

Is it Possible to Arbitrarily Slow Down Time in a Limited Volume With an Energy-Impulse Tensor Whose Components Can be Reduced Arbitrarily?

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Abstract

A solution is presented that describes a region of space, box or warp bubble, where time gets slowed down by an arbitrary factor, while reducing the components of the energy-impulse tensor by any chosen amount.

1 Introduction:

While watching sci-fi films I noticed situations in which it was possible to slow down time by an arbitrary factor with respect to an observer in an inertial reference frame within a warp bubble. This paper asks if this is possible in the framework of general relativity, if the energy-impulse tensor has negative or positive components and if they can be reduced by an arbitrary value and a way to test this in a laboratory.

Note: all notations here are those used by Landau and Lifshitz in the second book (“The Classical Theory of Fields”) of their well known Course of Theoretical Physics [2].

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We start with the metric

$$ds^2 = a(x, y, z) dt^2 - b(x, y, z)^2 dx^2 - b(x, y, z)^2 dy^2 - b(x, y, z)^2 dz^2 \quad (1)$$

The differential proper time is:

$$d\tau = a(x, y, z) dt \quad (2)$$

$a(x, y, z)$ is chosen in a simplified way (for a sphere with radius R and thickness $\Delta \ll 1$) as:

- 1)- $a(x, y, z) = 1$ for every $r > R + \frac{\Delta}{2}$
- 2)- $a(x, y, z) = 1$ for every r such that $R - \frac{\Delta}{2} < r < R + \frac{\Delta}{2}$ (cavity)
- 3)- $a(x, y, z) = A = \text{constant} \ll 1$ for $0 < r < R - \frac{\Delta}{2}$

then we get:

- 1)- $\tau = A t$ within the ball $0 < r < R - \frac{\Delta}{2}$, (zero initial conditions)
- 2)- Where $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$

t is the coordinate time in the exterior of the warp bubble (that is, proper time for observers whose reference frame is inertial, i.e., moving in a very weak gravitational field at a speed which is far smaller than the speed of light). τ tau is the proper time within the warp bubble (in our case).

When $A \ll 1$ the proper time becomes very small and it is like an hibernation of time within the warp bubble, we could call a chamber in this situation a “stasis chamber”.

2 Appendix I:

We choose a spherically symmetric warp bubble with a shell having radius R and thickness $\Delta \ll 1$

with the following convergence values , $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$.

for $b(x, y, z)$:

- 1)- $b(x, y, z) = 1$ for every r such that $r > R + \frac{\Delta}{2}$
- 2)- $b(x, y, z) \gg 1$ for every r such that $R - \frac{\Delta}{2} < r < R + \frac{\Delta}{2}$ (cavity wall)
- 3)- $b(x, y, z) = 1$ for every r such that $0 < r < R - \frac{\Delta}{2}$

3 The Einstein Tensor in contravariant form is:

$$G^{tt} = -\frac{1}{a(x, y, z)^2 b(x, y, z)^4} \left(2 \left(\frac{\partial^2}{\partial y^2} b(x, y, z) \right) b(x, y, z) - \left(\frac{\partial}{\partial x} b(x, y, z) \right)^2 \right. \\ \left. + 2 \left(\frac{\partial^2}{\partial x^2} b(x, y, z) \right) b(x, y, z) + 2 \left(\frac{\partial^2}{\partial z^2} b(x, y, z) \right) b(x, y, z) - \left(\frac{\partial}{\partial y} b(x, y, z) \right)^2 \right. \\ \left. - \left(\frac{\partial}{\partial z} b(x, y, z) \right)^2 \right)$$

$$G^{xx} = \frac{1}{b(x, y, z)^6 a(x, y, z)} \left(2 \left(\frac{\partial}{\partial x} a(x, y, z) \right) \left(\frac{\partial}{\partial x} b(x, y, z) \right) b(x, y, z) \right)$$

$$+ \left(\frac{\partial^2}{\partial y^2} b(x, y, z) \right) a(x, y, z) b(x, y, z) + \left(\frac{\partial}{\partial x} b(x, y, z) \right)^2 a(x, y, z)$$

$$+ \left(\frac{\partial^2}{\partial z^2} b(x, y, z) \right) a(x, y, z) b(x, y, z) + \left(\frac{\partial^2}{\partial y^2} a(x, y, z) \right) b(x, y, z)^2 - \left(\frac{\partial}{\partial y} b(x, y, z) \right)^2 a(x, y, z)$$

$$+ \left(\frac{\partial^2}{\partial z^2} a(x, y, z) \right) b(x, y, z)^2 - \left(\frac{\partial}{\partial z} b(x, y, z) \right)^2 a(x, y, z)$$

$$G^{xy} = - \frac{1}{b(x, y, z)^6 a(x, y, z)} \left(\left(\frac{\partial^2}{\partial y \partial x} a(x, y, z) \right) b(x, y, z)^2 - \left(\frac{\partial}{\partial y} b(x, y, z) \right) \left(\frac{\partial}{\partial x} a(x, y, z) \right) b(x, y, z) \right.$$

$$\left. - \left(\frac{\partial}{\partial x} b(x, y, z) \right) \left(\frac{\partial}{\partial y} a(x, y, z) \right) b(x, y, z) - 2 \left(\frac{\partial}{\partial y} b(x, y, z) \right) \left(\frac{\partial}{\partial x} b(x, y, z) \right) a(x, y, z) \right.$$

