Neutrino magnetic moment and mass

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ABSTRACT

An expression for the magnetic moment of a massive Dirac neutrino was deduced in the context of the electroweak interactions at the one-loop level in 1977. A linear dependence on the neutrino mass was found. In addition, a magnetic moment for a massive neutrino arising from gravitational origin is predicted by the so-called Wilson-Blackett law. The latter relation may also be deduced from a gravitomagnetic interpretation of the Einstein equations. Both formulas for the magnetic moment can be combined, yielding the value of the neutrino mass.

The gravitomagnetic moment, i.e., the magnetic moment from gravitational origin, may contain different g-factors for the massive neutrino eigenstates m_1, m_2 and m_3, respectively. Starting from the Dirac equation, a g-factor g = 2 has been deduced for a neutrino in first order, related to the derivation of the g-factor of charged leptons. When a value g = 2 is inserted, a value 1.530 meV results for the lightest neutrino mass m_1, the main result of this work. In addition, the remaining neutrino masses can be calculated from observed neutrino oscillations.

Our results for the neutrino masses are compatible with the three-parameter semi-empirical neutrino mass formulas obtained by Królikowski. In addition, an empirical relation between the three neutrino masses proposed by Sazdović yields neutrino masses in fair agreement with our results.

1. INTRODUCTION

Although a massive neutrino is electrically neutral, it may have electromagnetic properties through electroweak interactions with photons. The neutrino magnetic moment arises at the one-loop level from a minimal extension of the standard model with right-handed neutrinos [1, 2]. For a left-handed Dirac neutrino with a positive mass m_ν the following electromagnetic moment $\boldsymbol{\mu}_\nu$(em) was deduced

$$\boldsymbol{\mu}_\nu(\mathrm{em}) = \frac{3|e|G_\mathrm{F} m_\nu c^4 \hbar}{8\pi^2\sqrt{2}}\mathbf{s} = \frac{3G_\mathrm{F} m_\nu m_e c^4 \mu_B}{4\pi^2\sqrt{2}}\mathbf{s} = 3.2\times10^{-22}\left(\frac{m_\nu}{\mathrm{meV}}\right)\mu_B\mathbf{s}, \tag{1.1}$$

where G_F = 1.16638×10^{-5} GeV^{-2} is the Fermi coupling constant, c is the velocity of light, $\hbar$ is the Planck constant divided by 2π and $\mu_B = |e|\hbar/2m_e$ is the Bohr magneton. The unit vector **s** lying along the rotation axis of the neutrino of mass m_ν and the direction of $\boldsymbol{\mu}_\nu$(em) are found to be parallel. Note that the neutrino magnetic moment is proportional to the neutrino mass m_ν, but the value of m_ν does not follow from the calculation. The value of $\boldsymbol{\mu}_\nu$(em) has been calculated from the one-loop contributions to the neutrino electromagnetic vertex function. To leading order in m_l^2/m_W^2, the result is independent of the charged lepton masses m_l ($l = e, \mu, \tau$) and of the lepton matrix U [1, 2].

Observed neutrino oscillations from different sources (Sun, Earth 's atmosphere and in the laboratory) provide strong indications for the existence of massive neutrinos. A description of neutrino oscillations [3, 4] is possible by connecting three neutrino flavour states neutrinos ν_α ($\alpha = e, \mu, \tau$) to three massive eigenstates ν_i with active masses m_i (i = 1, 2, 3). In that case three different magnetic moments $\boldsymbol{\mu}_\nu$(em) may exist, corresponding to three neutrino masses m_i.

In this work the electromagnetic moment $\boldsymbol{\mu}_\nu$(em) of (1.1) for the neutrino will be compared with the so-called gravitomagnetic moment $\boldsymbol{\mu}_\nu$(gm). As will be discussed in section 2, it is assumed that $\boldsymbol{\mu}_\nu$(em) and $\boldsymbol{\mu}_\nu$(gm) are equal. For an elementary particle like a neutrino $\boldsymbol{\mu}_\nu$(gm) may be written as

$$\boldsymbol{\mu}_\nu(\mathrm{gm}) = -\frac{g_\nu \beta}{4}\left(\frac{G}{k}\right)^{1/2} \hbar \mathbf{s}, \tag{1.2}$$

where G is the gravitational constant and $k = (4\pi\varepsilon_0)^{-1}$ is the constant in Coulomb's law. The parameter g_ν or g_i (i = 1, 2, 3) is a dimensionless quantity of order unity, related to the g_l -factor for charged leptons ($l = e, \mu, \tau$). Note that $\boldsymbol{\mu}_\nu$(gm) does not explicitly depend on neutrino mass.

The gravitomagnetic moment $\boldsymbol{\mu}_\nu$(gm) of (1.2) for a neutrino with mass m_i (i = 1, 2, 3) may be distinguished by a different $g_\nu = g_i$-factor. Starting from the Dirac equation, however, the same factor $g_\nu = +2$ is deduced in section 3 for all neutrinos m_i, analogously to the factor $g_l = +2$ for all charged leptons. Additional contributions may cause deviations from the canonical value $g_\nu = +2$, however. Note that the formula for $\boldsymbol{\mu}_\nu$(gm) also contains an additional unknown dimensionless constant β.

