(t))^2}\right)dt^2+2v\frac{f(x,y,z-k(t))}{a(x,y,z-k(t))}dt\,dz-dx^2-dy^2-dz^2$$

From the components of the Einstein tensor in contravariant form [11] for

$a=a(r)=a(x,y,z-k(t))\gg 1$ in the Pfenning zone [4] (source of esotic matter), i. e., for

$$r\ :\ R-\frac{\Delta}{2}<r<R+\frac{\Delta}{2}\quad \text{where}\quad \Delta\ll 1$$

where $R$ is the warp bubble radius, $\Delta$ the wall thickness of the warp bubble with $R\gg\Delta$

$$r=\left(x^2+y^2+(z-k(t))^2\right)^{\frac{1}{2}}$$

and

$$\left(\ D_i=\frac{\partial}{\partial x^i},\ x^i=x,y,z\,(i=1,2,3)\ \text{e}\ \ D_{i,k}=\frac{\partial^2}{\partial x^i\,\partial x^k},\ x^i,x^k=x,y,z\,(i,k=1,2,3)\ \right)$$

$$z_0(t)=k(t)\quad \text{and}\quad \frac{dk(t)}{dt}=v=cost$$

can be reduced by an arbitrary value. $G^{tt}$ is [11]:

$$G^{tt}=-\frac{1}{4}\frac{1}{a(x,y,z-k(t))^4}\left(v^2\left(f(x,y,z-k(t))^2\,D_2(a)(x,y,z-k(t))^2+f(x,y,z-k(t))^2\,D_1(a)(x,y,z-k(t))^2\right.\right.$$

$$+D_1(f)(x,y,z-k(t))^2\,a(x,y,z-k(t))^2+D_2(f)(x,y,z-k(t))^2\,a(x,y,z-k(t))^2$$

$$-2\,D_1(f)(x,y,z-k(t))\,a(x,y,z-k(t))\,f(x,y,z-k(t))\,D_1(a)(x,y,z-k(t))$$

$- 2\, D_2(f)(x, y, z - k(t))\, a(x, y, z - k(t))\, f(x, y, z - k(t))\, D_2(a)(x, y, z - k(t))))$

Einstein Equations: $G^{ik}=\frac{8\pi G}{c^4}T^{ik}$

$G^{ik}$ proportional to $T^{ik}$ (energy-impulse tensor)

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$r=(x^2+y^2+(z-k(t))^2)^{\frac{1}{2}}$ and where $\frac{dk(t)}{dt}=v$

The function $f(r)$ assumes the following values:

$f=f(r)=0$ for every r such that $r>R+\frac{\Delta}{2}$

$0<f=f(r)<1$ for every r such that $r-\frac{\Delta}{2}<r<R+\frac{\Delta}{2}$ ,

e.g.: $f=-\frac{(r-R-\frac{\Delta}{2})}{\Delta}$ [4] (Pfenning zone)

$f=f(r)=1$ for every r such that $0<r<R-\frac{\Delta}{2}$

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$a(x,y,z-k(t))$ assumes the following values:

$a=a(x,y,z-k(t))=1$ for every r such that $r>R+\frac{\Delta}{2}$

$a(x,y,z-k(t))\gg 1$ for every r such that $R-\frac{\Delta}{2}<r<R+\frac{\Delta}{2}$ (Pfenning zone)

$$a(x,y,z-k(t))=1 \quad \text{for every r such that} \quad 0<r<R-\frac{\Delta}{2}$$

INTERNAL METRIC OF THE WARP BUBBLE (which contains the spaceship):

$$0<r<R-\frac{\Delta}{2}$$

where R is the radius of the warp bubble.

$$ds^2=(1-v^2)dt^2+2\,v\,dtdz-dx^2-dy^2-dz^2 \quad \text{or} \quad ds^2=dt^2-(dz-vdt)^2-dx^2-dy^2$$

Moving with velocity v (multiple of the speed of light c) along the z-axis.

METRIC OUTSIDE OF THE BUBBLE BEYOND THE PFENNING ZONE:

$$r>R+\frac{\Delta}{2}$$

$$ds^2=dt^2-dx^2-dy^2-dz^2$$

i.e., Minkowski space.

