A Preliminary Explanation for the Pentaquark P_c^+ found by LHCb

Mario Everaldo de Souza

Departamento de Física, Universidade Federal de Sergipe,

Av. Marechal Rondon, s/n, Rosa Elze,

49100-000 São Cristóvão, SE, Brazil

Abstract

We propose that the two resonant states of the recently found pentaquark P_c^+ with masses of 4380 MeV and 4450 MeV are states of two hadronic molecules with similar properties to those of the Karliner-Lipkin pentaquark. Applying the Morse molecular potential to both molecules some important numbers are obtained for their sizes.

I. INTRODUCTION

The idea of the pentaquark was firstly proposed by Strottman in 1979 [1]. In 2004 Karliner and Lipkin proposed a very important model for a pentaquark in the description of the Θ^+ [2]. They arrived at the conclusion that the bag model commonly used for hadrons may not be adequate for the pentaquark. In their model they propose that the pentaquark system is composed of two clusters, a diquark and a triquark, in a relative P-wave state, and the clusters can be separated by a distance larger than the range of the color-magnetic force which had been proposed by De Rujula, Georgi and Glashow [3].

The LHCb Collaboration has recently [4] announced the discovery of pentaquark-charmonium states which resulted from the Λ_b^0 exotic decay $\Lambda_b^0 \to P_c^+ K^-$, in which P_c^+ is a pentaquark with the quark content $u\bar{c}cud$. The two resonant states have masses of about 4380 MeV and 4450 MeV in opposite parities with $J = 3/2, 5/2$ and with the corresponding Γ's of $205 \pm 18 \pm 86$ MeV and $39 \pm 5 \pm 19$ MeV, which correspond to lifetimes $\tau_1 = (0.31 \pm 0.16) \times 10^{-22}$ s and $\tau_2 = (0.16 \pm 0.10) \times 10^{-21}$ s.

II. A SIMPLE MODEL FOR THE LHCb P_c^+

We propose that the recently found LHCb P_c^+ is composed of two colorless clusters, a meson and a baryon. The quark content of the P_c^+ pentaquark, $u\bar{c}cud$ allows the two possibilities $u\bar{c} \oplus cud$ and $c\bar{c} \oplus uud$, and that is why there are two resonant states. The symbols $\oplus$ is used to designate the binding between the two clusters. The two clusters

should be weakly bound because hadrons are colorless, but as they are bound, they form a hadronic molecule. In the case of the molecule $u\bar{c} \oplus cud$ the two masses are, respectively, 1864 MeV for $u\bar{c}$ and 2285 MeV for cud for their ground states. These two masses sum to 4149 MeV. There is, thus, a difference in mass of 301 MeV for the 4450 MeV state, and 231 MeV for the 4380 MeV state. In the case of the molecule $c\bar{c} \oplus uud$ the masses are 3096 MeV and 980 MeV for their ground state components. The sum of the masses is 4076 MeV. In this case there is a difference in mass of 374 MeV for the 4450 MeV state and 304 MeV for the 4380 MeV state. It is important to notice that the masses of the components of the two molecules 4149 MeV and 4076 MeV are very close.

Let us call P_{c1}^{+} and P_{c2}^{+} the molecules $u\bar{c} \oplus cud$ and $c\bar{c} \oplus uud$, respectively, δE_1 and δE_2 the two respective differences in mass, and μ_1 and μ_2 the corresponding reduced masses. Since the two clusters should be weakly bound, the differences in mass should mostly be due to rotational energies. Taking into account the *J* values of 3/2 and 5/2 for both states we see that the rotational quantum number *L* of the molecules should be 1 or 2. Taking into account the Karliner-Lipkin model in which the clusters are in a relative P-wave state, we can propose that the P_{c1}^{+} and P_{c2}^{+} systems are in relative P-wave and D-wave states. As for the differences in mass there are the two respective possibilities $\delta E_1 = 301\,\text{MeV}$; $\delta E_2 = 304\,\text{MeV}$ and $\delta E_1 = 231\,\text{MeV}$; $\delta E_2 = 374\,\text{MeV}$. And for the reduced masses we have μ_1 = 1026.57 MeV and μ_2 = 744.38 MeV which are of the same order of magnitude.