When the magnetic moments $\boldsymbol{\mu}_\nu$(em) from (1.1) and $\boldsymbol{\mu}_\nu$(gm) from (1.2) are taken equal, the following expression for mass m_ν results

$$m_\nu = -\frac{2\pi^2\sqrt{2}}{3|e|G_F c^4}\, g_\nu \beta \left(\frac{G}{k}\right)^{1/2}. \tag{1.3}$$

Note that $\boldsymbol{\mu}_\nu$(em) of (1.1) and $\boldsymbol{\mu}_\nu$(gm) of (1.2) have the same direction for a negative value of the product $g_\nu\beta$. Since g_ν is positive (a factor $g_\nu = +2$ is deduced from the Dirac equation in section 3), β must be negative. This is already an important result, for the sign of the β-factor was unknown, so far. Insertion of the value $g_1 = +2$ and a value $\beta = -1$ into (1.3) yields a value of 1.530 meV for neutrino mass m_1, the main result of this work.

At present, no magnetic moment of any neutrino has been measured. The tightest constraint on μ_ν comes from studies of a possible delay of helium ignition in the core of red giants in globular clusters. From the lack of observational evidence of this effect a limit of $\mu_\nu < 3\times10^{-12}\mu_B$ has been extracted [5]. This limit still exceeds the value $\mu_\nu = 3.2\times10^{-22}(m_\nu/\mathrm{meV})\mu_B$ from (1.1) by many orders of magnitude. Conformation of the proposed value of mass m_1, however, may provide a first indication of the existence of non-zero neutrino magnetic moments (1.1) and (1.2).

According to the neutrino oscillation theory [3, 4], the masses of the three neutrino flavour states ν_α ($\alpha = e, \mu, \tau$) can be expressed as superpositions of three massive eigenstates ν_i with masses m_i (i = 1, 2, 3). In addition, mass-squared splittings $\Delta m_{21}^2 \equiv m_2^2 - m_1^2$ and $\Delta m_{31}^2 \equiv m_3^2 - m_1^2$ follow from observations. So, two relations between the masses m_1, m_2, and m_3 are available, whereas three masses m_i are initially unknown. Thus, if the neutrino mass m_1 is known, the remaining masses m_2 and m_3 can be calculated. In section 4 such a calculation has been performed. In section 5 our results are compared with results following from relations between neutrino masses proposed by other authors. Conclusions are drawn in section 6. In section 2, however, we first consider the deduction of relation (1.2) and the corresponding electromagnetic magnetic moments.

2. GRAVITO- AND ELECTROMAGNETIC MAGNETIC MOMENTS

Since 1891 many authors have discussed a gravitational origin of the magnetic field of rotating celestial bodies. Particularly, the so-called Wilson-Blackett formula has often been considered [6–14]

$$\boldsymbol{\mu}(\mathrm{gm}) = -\frac{\beta}{2}\left(\frac{G}{k}\right)^{1/2}\mathbf{S}, \tag{2.1}$$

where $\boldsymbol{\mu}$(gm) is the gravitomagnetic dipole moment of the massive body with angular momentum **S**. For a sphere with a homogeneous mass density the angular momentum **S** is given by $\mathbf{S} = 2/5\ mr^2\boldsymbol{\omega}$, where m is the mass of the sphere of radius r and $\boldsymbol{\omega}$ its angular velocity. Note that $\boldsymbol{\mu}$(gm) is proportional to the mass m. The parameter β is an unknown dimensionless constant of order unity. See ref. [14] for an ample discussion of its value.

Analogously to its electromagnetic counterpart $\boldsymbol{\mu}$(em) of (1.1), the gravitomagnetic moment $\boldsymbol{\mu}$(gm) leads to a dipolar gravitomagnetic field at distance **R** of magnitude

$$\mathbf{B}(\mathrm{gm}) = \frac{\mu_0}{4\pi}\left(\frac{3\boldsymbol{\mu}(\mathrm{gm})\cdot\mathbf{R}}{R^5}\mathbf{R} - \frac{\boldsymbol{\mu}(\mathrm{gm})}{R^3}\right). \tag{2.2}$$

According to the Wilson-Blackett relation, the field **B**(gm) may be identified as an electromagnetic induction field. In that case the magnetic moments $\boldsymbol{\mu}$(em) of (1.1) and $\boldsymbol{\mu}$(gm) of (1.2) are equivalent.

Attempts to derive (2.1) from a more general theory have been given by several authors (see, e.g., [10–15] and references therein). For example, Bennet *et al.* [10] already gave a five-dimensional theory resulting into (2.1). Luchak [11] found a relation related to (2.1) by proposing a five-dimensional theory, based on a relativistic generalization of the Maxwell equations, in order to include gravitational fields. Other authors like Biemond [12–14] and Widom and Ahluwalia [15], tried to explain equation (2.1) as a consequence of general relativity. The former author [12–14] started from the Einstein equations in the slow motion and weak field approximation and deduced a set of four gravitomagnetic equations, analogous to the four Maxwell equations. The so-called "magnetic-type" gravitational field in these equations is identified as a magnetic induction field, resulting into the gravitomagnetic dipole moment $\boldsymbol{\mu}$(gm) of (2.1).

Since charges in rotating bodies may affect the value of the parameter β in many different ways, one can hardly expect that the observed value of β is a constant. Different values for the empirical value of β have indeed been found for about fourteen rotating bodies: metallic cylinders in the laboratory, moons, planets, stars and the Galaxy [13, ch. 1]. For pulsars a separate analysis has been given in ref. [16]. From a linear regression analysis of the series of the fourteen rotating bodies an almost linear relationship between the observed magnetic moment $|\boldsymbol{\mu}(\mathrm{obs})|$ and the angular momentum $|\mathbf{S}|$ was found. Such a linear relationship between $\boldsymbol{\mu}$(gm) and **S** is predicted by (2.1). From this analysis an average value of $|\beta| = 0.076$ was calculated. Although this result is distinctly different from a gravitomagnetic prediction for a theoretical value like $|\beta| = 1$ in (2.1), the correct order of magnitude of β for so many, *strongly different*, rotating bodies is amazing ($|\boldsymbol{\mu}(\mathrm{obs})|$ and $|\mathbf{S}|$ vary over an interval of *sixty decades!*). So, the gravitomagnetic hypothesis, embodied in the Wilson-Blackett law (2.1), may basically be valid.