**Computation of the negative energy in the Pfenning zone, its comparison with the Casimir effect (plane-parallel condenser) and quantum inequalities.**

The energy is: (1) $E=\int\int\int(-g)^{1/2}\,T^{tt}\,dx^3$ where the triple integral extends over all the volume

and the energy density is $T^{tt}=k\,G^{tt}$ $(\textit{determinant of the spacial metric})$ and

$k=\frac{c^4}{8\pi G}$ . In our case we get, in xyz-coordinates:

$$G^{tt}=-\frac{1}{4}\frac{1}{a(x,y,z-k(t))^4}(v^2(f(x,y,z-k(t))^2\,\mathrm{D}_2(a)(x,y,z-k(t))^2+f(x,y,z-k(t))^2\,\mathrm{D}_1(a)(x,y,z-k(t))^2$$

$+ D_1(f)(x, y, z - k(t))^2\, a(x, y, z - k(t))^2 + D_2(f)(x, y, z - k(t))^2\, a(x, y, z - k(t))^2$

$- 2\, D_1(f)(x, y, z - k(t))\, a(x, y, z - k(t))\, f(x, y, z - k(t))\, D_1(a)(x, y, z - k(t))$

$- 2\, D_2(f)(x, y, z - k(t))\, a(x, y, z - k(t))\, f(x, y, z - k(t))\, D_2(a)(x, y, z - k(t))))$

which, written in Alcubierre form becomes:

$$(2) \quad G^{tt} = -\frac{1}{4} v^2 \frac{x^2+y^2}{r^2} g(r)$$

where $g(r)$ is given by:

$$(3) \quad g(r) = [\frac{1}{a(r)^2} (\frac{df(r)}{dr})^2 + (\frac{f(r)^2}{a(r)^4})(\frac{da(r)}{dr})^2 - 2 \frac{df(r)}{dr} \frac{f(r)}{a(r)^3} \frac{da(r)}{dr}]$$

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$f = f(r)$ assumes the values:

$f = f(r) = 0$ for every r such that $r > R + \frac{\Delta}{2}$

$f = -\frac{(r - R - \frac{\Delta}{2})}{\Delta}$ in the Pfenning zone (i.e., in the spacial interval

$R - \frac{\Delta}{2} < r < R + \frac{\Delta}{2}$ ), see [4],

$f = f(r) = 1$ for every r such that $0 < r < R - \frac{\Delta}{2}$

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In the simplified case:

$a = a(r) = a(x, y, z-k(t))$ is chosen under the following conditions

$a = a(r) = a(x, y, z-k(t)) = 1$ for every r such that $r > R + \frac{\Delta}{2}$

$a(r) = a(x, y, z-k(t)) = A = cost \gg 1$ for every r such that $R - \frac{\Delta}{2} < r < R + \frac{\Delta}{2}$

$a(r) = a(x, y, z-k(t)) = 1$ for every r such that $0 < r < R - \frac{\Delta}{2}$

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**The energy E in the Pfenning zone in our case (see [4] for an example)**

The energy given by (1), taking into account the conditions set therein, becomes:

$E = -\frac{8\pi k}{12} v^2 \frac{(\frac{R^2}{\Delta} + \frac{\Delta}{12})}{A^2}$ se $R \gg \Delta$ (very likely), R being the radius of the Warp bubble.

If $\Delta$ is about $1.6 \cdot 10^{-35} m$ (Planck lenght) as an absurd, setting $A = 10^{50}$ and

R = 100 m, the energy E gets reduced significantly. Using the chosen data the energy is

$E = -3 \cdot 10^{-17} v^2 \, joule$ quite smaller than the U(d) due the the Casimir effect.

In the case in which $\Delta$ is about 10 nm very similar results can be obtained with a smaller A, so

these results are not shown here.

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**The energy U(d) between the two plates in a plane-parallel condenser in empty space, due to the Casimir effect, is:**

$U(d) = -\pi^2 \left[ (\frac{h}{2\pi}) \frac{c}{720 d^3} \right] L^2$ where d is the distance between the plates and L is the side of the

square conducting plate. As can be seen, the energy is negative and this implies that the force (equal to the opposite of the derivative with respect to d) is attractive, as has been experimentally found

("Lamoreaux" [12]). In the case $d = 1\,\mu m$ and L = 1 m (L is chosen to be quite large, but not as large as the Pfenning zone) it is found that $U(d) = -4\,10^{-10}\,joule$ also L should be L=200 m given that the radius of the bubble which contains the spaceship is R=100 m, but is chosen to be 1 m here.

