Two conjectures in number theory

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Abstract

In this note, I propose a conjecture of generalization of the Lander, Parkin, and Selfridge conjecture; and a conjecture of generalization of the Beal's conjecture.

Conjecture 1. *Let k, n, m be three positive integers such that* $k > m + n$ *and* $m \neq n$. *Let* $a_1, a_2, a_3, ..., a_k$ *be the integers, with* $a_k \geq |a_{k-1}| \geq ... \geq |a_0|$. *Let a polynomial* $f(x) = a_k x^k + a_{k-1}x^{k-1} + ... + a_0$, *then no (n+m) positive integers* x_1, x_2, x_n, y_1, y_2,..., y_m *greater than* a_k *can satisfy an equation as follows:*

$$f(x_1) + f(x_2) + ... + f(x_n) = f(y_1) + f(y_2) + ... + f(y_m) \tag{1}$$

Conjecture 2. *Let n, m be two positive integers such that* $m \neq n$. *Let* k_1, k_2,..., k_n, h_1, h_2,..., h_m *be* $(n + m)$ *positive integers, such that* $k_i > n + m$ *for* $i = 1, 2, ..., n$ *and* $h_j > n + m$ *for* $j = 1, 2, ..., m$. *Let* x_1, x_2,..., x_n, y_1, y_2,..., y_m *be* $(n + m)$ *positive integers satisfy an equation as follows:*

$$x_1^{k_1} + x_2^{k_2} + ... + x_n^{k_n} = y_1^{h_1} + y_2^{h_2} + ... + y_m^{h_m} \tag{2}$$

Then x_1, x_2,...x_n, y_1, y_2,..., y_m *have a common prime factor.*

References

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