For a macroscopic rotating sphere of mass m with a homogeneous charge density, the magnetic dipole moment from the total charge Q is given by (see, e.g., ref. [17, 18])

$$\boldsymbol{\mu}(\mathrm{em}) = \frac{Q}{2m}\mathbf{S}. \tag{2.3}$$

It is noted that the derivation of (2.3) from the Maxwell equations and the deduction of (2.1) from the gravitomagnetic equations are analogous.

For elementary particles like charged leptons l ($l = e, \mu, \tau$) the angular momentum **S** is given by $\mathbf{S} = \frac{1}{2}\hbar\mathbf{s}$, where **s** is a unit vector in the direction of **S**, as has been discussed by Pauli [19]. As an example, for an electron with mass m_e and charge e the electromagnetic moment $\boldsymbol{\mu}$(em) of (2.3) transforms into

$$\boldsymbol{\mu}(\mathrm{em}) = \frac{g_e e}{2m_e}\frac{\hbar}{2}\mathbf{s}, \tag{2.4}$$

where $\boldsymbol{\mu}$(em) and **s** are taken antiparallel for an electron ($e < 0$). Since more contributions to the dimensionless factor g_l can be distinguished, the g_l-factor is usually written as a series expansion

$$g_l = 2 + \frac{\alpha}{\pi} + ... = 2\left(1 + \frac{\alpha}{2\pi} + ...\right) = 2(1 + 0.00116141 + ...), \tag{2.5}$$

where $\alpha = ke^2/\hbar c = 1/137.036$ is the fine-structure constant. The leading term in the series expansion of g_l, $g_l = +2$, has been deduced by Dirac [20]. Later on, Schwinger [21] deduced the first and largest one-loop correction $\alpha/(2\pi)$ to g_l that follows from quantum electrodynamics (QED).

Analogous to $\boldsymbol{\mu}$(em) of (2.4), the following gravitomagnetic moment $\boldsymbol{\mu}$(gm) can be obtained from (2.1), both for charged leptons and neutrinos

$$\boldsymbol{\mu}(\mathrm{gm}) = -\frac{\beta g}{2}\left(\frac{G}{k}\right)^{1/2}\frac{\hbar}{2}\mathbf{s}. \tag{2.6}$$

Whereas $\boldsymbol{\mu}_\nu$(em) of (1.1) is proportional to neutrino mass m_ν, $\boldsymbol{\mu}$(gm) of (2.6) does not explicitly depend on mass. Other effects that influence the g-factor may be present, however.

As an example, for neutrino mass m_1 we choose $\beta = -1$ and $g = g_1 = +2$ in (2.6), and for an electron $g_e = +2$ in (2.4). One then obtains for the ratio $|\boldsymbol{\mu}(\mathrm{gm})|/|\boldsymbol{\mu}(\mathrm{em})|$

$$\frac{|\boldsymbol{\mu}(\mathrm{gm})|}{|\boldsymbol{\mu}(\mathrm{em})|} = \left(\frac{G}{k}\right)^{1/2}\frac{m_e}{|e|} = +4.899\times 10^{-22}. \tag{2.7}$$

It appears that the gravitomagnetic moment $|\boldsymbol{\mu}(\mathrm{gm})|$ for the neutrino is extremely small compared to the electromagnetic moment $|\boldsymbol{\mu}(\mathrm{em})|$ of the electron. Note that $G^{1/2}m_e$ and $k^{1/2}e$ in (2.7) have the same dimension.

The leading QED correction to $\boldsymbol{\mu}$(em) in (2.4) (see (2.5)) equals to $\delta\boldsymbol{\mu}(\mathrm{em}) \approx (\alpha/2\pi)$ $\boldsymbol{\mu}$(em). When $\beta = -1$ and $g_1 = +2$ are again substituted into (2.6), and $g_e = +2$ is again inserted into (2.4), the ratio $|\boldsymbol{\mu}(\mathrm{gm})|/|\delta\boldsymbol{\mu}(\mathrm{em})|$ appears to be

$$\frac{|\boldsymbol{\mu}(\mathrm{gm})|}{\delta|\boldsymbol{\mu}(\mathrm{em})|} \approx \left(\frac{G}{k}\right)^{1/2}\frac{m_e}{|e|}\frac{2\pi}{\alpha} = +4.22\times 10^{-19}. \tag{2.8}$$

So, for the electron the gravitomagnetic moment $|\boldsymbol{\mu}(\mathrm{gm})|$ is still much smaller than the contribution $\delta\boldsymbol{\mu}$(em). In next section the g_ν-factor for neutrinos will, however, be considered more in detail.