$$\left. + \left(\frac{\partial^2}{\partial y \partial x} b(x, y, z) \right) a(x, y, z) b(x, y, z) \right)$$

$$G^{xz} = - \frac{1}{b(x, y, z)^6 a(x, y, z)} \left(\left(\frac{\partial^2}{\partial z \partial x} a(x, y, z) \right) b(x, y, z)^2 - \left(\frac{\partial}{\partial z} b(x, y, z) \right) \left(\frac{\partial}{\partial x} a(x, y, z) \right) b(x, y, z) \right.$$

$$\left. - \left(\frac{\partial}{\partial x} b(x, y, z) \right) \left(\frac{\partial}{\partial z} a(x, y, z) \right) b(x, y, z) - 2 \left(\frac{\partial}{\partial z} b(x, y, z) \right) \left(\frac{\partial}{\partial x} b(x, y, z) \right) a(x, y, z) \right.$$

$$\left. + \left(\frac{\partial^2}{\partial z \partial x} b(x, y, z) \right) a(x, y, z) b(x, y, z) \right)$$

$$\begin{aligned}
G^{yy} = & \frac{1}{b(x, y, z)^6 a(x, y, z)} \left(2 \left(\frac{\partial}{\partial y} a(x, y, z) \right) \left(\frac{\partial}{\partial y} b(x, y, z) \right) b(x, y, z) \right. \\
& + \left(\frac{\partial^2}{\partial x^2} b(x, y, z) \right) a(x, y, z) b(x, y, z) + \left(\frac{\partial}{\partial y} b(x, y, z) \right)^2 a(x, y, z) \\
& + \left(\frac{\partial^2}{\partial z^2} b(x, y, z) \right) a(x, y, z) b(x, y, z) + \left(\frac{\partial^2}{\partial x^2} a(x, y, z) \right) b(x, y, z)^2 - \left(\frac{\partial}{\partial x} b(x, y, z) \right)^2 a(x, y, z) \\
& \left. + \left(\frac{\partial^2}{\partial z^2} a(x, y, z) \right) b(x, y, z)^2 - \left(\frac{\partial}{\partial z} b(x, y, z) \right)^2 a(x, y, z) \right)
\end{aligned}$$

$$\begin{aligned}
G^{yz} = & - \frac{1}{b(x, y, z)^6 a(x, y, z)} \left(\left(\frac{\partial^2}{\partial z \partial y} a(x, y, z) \right) b(x, y, z)^2 - \left(\frac{\partial}{\partial z} b(x, y, z) \right) \left(\frac{\partial}{\partial y} a(x, y, z) \right) b(x, y, z) \right. \\
& - \left(\frac{\partial}{\partial y} b(x, y, z) \right) \left(\frac{\partial}{\partial z} a(x, y, z) \right) b(x, y, z) - 2 \left(\frac{\partial}{\partial y} b(x, y, z) \right) \left(\frac{\partial}{\partial z} b(x, y, z) \right) a(x, y, z) \\
& \left. + \left(\frac{\partial^2}{\partial z \partial y} b(x, y, z) \right) a(x, y, z) b(x, y, z) \right)
\end{aligned}$$

$$\begin{aligned}
G^{zz} = & \frac{1}{b(x, y, z)^6 a(x, y, z)} \left(2 \left(\frac{\partial}{\partial z} a(x, y, z) \right) \left(\frac{\partial}{\partial z} b(x, y, z) \right) b(x, y, z) \right. \\
& + \left(\frac{\partial^2}{\partial x^2} b(x, y, z) \right) a(x, y, z) b(x, y, z) + \left(\frac{\partial}{\partial z} b(x, y, z) \right)^2 a(x, y, z)
\end{aligned}$$

$$\begin{aligned}
& + \left(\frac{\partial^2}{\partial y^2} b(x, y, z) \right) a(x, y, z) b(x, y, z) + \left(\frac{\partial^2}{\partial x^2} a(x, y, z) \right) b(x, y, z)^2 - \left(\frac{\partial}{\partial x} b(x, y, z) \right)^2 a(x, y, z) \\
& + \left(\frac{\partial^2}{\partial y^2} a(x, y, z) \right) b(x, y, z)^2 - \left(\frac{\partial}{\partial y} b(x, y, z) \right)^2 a(x, y, z)
\end{aligned}$$

Einstein Equation

$$G^{ik} = \frac{8\pi G}{c^4} T^{ik} \quad [2]$$

If $b = b(x, y, z) \gg 1$ and $\partial b / \partial x^i \leq b$, $\partial^2 b / \partial x^i \partial x^k \leq b$, $x^i, x^k = x, y, z$, $i, k = 1, 2, 3$ in the cavity wall of the warp bubble or box the components of the impulse-energy tensor can be reduced by an arbitrary amount.

4 Conclusion:

This paper shows that it is possible to slow down time by an arbitrary factor up to approaching the “freezing” of time (hibernation of time) in a limited volume (a warp bubble or box) with respect to an inertial observer and to reduce the components of the energy-impulse tensor by an arbitrary value within the cavity wall of the warp bubble or box (with components which are partially negative, thus requiring esotic matter). The speed of light in the volume containing exotic matter is strongly slowed down (something that can be compensated) and this can be a problem depending on how much one wants to reduce the components of the energy-impulse tensor.

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