The most famous hadronic molecule is the deuteron, also constituted of colorless particles. Because the two particles are colorless the binding is very weak, just about 2.2 MeV. This is a very important fact and tells us that in the case of the P_{c1}^{+} and P_{c2}^{+} systems the binding energy should also be just a couple of MeVs which is only about 1% of the differences in mass δE_1 and δE_2, and thus these should come from rotational energy contributions. As the nuclear force is a residual effect of the more fundamental forces of the color field in the quark systems, the force responsible for the binding between each meson and 0the baryon in the molecules $u\bar{c} \oplus cud$ and $c\bar{c} \oplus uud$ must also have the same nature. These residual forces are analogues of the London forces between neutral atoms and molecules [5,6,7].

III. POSSIBILITIES FOR THE ANGULAR MOMENTA

When the system cud is alone it is the baryon Λ_c^{+} which has spin equal to 1/2, but in the system P_{c1}^{+} we can assume that it can have spin equal to 3/2 or 1/2. It is well known that the spin-spin interaction in baryons is small, below 30 MeV [8], and thus we assume that during the decay $\Lambda_b^0 \to P_c^{+} K^{-}$ the cud system may have spin 3/2 too, besides 1/2. The same should hold for the system uud. We consider below just the preferred *J* values found in the experiment. Considering the experimental *J* values of 3/2 and 5/2 we can have the following possibilities for the spins of the molecules and their components:

a) Molecule $u\bar{c} \oplus cud$
As $u\bar{c}$ has spin 0 and cud can have spins 1/2 or 3/2, the molecule can have $J = 1/2, 3/2, 5/2$ for $L = 1$ and thus negative parity. And hence the observed preferred possibilities $J = 3/2^-, 5/2^-$ are taken care of. For positive parity we should have $L = 2$ and thus we obtain $J = 7/2^+, 5/2^+, 3/2^+, 1/2^+$ that includes the preferred observed values $J = 3/2^+, 5/2^+$.

b) Molecule $c\bar{c} \oplus uud$
As $c\bar{c}(J/\Psi)$ has spin 1 and uud can have spins 1/2 and 3/2, the molecule can have $J = 1/2^-, 3/2^-, 5/2^-$ for $L = 1$, and $J = 1/2^+, 3/2^+, 5/2^+, 7/2^+, 9/2^+$ for $L = 2$ that includes the preferred observed values $J = 3/2^-, 5/2^-$ and $J = 3/2^+, 5/2^+$.

We observe that the other possibilities $J = 1/2^-, 1/2^+, 7/2^-, 7/2^+, 9/2^+$ above found for J were observed by LHCb [4]. We should investigate further and find out why the values of 3/2 and 5/2 are preferred for *J*.

Relative S-waves for the molecules should be excluded because in this case we would not have $J = 5/2^-, 5/2^+$ for the system $u\bar{c} + cud$.

As the two molecules are possible states, the P_c^+ eigenstates should be mixtures of P_{c1}^+ and P_{c2}^+ states, that is, the state of the pentaquark P_c^+ should be given by

$$|\Psi\rangle = c_1|1\rangle + c_2|2\rangle \tag{1}$$

where $|1\rangle$ and $|2\rangle$ are the states of P_{c1}^+ and P_{c2}^+, respectively, and c_1 and c_2 are constants. Of course, $|c_1|^2$ and $|c_2|^2$ are the probabilities for the P_c^+ system being each of the two molecules $u\bar{c} \oplus cud$ and $c\bar{c} \oplus uud$, respectively.

IV. THE APPROXIMATE SIZES OF THE TWO MOLECULES

We have no idea yet on the kind of effective potential that exists in the P_{c1}^+ and P_{c2}^+ systems. Assuming that there is a shallow potential well we may model the system through a molecular potential and obtain some important numbers. It is important to note that these systems are heavy and, thus, we can use the Schrödinger equation for them. Recently, de Souza [9,10] has used the Morse molecular potential in the description of charmonium and bottomonium states. Let us try to model the two molecules P_{c1}^+ and P_{c2}^+ using the Morse molecular potential which can be expressed as [11]

$$V(r) = D\left(e^{-2\alpha x} - 2e^{-\alpha x}\right) \tag{2}$$

where $-D$ is the minimum of the well, a is the distance where $V = -D$, and $x = (r - a)/a$.

For $|x|<1$ this potential can be expanded around the minimum up to order 3 in x and the expression

$$V(x) = -D + \frac{1}{2}ka^2x^2 - \lambda ka^3x^3 \tag{3}$$

is obtained where $\lambda = \alpha / 2a$.