3. CALCULATION OF THE *g*-FACTOR FOR LEPTONS

For comparison, we first review the derivation of the g_l-factor for a charged lepton before considering the g_ν-factor of a neutrino. For a system of particles of mass m and charge e moving with speed $\mathbf{v}$ in an external constant uniform magnetic induction field

B(em), the following term has to be added to the Lagrangian (see, e.g., Landau and Lifshitz [22, ch. 5]

$$L' = \sum e\mathbf{A}(\mathrm{em})\cdot\mathbf{v} = \sum \frac{e}{2}\{\mathbf{B}(\mathrm{em})\times\mathbf{r}\}\cdot\mathbf{v} = \sum \frac{e}{2}(\mathbf{r}\times\mathbf{v})\cdot\mathbf{B}(\mathrm{em}). \tag{3.1}$$

In deriving (3.1), use has been made of the relation **B**(em) = $\nabla\times$**A**(em) and of the expression for the external uniform electromagnetic vector potential **A**(em) = ½**B**(em)×**r**. If all charges of the system have the same ratio of charge to mass and the velocities |**v**| of all charges are much smaller than c, then $m\mathbf{v}$ is the momentum of the particle with charge e. In that case (3.1) can be rewritten as

$$L' = \sum \frac{e}{2m}(\mathbf{r}\times m\mathbf{v})\cdot\mathbf{B}(\mathrm{em}) = \frac{e}{2m}\mathbf{S}\cdot\mathbf{B}(\mathrm{em}) = \boldsymbol{\mu}(\mathrm{em})\cdot\mathbf{B}(\mathrm{em}), \tag{3.2}$$

where **S** is the angular momentum of the system and **μ**(em) its magnetic moment.

For a lepton with charge e ($e < 0$) and mass m the contribution to the Hamiltonian, H', corresponding to (3.2), is given by

$$H' = -\frac{g_l e}{2m}\mathbf{S}\cdot\mathbf{B}(\mathrm{em}) = -\boldsymbol{\mu}_l(\mathrm{em})\cdot\mathbf{B}(\mathrm{em}), \tag{3.3}$$

where **S** = ½$\hbar$**s** is now the angular momentum of the charged lepton and $\boldsymbol{\mu}_l$(em) its magnetic moment. In order to calculate g_l-value in (3.3) the Dirac equation will now be considered below.

The Dirac equation in the presence of an external electromagnetic covariant four-vector potential A_μ(em) = {A_0(em), – **A**(em)} (or alternatively written in terms of the contravariant four-vector A^μ(em) = {A^0(em), **A**(em)}) for a charged lepton is given by [20]

$$\left[\gamma^\mu\left\{p_\mu - \frac{e}{c}A_\mu(\mathrm{em})\right\} - mc\right]\psi = 0. \tag{3.4}$$

Here the matrices γ^μ (μ = 0, 1, 2, 3) are 4×4 matrices and p_μ is a four-vector defined by $p_\mu \equiv i\hbar\partial/\partial x^\mu$ with x^μ = (ct, **r**), whereas the wave function ψ is a four-component column matrix. When the electromagnetic vector potential **A**(em) = {A^1(em), A^2(em), A^3(em)} and the scalar potential A_0(em) = A^0(em) = ϕ in (3.4) are relatively small, the components of the wave functions ψ are approximately solutions of the Dirac equation for the free particle (this approximation is usually denoted as the principle of minimal coupling). Analogously to the Schrödinger equation, the Dirac equation (3.4) can be written as a differential equation first order in time

$$i\hbar\frac{\partial\psi}{\partial t} = \left[c\boldsymbol{\alpha}\cdot\left\{\mathbf{p} - \frac{e}{c}\mathbf{A}(\mathrm{em})\right\} + e\phi + \beta mc^2\right]\psi = H_\mathrm{D}\psi. \tag{3.5}$$

Here the components of **α** are defined by the 4×4 matrices $\alpha^i \equiv \gamma^0\gamma^i$ (i = 1, 2, 3) and the components of momentum **p** by $p^i \equiv (p^1, p^2, p^3)$, respectively. Note that α^i is no spatial part of a four-vector (there is no α^0), so, its superscript index is no contravariant index. The 4×4 matrix β is defined by $\beta \equiv \gamma^0$ and **A**(em) is the vector potential. From (3.5) the expression for the Dirac Hamiltonian H_D follows.

Since the time dependence of the wave function is governed by the energy (energy eigenstates have the time dependence $e^{-iEt/\hbar}$), a factor of $e^{-imc^2t/\hbar}$ may be split off from the Dirac wave function ψ in first order. In addition, the four-component spinor ψ may be decomposed into two two-component spinors φ and χ. So, ψ will be written as

$$\psi = e^{-imc^2t/\hbar}\begin{pmatrix}\varphi\\ \chi\end{pmatrix}. \tag{3.6}$$

In addition, the four 4×4 matrices γ^μ in the so-called Dirac representation and α^i are partitioned into 2×2 matrices

$$\gamma^0 = \begin{pmatrix}1_2 & 0\\ 0 & -1_2\end{pmatrix} \equiv \beta, \quad \gamma^i = \begin{pmatrix}0 & \sigma_i\\ -\sigma_i & 0\end{pmatrix}; \qquad \alpha^i \equiv \gamma^0\gamma^i = \begin{pmatrix}0 & \sigma_i\\ \sigma_i & 0\end{pmatrix}, \tag{3.7}$$

where σ_i are the Pauli 2×2 matrices and 1_2 is a 2×2 unit matrix. These matrices are given by

$$\sigma_1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \quad \sigma_2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, \quad \sigma_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \quad \text{and} \quad 1_2 = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}. \tag{3.8}$$