Knowing that $E = mc^2$ the negative mass is: $m = -4 \bullet 10^{-27}\,kg = -4 \bullet 10^{-24}\,g = -4 \bullet 10^{-15}\,pg$

In the case L=200 m and d=10 nm a favourable increase in negative energy is obtained, and the calculations are omitted here because they are very similar.

**Quantum inequalities. Calculation for our solution:**

The quantum inequalities (4) are $\frac{t_0}{\pi} \int_{-\infty}^{+\infty} \frac{<T^{ik} u_i u_k>}{t^2 + t_0^2} dt \geq -\frac{3}{32 \pi^2 t_0^4}$ [13] This is the general solution using natural units $(h, c, G = 1)$ and $u_i$ is the quadrivelocity in a Eulerian or moving reference system. In our case using the International System of Units we get: (5) $\Delta \leq \frac{1}{\alpha^2} v^2 L_{Planck}$ where Pfenning in his paper [4] chooses $\alpha = \frac{1}{10}$ and therefore $\Delta \leq 10^2 v^2 L_{Planck}$ . I believe that the concentration of energy in a very small volume is due to the bad choice of the $f = f(r)$ function, (but $\Delta$ can be about 1 m ), or of the parameter $\alpha$ by Pfenning [4], since the quantum inequalities are valid for all values of $t_0$ [13] and this leads to $0<\alpha\ll 1$ therefore $\alpha$ to an arbitrary value. We can conclude that $\alpha$ can assume an arbitrarily small value, and the results presented in [13] provide indirect evidence of this.

**Appendix:**

The components of the Einstein tensor proportional to the energy-impulse tensor have been calculated with reference to an observer whose gravitational field is very weak and whose speeds are far lower than the speed of light, observing the spaceship and the warp bubble moving at speed v, i.e., an inertial reference frame in which the spaceship is moving at speed v). If we want to calculate in the Eulerian reference frame, that is moving with the spaceship, we get for each component of the energy-impulse tensor in implicit form the following:

$$(energy\ density)=k\,G^{tt}$$

$$(impulse\ density\ x)=-k\,G^{tx}$$

$$(impulse\ density\ y)=-k\,G^{ty}$$

$$(impulse\ density\ z)=k\left[G^{tt}\,v\left(\frac{f}{a}\right)-G^{tz}\right]$$

$$(stress\ xx)=k\,G^{xx}$$

$$(stress\ yy)=k\,G^{yy}$$

$$(stress\ zz)=k\left[G^{tt}\,v^2\left(\frac{f}{a}\right)^2-2\mathrm{v}\left(\frac{f}{a}\right)G^{tz}+G^{zz}\right]$$

$$(stress\ xy)=k\,G^{xy}$$

$$(stress\, xz)=k\left[-v\left(\frac{f}{a}\right)G^{xt}+G^{xz}\right]$$

$$(stress\, yz)=k\left[-v\left(\frac{f}{a}\right)G^{yt}+G^{yz}\right]$$

The various stress xx, stress yy, stress zz, stress xy, stress xz, stress yz and their symmetric counterparts are the components of the stress tensor [15], and $k=\frac{c^4}{8\pi G}$ $c$ is the speed of light and G is Newton's gravitational constant and $a=a(x,y,z-k(t))\gg 1$ in Pfenning zone where the energy-impulse tensor is not zero.

**Conclusion:** The calculations seem to suggest that the modified Alcubierre propulsion system allows to reach superluminal speeds without problems in the energy density and components of the energy-impulse tensor, and that the negative energy can be arbitrarily reduced. This energy is compared with that of the Casimir effect in a parallel plane condenser, to investigate the experimental feasibility. Quantum inequalities are then calculated in orded to investigate the spacial distribution of the negative energy in the Pfenning zone. In the Alcubierre solution a event horizon is present at $v>1$ while in our solution it is present at $v\geq 1$ . In our next paper this problem will be investigated [16].

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