For this potential the solution of the Schrödinger equation yields the expression [11,12]

$$\begin{aligned} E_{nl} &= \hbar\omega\left(\upsilon + \frac{1}{2}\right) - A\left(\upsilon + \frac{1}{2}\right)^2 + B_L L(L+1) \\ &-D_L L^2(L+1)^2 - C_{\upsilon L}\left(\upsilon + \frac{1}{2}\right)L(L+1) + ... \end{aligned} \tag{4}$$

for the vibrational and rotational levels above the minimum of the potential, where the quantum numbers $\upsilon, L = 0,1,2,3,...$ In Eq. 3 the first term describes harmonic vibrations, the second term takes into account the anharmonicity of the potential, the third term describes rotations with constant moment of inertia, the fourth term represents the centrifugal distortion and the fifth term represents the coupling between vibration and rotation. The constant B_L is given by $B_L = \hbar^2 / 2\mu a^2$ where μ is the reduced mass of the system.

As the binding is weak the first term in Eq. 3 is approximately equal to the reduced mass of the system for $\nu = 0$ and the second term is small. Also the 4th and 5th terms are smaller than the third term. Thus, due to a lack of more information we can make the rough estimate

$$\delta E \approx \frac{\hbar^2}{2\mu a^2} L(L+1) \tag{5}$$

and estimate the sizes of the two molecules by means of *a*. Let us call a_1 the value of *a* for $L = 1$ and a_2 for $L = 2$.

From Eq. 4 we obtain the values below shown in Tables 1 and 2 for *a*.

δE_1	a_1 (fm)	a_2 (fm)
231	2.55	4.40
301	2.23	3.86

Table 1. Calculation of the possible values of *a* according to Eq. 4 for P_{c1}^+ for $L = 1$ and $L = 2$.

δE_2	a_1 (fm)	a_2 (fm)
304	2.61	4.52
374	2.35	4.08

Table 2. Calculation of the possible values of *a* according to Eq. 4 for P_{c2}^{+} for $L=1$ and $L=2$.

As a_2 are very large for both molecules, the D-waves should be very unstable and thus the largest contributions should come from P-waves. In the P-wave states both molecules are larger than the deuteron which has a charge radius of about 2.13 fm [13] and a matter radius of about 1.975 fm [14,15].

There are some available data for the radii of the proton and charmonium in their ground states that allows us to have more information on the molecule $c\bar{c} \oplus uud$. The size of $c\bar{c}$ in its ground state is about 0.35 ± 0.06 fm [10] and the proton radius (charge radius) has been reported as being 0.8879 fm [16], 0.8775 fm [17], and 0.84087 fm [18]. Summing the radii of $c\bar{c}$ and the proton we obtain about 1.2 fm which means that the meson $c\bar{c}$ and the proton are more than a fermion apart. This means that the P_c^+ system is a very large system in the $c\bar{c} \oplus uud$ configuration.

In the case of D-waves the meson and the baryon are more than 3 fm apart from each other. That is very far, but let us have in mind that the color field is an infinite range interaction, and thus residual interactions generated by the different color fields from the two clusters also have an infinite range. Between two nucleons the nuclear force becomes negligible at distances beyond 2.5 fm [19], but we do not know its range when it acts between a baryon and a meson. The pentaquark system is giving us this opportunity.

Let us analyze how good is our approximation for a_1 and a_2. As the binding is very weak the second term in Eq. 3 is very small. The 4th term, which is the centrifugal distortion, is in general much smaller than the 3rd term. In charmonium the 4th term is only about 10% of the 3rd term [10], and in bottomonium it is only 8% of the 3rd term [9]. It is reasonable to assume the same kind of trend for the molecule $c\bar{c} \oplus uud$. It is also important to notice that the 4th term is negative, and thus it lowers the values of a_1 and a_2, and thus the calculated values in Tables 1 and 2 are upper limits, but should be close to the actual values.

V. CONCLUSION

It is proposed that the recently found pentaquark P_c^+ is a system composed of the two molecules $u\bar{c} \oplus cud$ and $c\bar{c} \oplus uud$, and is similar to the pentaquark system proposed by Karliner and Lipkin [2]. It is shown how the observed angular momenta are generated, and approximate values for the sizes of the molecules are estimated.

Although still preliminary, the work is relevant and shows a consistent possibility for this recently discovered remarkable system of 5 quarks. More data is

needed to improve the model and obtain more precise values for a_1 and a_2. It is important to find out, for example, if the system has or not excited states.

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