Utilizing (3.6), (3.7) and (3.8), equation (3.5) transforms into

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\varphi\\ \chi\end{pmatrix} = \left\{c\begin{pmatrix}0 & \boldsymbol{\sigma}\cdot\boldsymbol{\pi}\\ \boldsymbol{\sigma}\cdot\boldsymbol{\pi} & 0\end{pmatrix} + e\phi + mc^2\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\right\}\begin{pmatrix}\varphi\\ \chi\end{pmatrix}, \tag{3.9}$$

where the 4×4 matrices of $\boldsymbol{\alpha}$ are replaced by the 2×2 Pauli matrices σ_1, σ_2 and σ_3, or by a vector $\boldsymbol{\sigma}$, and the 4×4 matrix β is replaced by the 2×2 matrix 1_2. The generalized momentum $\boldsymbol{\pi}$ is defined by $\boldsymbol{\pi} \equiv \mathbf{p} - (e/c)\,\mathbf{A}$(em). Evaluation of (3.9) leads to two coupled equations

$$i\hbar\frac{\partial\varphi}{\partial t} = c(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})\chi + e\phi\varphi, \tag{3.10}$$

$$i\hbar\frac{\partial\chi}{\partial t} = c(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})\varphi + e\phi\chi - 2mc^2\chi. \tag{3.11}$$

When in the weak field limit $e\phi$ is small compared to rest energy mc^2 and the function χ slowly varies in time, i.e. $i\hbar\partial\chi/\partial t \approx 0$, relation (3.11) reduces to

$$\chi \approx \frac{1}{2mc}(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})\varphi. \tag{3.12}$$

In the non-relativistic limit $\boldsymbol{\sigma}\cdot\boldsymbol{\pi} << mc$, so that $\chi << \varphi$. Substitution of (3.12) into (3.10) yields

$$i\hbar\frac{\partial\varphi}{\partial t} = \frac{1}{2m}(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})^2\varphi + e\phi\varphi. \tag{3.13}$$

This differential equation is known as the Pauli equation [19].

Equation (3.13) can be evaluated by the Pauli vector identity

$$(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})^2 = \boldsymbol{\pi}^2 + i\boldsymbol{\sigma}\cdot(\boldsymbol{\pi}\times\boldsymbol{\pi}). \tag{3.14}$$

In addition, the quantity $(\boldsymbol{\pi}\times\boldsymbol{\pi})\varphi$ can be shown to be

$$\left(\boldsymbol{\pi}\times\boldsymbol{\pi}\right)\varphi = ie\hbar\nabla\times\mathbf{A}(\text{em})\,\varphi = ie\hbar\,\mathbf{B}(\text{em})\,\varphi. \tag{3.15}$$

Combining equations (3.13), (3.14) and (3.15) yields

$$i\hbar\frac{\partial\varphi}{\partial t} = \left\{\frac{\boldsymbol{\pi}^2}{2m} + e\phi - \frac{e\hbar}{2m}\boldsymbol{\sigma}\cdot\mathbf{B}(\text{em})\right\}\varphi. \tag{3.16}$$

Insertion of $\mathbf{S} = (\hbar/2)\boldsymbol{\sigma}$ into (3.16) leads to the Pauli Hamiltonian H_{P}

$$H_{\text{P}} = \frac{\boldsymbol{\pi}^2}{2m} + e\phi - \frac{e}{m}\mathbf{S}\cdot\mathbf{B}(\text{em}) = \frac{\boldsymbol{\pi}^2}{2m} + e\phi - \boldsymbol{\mu}_l(\text{em})\cdot\mathbf{B}(\text{em}). \tag{3.17}$$

From comparison of the spin-dependent term in H_{P} of (3.17a) and that of H' in (3.3a) follows that for a charged lepton $g_l = +2$ in first order.

As follows from section 2.2 in ref. [13] and section 2 in ref. [14], the Lagrangian for a system of masses m in an external constant uniform gravitomagnetic vector potential $\mathbf{A}$(gm) contains the additional term

$$L' = \frac{4m}{\beta}\left(\frac{G}{k}\right)^{1/2}\mathbf{A}(\text{gm})\cdot\mathbf{v}, \quad A^{\alpha}(\text{gm}) \equiv -\frac{\beta}{4}\left(\frac{k}{G}\right)^{1/2} g_{0\alpha}, \tag{3.18}$$

where the three components of $\mathbf{A}$(gm) are given by A^{α}(gm) ($\alpha = x, y, z$). The components A^{α}(gm) can be connected to the metric components $g_{0\alpha}$ and have been deduced in the weak field and slow motion limit [13, 14].

Utilizing the relation for the gravitomagnetic induction field $\mathbf{B}(\text{gm}) = \nabla\times\mathbf{A}(\text{gm})$, the expression for the vector potential $\mathbf{A}(\text{gm}) = \tfrac{1}{2}\mathbf{B}(\text{gm})\times\mathbf{r}$ and (2.1), equation (3.18) can be rewritten as

$$L' = \frac{2}{\beta}\left(\frac{G}{k}\right)^{1/2}\mathbf{S}\cdot\mathbf{B}(\text{gm}) = -\frac{4}{\beta^2}\boldsymbol{\mu}(\text{gm})\cdot\mathbf{B}(\text{gm}), \tag{3.19}$$

where $\mathbf{S}$ is the angular momentum of the system and $\boldsymbol{\mu}$(gm) its gravitomagnetic moment.

