

The Sums of the Odd Number Series Are Squares

Ben Steber

The author will demonstrate that the sums of odd numbers to an nth value equals that nth value squared. A geometric proof will be provided to demonstrate the principle of sum odds equaling squares.

To begin let us consider a number line of n values which will correspond to the odd number series

$$n_{\text{odd}} = 1, 2, 3, 4, 5, 6, 7, 8, 9...$$

This will correspond to the odd number series

$$\text{odd} = 1, 3, 5, 7, 9, 11, 13, 15, 17...$$

We now will consider the series of squares...

$$\text{Squares} = 1, 4, 9, 16, 25, 36, 49, 64, 81$$

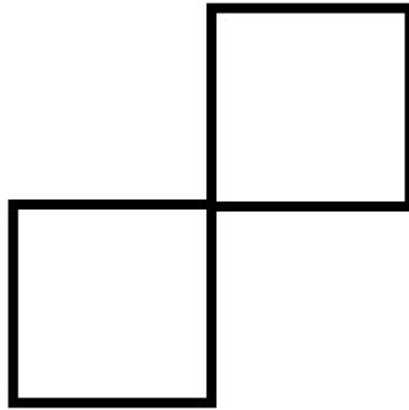
It can be seen that each square is the value of the sum of all preceding odd numbers. This presents the equation

$$\sum_{n=1}^{n_{\text{odd}}} \text{odd} = n_{\text{odd}}^2$$

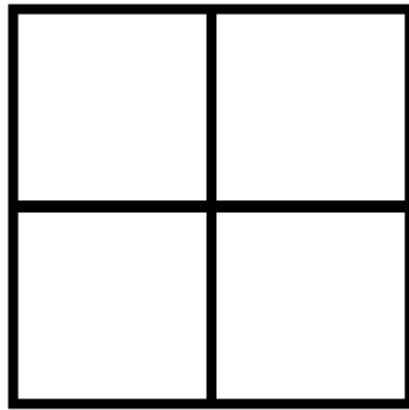
This may be visualized and proved geometrically by using unit squares. Start with a single unit square.



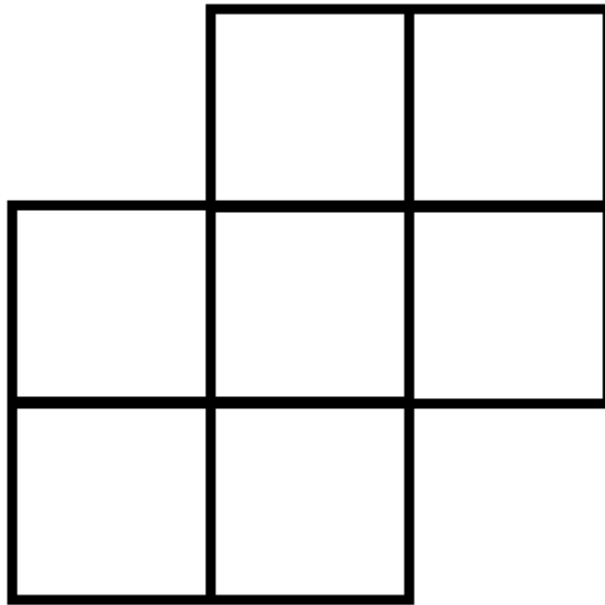
This unit square which will be called the previous value when it is duplicated and relocated diagonally to the original square.



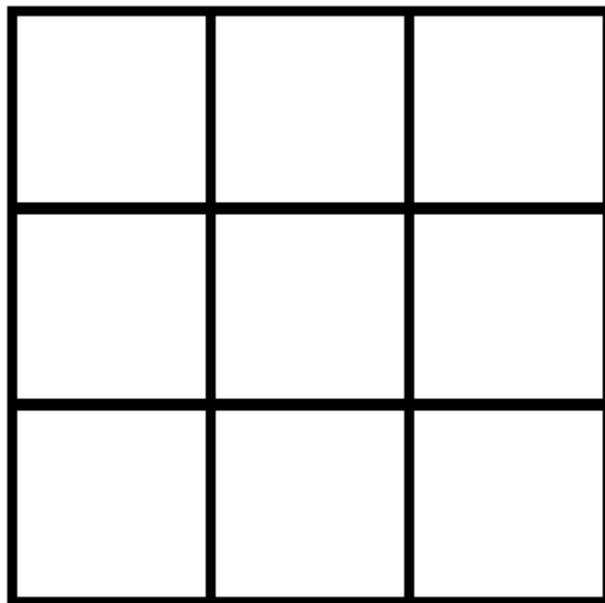
It is plain to see that there are two positions that may be filled by unit squares to complete the next complete square.



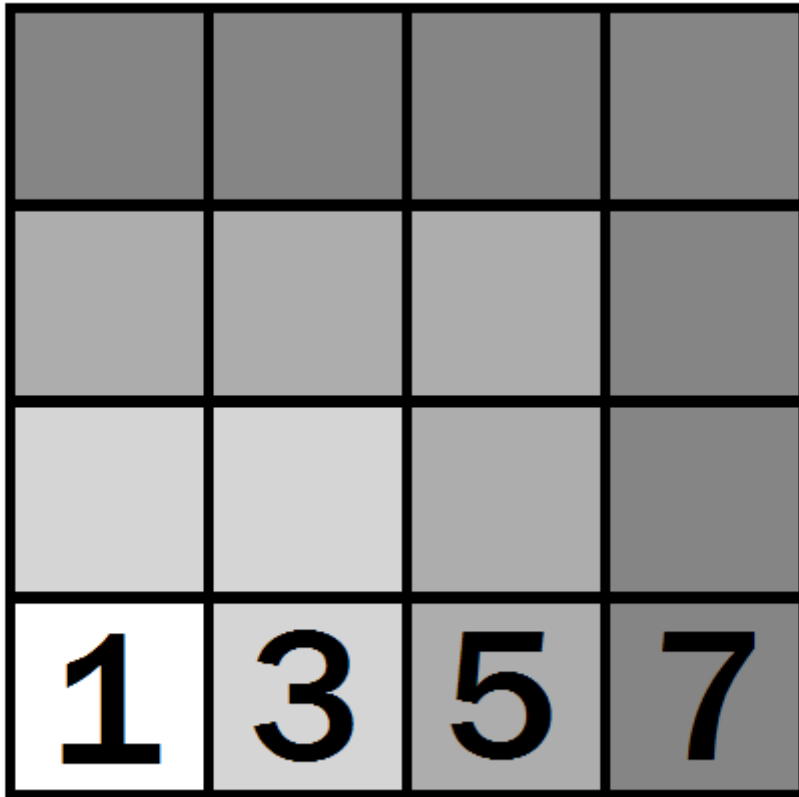
The process is repeated and the previous value is duplicated and relocated one diagonal unit.



Again, the result produces two empty slots to be filled by unit squares.



It is now plain to see that the n th iteration produces the square of that n value by reproducing the previous value and adding two with each iteration of the process. Since the odd number series is based on increasing by increments of two the equation is proved. The sums of the odd number series produce the squares of each n th value of odd numbers in the series.



$$1^2 = 1$$

$$2^2 = 4$$

$$3^2 = 9$$

$$4^2 = 16$$