For an elementary particle like a neutrino, the contribution to the Hamiltonian, H', corresponding to (3.19) is given by

$$H' = -\frac{2}{\beta}\left(\frac{G}{k}\right)^{1/2}\mathbf{S}\cdot\mathbf{B}(\text{gm}) = \frac{4}{\beta^2}\boldsymbol{\mu}_{\nu}(\text{gm})\cdot\mathbf{B}(\text{gm}), \tag{3.20}$$

where $\mathbf{S}$ is now the angular momentum and $\boldsymbol{\mu}_{\nu}$(gm) the gravitomagnetic moment of the neutrino, respectively. When $\boldsymbol{\mu}_{\nu}$(gm) and $\mathbf{B}$(gm) are equivalent to their electromagnetic counterparts $\boldsymbol{\mu}_{\nu}$(em) and $\mathbf{B}$(em), a deduction analogous to the present one will lead to gravitomagnetic results, analogous to (3.3a) and (3.17a). In that case a value $g_{\nu} = +2$ will be obtained for all neutrinos in first order. The g_{ν}-factor, however, may deviate from this value by additional contributions.

4. CALCULATION OF THE NEUTRINO MASSES

In table 1 data deduced from the framework of three-neutrino oscillations are summarized. Best fit values are given for the two squared-mass differences Δm_{21}^2 and Δm_{31}^2 from Gonzalez-Garcia *et al.* [23], Forero *et al.* [24] and Capozzi *et al.* [25]. Three data series for the normal hierarchy are shown and one for the inverted hierarchy.

Table 1. Best fit Δm^2 values ($\pm 1\sigma$) from three-neutrino oscillation analyses.

	normal hierarchy [23]	inverted hierarchy [23]	normal hierarchy [24]	normal hierarchy [25]
Δm^2	best fit $\pm 1\sigma$	best fit $\pm 1\sigma$	best fit $\pm 1\sigma$	best fit $\pm 1\sigma$
$\Delta m_{21}{}^2/10^{-5}$ eV^2	7.45	7.45	7.60	7.54
$\Delta m_{31}{}^2/10^{-3}$ eV^2	+ 2.417	– 2.411	+ 2.48	+ 2.468

Choosing a value $\beta = -1$ and a g_ν-factor $g_1 = +2$ for the neutrino mass $m_\nu = m_1$ in (1.3), yields a value of $m_1 = +1.530$ meV. Subsequently, the masses m_2 and m_3 have been calculated from the values of $\Delta m_{21}{}^2 \equiv m_2{}^2 - m_1{}^2$ and $\Delta m_{31}{}^2 \equiv m_3{}^2 - m_1{}^2$ for the three data series in the case of normal hierarchy given in table 1. Results are summarized in table 2. Note that all masses m_i have a positive sign. Since the mass m_1 is relativity small compared with masses m_2 and m_3, comparison of tables 1 and 2 shows, that the masses m_2 and m_3 are approximately given by the roots of $\Delta m_{21}{}^2$ and $\Delta m_{31}{}^2$, respectively. In the case of normal hierarchy the alternative choices $m_2 = +1.530$ meV or $m_3 = +1.530$ meV can both be excluded, for they both lead to negative values for $m_1{}^2$.

When the calculated masses m_2 and m_3 are subsequently inserted into (1.3) and a value $\beta = -1$ is chosen, the empirical g_i-factors g_2 and g_3 can be calculated, respectively. For comparison, the g_i-factors have been expressed relative to $g_1 = +2$ by defining the relative factor $g'_i \equiv g_i/g_1$. So, $g'_1 = 1$ and from (1.3) follows that $g'_i = m_i/m_1$. In addition, the quantity $g'_i - g'_1$ can be expressed in units of the electroweak coupling constant $\alpha_\mathrm{W} = kg^2/\hbar c$, where the charge g is connected to the charge e and the weak angle θ_W by the relation $g \sin\theta_W = e$ and $\Delta g'_i(\alpha_\mathrm{W})^{-1}$ is defined by $\Delta g'_i(\alpha_\mathrm{W})^{-1} \equiv (g'_i - g'_1)\times\alpha_\mathrm{W}$. The on-shell value $\sin^2\theta_W = 0.22333$ has been taken from ref. [26], resulting into a value of $(\alpha_\mathrm{W})^{-1} = 30.60$. The values of g'_i and $\Delta g'_i(\alpha_\mathrm{W})^{-1}$ are also shown in table 2.

Table 2. Calculated neutrino masses m_2, and m_3 from data in table 1 for the normal hierarchy. All masses are given in units of meV. The g'_i-factor is given by $g'_i = m_i/m_1$ and the quantity $\Delta g'_i(\alpha_\mathrm{W}{}^{-1})$ by $\Delta g'_i(\alpha_\mathrm{W}{}^{-1}) \equiv (g'_i - g'_1)\times\alpha_\mathrm{W}$, respectively.

ref.	[23]	[24]	[25]		[23]	[24]	[25]		[23]	[24]	[25]
mass								a)			
m_1/meV	1.530	1.530	1.530	g'_1	1	1	1	$\Delta g'_1(\alpha_\mathrm{W})^{-1}$	0	0	0
m_2/meV	8.77	8.85	8.82	g'_2	5.73	5.79	5.76	$\Delta g'_2(\alpha_\mathrm{W})^{-1}$	0.15	0.16	0.16
m_3/meV	49.2	49.8	49.7	g'_3	32.2	32.6	32.5	$\Delta g'_3(\alpha_\mathrm{W})^{-1}$	1.02	1.03	1.03

[a] The reciproke electroweak coupling constant $(\alpha_\mathrm{W})^{-1} = 30.60$ has been calculated from ref. [26].

In case of the inverted hierarchy the choices $m_1 = 1.530$ meV or $m_2 = 1.530$ meV must be excluded, for the data from [23] in table 1 lead to negative values for $m_3{}^2$ and $m_1{}^2$, respectively. The choice $m_3 = 1.530$ meV, however, leads to the values of $m_1 = 49.1$ meV and $m_3 = 49.9$ meV, respectively, but we will not discuss this possibility further.

From calculated values m_i in table 2 an average sum $\Sigma_i\ m_i = 60$ meV can be calculated. This value can be compared with the sum $\Sigma_i\ m_i < 140$ meV at 95% C. L. deduced by Palanque-Delabrouille *et al*. [27]. The latter figure is extracted by combining Baryon Oscillation Spectroscopic Survey (BOSS) and Planck Cosmic Microwave Background (CMB) data.

5. DISCUSSION OF THE RESULTS

The origin of mass of elementary particles remains an important unsolved problem of physics. In order to find that origin, many relations between the masses of leptons and quarks have been proposed in the past. To start with an own suggestion, one may write the averaged values of the masses m_2 and m_3 obtained from table 2 in terms of the electroweak coupling constant α_W

$$m_2 = (1+0.16\,\alpha_W^{-1})\,m_1, \tag{5.1}$$

$$m_3 = (1+1.03\alpha_W^{-1})\,m_1. \tag{5.2}$$

It appears that mass m_3, apart from a unity term, contains a contribution 1.03 $(\alpha_W)^{-1}$ = 1.03×30.60 = 30.9, close to the value of $(\alpha_W)^{-1}$. This coincidence suggests an electroweak interaction within the neutrino, although no mechanism is known at present.

Another attempt to find relations between the three masses of charged leptons, neutrinos and up- and down quarks was given by Królikowski [28]. Using generalized Dirac-type equations in the Weyl representation and an extended Pauli exclusion principle, he obtained the following semi-empirical mass formula

$$m_i = \mu\rho_i\left(-\xi + N_i^2 + \frac{\varepsilon-1}{N_i^2}\right) \quad i = 1,2,3, \tag{5.3}$$

where μ, ξ and ε are free parameters. The quantities N_i and ρ_i are given by

$$N_1 = 1, \quad N_2 = 3, \quad N_3 = 5, \qquad \rho_1 = \frac{1}{29}, \quad \rho_2 = \frac{4}{29}, \quad \rho_3 = \frac{24}{29}. \tag{5.4}$$

The quantities ρ_i ($\Sigma_i\,\rho_i$ = 1) are the squares of the normalization constants of the three wave functions connected to N_1, N_2 and N_3, respectively.

We first consider (5.3) for the case of the three charged leptons. Combination of (5.3) and (5.4) yields

$$m_e = \frac{\mu}{29}(\varepsilon-\xi), \quad m_\mu = \frac{\mu}{29}\frac{4}{9}(80+\varepsilon-9\xi), \quad m_\tau = \frac{\mu}{29}\frac{24}{25}(624+\varepsilon-25\xi). \tag{5.5}$$

When ξ is neglected and μ and ε are eliminated from the relations in (5.5), the following approximate expression for the tauon mass m_τ is obtained

$$m_\tau = \frac{6}{125}(351m_\mu - 136\,m_e) = 1776.80\,\text{MeV}, \tag{5.6}$$

where the electron mass m_e = 0.51099893 MeV and the muon mass m_μ = 105.65837 MeV are used as the only input (see data in ref. [26] and the discussions in refs. [28, 29]) for the calculation of mass m_τ. The observed value m_τ = 1776.82 MeV is close to the predicted one and is a justification for the approximation ξ = 0. From the observed masses m_e, m_μ and m_τ the values for μ, ε and ξ can also be calculated from the full relations of (5.5) (see ref. [28] for the explicit expressions of μ, ε and ξ) resulting into: μ = 85.9942 MeV, ε = 0.172541 and $\xi = 2.157\times10^{-4}$.

In addition, the following ratio R_l for the triplet of charged leptons can be calculated from (5.5)

$$R_l \equiv \frac{m_\mu^2}{m_e m_\tau} = \frac{50}{243}\frac{\left(80+\varepsilon-9\xi\right)^2}{(\varepsilon-\xi)\left(624+\varepsilon-25\xi\right)}, \tag{5.7}$$

where the parameter μ drops out of the relation for R_l. Substitution of the values of observed masses m_e, m_μ and m_τ, or the values ε = 0.172541 and $\xi = 2.157\times10^{-4}$ into (5.7), results into the same value R_l = 12.2954.

Recently, Sazdović [30] also proposed empirical relations between the three masses of the leptons and quarks with the same electric charge, respectively, and between the three active masses of neutrinos. For the charged leptons he considered the empirical relation

$$R_l = \frac{m_\mu^2}{m_e m_\tau} = e^{5/2} = 12.1825. \tag{5.8}$$

Compared to the value R_l = 12.2954 calculated from the observed masses m_e, m_μ and m_τ, Sazdović's prediction (5.8) is 0.918 % lower.

For the mass of Dirac neutrinos m_i^D, the quantity ξ cannot be neglected and relation (5.3) can be rewritten as (compare to Królikowski's eq. (31) in [28])

$$m_i^D = -\mu\xi\rho_i\left(1 - \frac{N_i^2}{\xi} - \frac{\varepsilon - 1}{\xi N_i^2}\right) \quad i = 1, 2, 3. \tag{5.9}$$

Substitution of $\mu' \equiv -\mu\xi$ and (5.4) into (5.9) then yields the following expressions for the active neutrino masses $m_i^{\mathrm{D}} = m_i$

$$m_1 = \frac{\mu'}{29}\left(1 - \frac{1}{\xi} - \frac{\varepsilon-1}{\xi}\right), \quad m_2 = \frac{4\mu'}{29}\left(1 - \frac{9}{\xi} - \frac{\varepsilon-1}{9\xi}\right), \quad m_3 = \frac{24\mu'}{29}\left(1 - \frac{25}{\xi} - \frac{\varepsilon-1}{25\xi}\right). \tag{5.10}$$

In this work it will now be assumed that the Dirac neutrino masses $m_i^{\mathrm{D}} = m_i$ are positive, so that the mass $m_\nu = m_i$ in relation (1.1) is also positive. As a consequence, for dominating unity terms in the expressions for m_i in (5.10) the sign of μ' must be positive. Alternatively, Królikowski [28] chose a negative sign for the Dirac masses m_i^{D}. In addition, he made use of the popular seesaw relation and obtained a mass formula for the light Majorana neutrinos, analogous to (5.10).

As an example, the masses m_1 = 1.530 meV, m_2 = 8.77 meV and m_3 = 49.2 meV from table 2 can be inserted into (5.10). The following values are then obtained for the parameters μ', ξ and ε (compare to ref. [28]): μ' = 69.7 meV, ξ = 189 and ε = 68.6, respectively. Substitution of these values into (5.10) yields

$$\begin{aligned} m_1 &= 2.40(1 - 0.005 - 0.358) = 1.53\,\text{meV}, \\ m_2 &= 9.61(1 - 0.048 - 0.040) = 8.77\,\text{meV}, \\ m_3 &= 57.7(1 - 0.132 - 0.014) = 49.2\,\text{meV}. \end{aligned} \tag{5.11}$$

Equation (5.11) shows, that the unity terms in the expressions for m_1, m_2 and m_3 dominate, but the other terms cannot be neglected. When the unity terms in (5.11) are only taken into account, one finds for the ratio m_3/m_1 = 24, whereas a ratio m_3/m_1 = 32.2 is found from the full expressions of (5.10).

In addition, from (5.10) the following ratio R_ν can be calculated for the three neutrinos

$$R_\nu \equiv \frac{m_2^2}{m_1 m_3} = \frac{50}{243}\frac{(9\xi - 80 - \varepsilon)^2}{(\xi - \varepsilon)(25\xi - 624 - \varepsilon)}, \tag{5.12}$$

Note that the parameter μ' drops out of equation (5.12). Direct substitution of m_1 = 1.530 meV, m_2 = 8.77 meV and m_3 = 49.2 meV from table 2 into (5.12) is completely compatible with substitution of the calculated values ε = 68.6 and ξ = 189 into (5.12). In both cases the same value R_ν = 1.02 is obtained.

For the triplet of neutrinos Sazdović [30] proposed the simple relation

$$R_\nu = \frac{m_2^2}{m_1 m_3} = 1, \tag{5.13}$$

without giving a theoretical basis for this relation. The ratio (5.13) provides a third relation between the masses m_1, m_2 and m_3, so that all masses can be calculated. Choosing the values for Δm_{21}^2 and Δm_{31}^2 for the normal hierarchy from [23] in our table 1, the following results can be calculated from (5.13): m_1 = 1.56 meV, m_2 = 8.77 meV and m_3 = 49.2 meV, close to the values following from our approach and given in our table 2. The proposed ratio R_ν = 1 almost equals to the value R_ν = 1.02 from (5.12), calculated from m_1 = 1.530 meV and Δm_{21}^2 and Δm_{31}^2 data for the normal hierarchy from [23] in our table 1.

Instead of introducing $\beta = -1$ into (1.3), one might choose a value like $\beta = -2$. In that case the value of m_1 doubles to m_1 = 3.06 meV. Combination of this value for m_1 and data from [23] in table 1 yields the values m_2 = 9.16 meV and m_3 = 49.3 meV. In that case a value of R_ν = 0.56 follows from (5.12). If Sazdović 's relation (5.13) is approximately valid, however, the choice $\beta = -2$ must be rejected, whereas the value $\beta = -1$ is acceptable.

Królikowski [28] and Sazdović [30] also gave related expressions for the masses of the up- and down quarks. Since the masses of the quarks are less accurately known in general, we will not consider these relations further.

6. CONCLUSIONS

A value of 1.530 meV/c^2 = 2.727×10^{-39} kg for the mass of the lightest neutrino m_1 is obtained in this work. This value is extracted from a combination of the magnetic moment of a massive Dirac neutrino [1, 2], deduced in the context of electroweak interactions at the one-loop level, ànd the magnetic moment from gravitational origin proposed by Wilson and Blackett [6–12]. The latter relation has also been obtained from a gravitomagnetic interpretation of the Einstein equations [12–14]. Combination with neutrino oscillation data yields the other masses m_2 and m_3 (see table 2). Note that the ratio m_3/m_1 is in the range of the reciprocal value of the electroweak coupling constant.

Our results for the neutrino masses are compatible with the three-parameter semi-empirical neutrino mass formulas deduced by Królikowski [28], utilizing generalized Dirac-type equations in the Weyl representation and an extended Pauli exclusion principle.

Sazdović [30] recently proposed the empirical relation $R_\nu = (m_2)^2/(m_1 m_3) = 1$ for the three active masses of the neutrinos and found about the same set of masses m_1, m_2 and m_3 obtained in this work. From Królikowski's theoretical treatment also follows an expression for R_ν. His approach and ours yield the same value R_ν = 1.02, when our set of neutrino masses is chosen. It is remarkable that three different approaches lead to about the same value of R_ν.

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