

# DSmH Evidential Network for Target Identification<sup>★</sup>

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## Abstract

This paper proposes a model of evidential network based on Hybrid Dezert-Smarandache theory (DSmH) to improve target identification of multi-sensors. In the classification simulation, we compared the results obtained at the Target Type node and Foe-Ally node in evidential network by using Dempster-Shafer theory (DS) and using DSmH. The comparisons show that, when we use DSmH in the evidential network, we can assign more Basic Belief Assignments (BBA) to the focal element the target belongs to. Experiments confirm that the model of evidential network using DSmH is better than the one using DS.

*Keywords:* Evidential Network; DSmH; Transferable Belief Model (TBM); Target Identification

## 1 Introduction

The concept of evidential network was proposed by Xu Hong and Smets in 1994 [1]. It is a model related to Graph theory and Dempster-Shafer theory [2], which is composed of variable nodes, arcs with direction between nodes, and arguments of relationship between two nodes. Attoh-Okine and Bovee put forward the evidential network with Markov Tree theory [3, 4]. Srivastava and others created an evidential network with belief assignments and the Causality Diagram. They used the extension and marginalization methods to propagate the belief in the evidential network. In applications, evidential networks are used in the evaluation of degree of satisfaction [5], target threaten [6], reliability [7, 8, 9], intellectual control [10], diagnosis and nursing care [11].

In previous works, Conditional belief reasoning [7, 8, 9] and joint belief reasoning [5, 6, 10] was proposed with the DS model [12], which has focal elements with exhaustive and exclusive hypotheses. For the target identification evidential network, the DS model has a limitation in describing the intersections. Jean Deserts and Florentin Smarandache propose a new information

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<sup>★</sup>The author is partially supported by National Natural Science Foundation of China under grants (No. 60773012) and (No. 60573127), National Research Foundation for the Doctoral Program of Higher Education of China (No. 200405333036).

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fusion theory (DSm) [13, 14, 15] which is an extension of the DS theory. Compared with DS, DSm can express intersections with non-exclusive hypothesis. However, works about DSm are in the same frame of discernment. Few literatures introduce the DSm beliefs inference in multi-frames of discernment, Moreover, few scholars propose a model about evidential network using DSmH. In this paper, we proposed an evidential network using DSmH, and the methods about inference and combination. Then the DSmH evidential network is applied to target identification. Experiments confirm that the DSmH evidential network (evidential network using DSmH) is superior to the DS evidential network (evidential network using DS).

## 2 Theory DSm and DSmH

DSm was proposed by Jean Deserts and Florentin Smarandache, compared with DS theory which is based on the power set. DSm is based on the definition of hyper power set ( $D^\Theta$ ) of the frame  $\Theta$ . Hence, DSm can express contradiction using intersection within  $\Theta$ . There are two types of DSm: free DSm model and hybrid DSm model. The free DSm model is an opposite to the Shafer's model  $M^0(\Theta)$ , which requires the exclusivity and exhaustivity of all elements in  $\Theta$ . The DSm hybrid model  $M(\Theta)$  is derived from the free DSm model  $M^f(\Theta)$  by introducing some integrity constraints on some elements  $\theta_i \in D^\Theta$ , when there are some certain facts in accordance with the exact nature of the model related to the considered problem.

### 2.1 Hyper-power set

Let  $\Theta = \{\theta_1, \dots, \theta_n\}$  be the general frame of discernment, which is a finite set of  $n$  exhaustive elements,  $\varphi$  is a vacuous set. The hyper-power set  $D^\Theta$  is defined as the set of all compositions constructed from elements of  $\Theta$  with  $\cup$  and  $\cap$  operators such as:

- $\varphi, \theta_1, \dots, \theta_n \in D^\Theta$
- If  $A, B \in D^\Theta$ , then  $A \cap B \in D^\Theta$  and  $A \cup B \in D^\Theta$
- No other elements belong to  $D^\Theta$ , except those obtained by using rules listed above.

### 2.2 The combination rules of DSmH

Let  $M^h(\Theta)$  be a hybrid DSm model. Given that  $m_1$  to  $m_k$  are  $k$  independent gBBAs over the same frame  $\Theta$ . The DSmH rule of combination is given as follows:

$$m_M(A) \triangleq \varphi(A)[S_1(A) + S_2(A) + S_3(A)], \dots \forall A \in D^\Theta \quad (1)$$

$$\text{where } \varphi(A) = \begin{cases} 1, \dots A \notin \Phi \\ 0, \dots \text{else} \end{cases}$$

$$S_1(A) \triangleq \sum_{\substack{X_1, X_2, \dots, X_k \in D^\Theta \\ X_1 \cap X_2 \cap \dots \cap X_k = A}} \prod_{i=1}^k m_i(X_i) \quad (2)$$

$$S_2(A) \triangleq \sum_{\substack{X_1, X_2, \dots, X_k \in \Phi \\ [U=A] \vee [(U \in \Phi) \wedge (A=I_t)]}} \prod_{i=1}^k m_i(X_i) \quad (3)$$

$$S_3(A) \triangleq \sum_{\substack{X_1, X_2, \dots, X_k \in D^\Theta \\ X_1 \cup X_2 \cup \dots \cup X_k = A \\ X_1 \cap X_2 \cap \dots \cap X_k \in \Phi}} \prod_{i=1}^k m_i(X_i) \quad (4)$$

with  $U \triangleq u(X_1) \cup u(X_2) \cup \dots \cup u(X_k)$ ,  $u(X) = \bigcup_{\theta_i \in X} \theta_i$ ,  $I_t \triangleq \theta_1 \cup \dots \cup \theta_n$

$S_1(A)$  corresponds to the classic DSm combination rule for  $k$  independent sources based on the free DSm model  $M^f(\Theta)$ ;  $S_2(A)$  represents the mass of all relatively and absolutely empty sets which are transferred to the total or relative ignorance associated with non-existential constraints (if any, like in some dynamic problems);  $S_3(A)$  transfers the sum of empty sets directly into the (canonical) disjunctive form of non-empty sets.

### 3 Conditional Belief Reasoning in DSmH Evidential Network

Conditional Belief Reasoning is derived from Transferable Belief Model (TBM), which is used for representing quantified beliefs based on belief functions [16]. The disjunctive rule of the combination (DRC) and the generalized Bayesian theorem (GBT) are put forward within TBM [17]. The DRC rule allows the belief to be computed over  $X$  from the beliefs induced by two distinct pieces of evidence when one of the pieces of evidence is held, and it can be used for forward propagating in belief network. The GBT allows the computation of the belief over a frame of discernment  $\Theta$  given that  $x \subseteq X$  when the beliefs over  $X$  for every  $\theta_i \subseteq \Theta$  are known, and it can be used for backward propagating in belief network. Whatever DRC or GBT is, the basic belief assignments are over power set of  $\Theta$  with the constraints of the exclusivity and exhaustivity. For conditional reasoning in DSmH evidential network, it is required to generalise DRC and GBT over hyper power set. Therefore, we present some definitions and theorems to DDRC (DSm based DRC) and DGBT (DSm based GBT), which are the theoretical basements of reasoning in DSmH Evidential Network.

**Definition 1**  $\Theta$  is defined as a non-vacuous frame of discernment,  $m : D^\Theta \rightarrow [0, 1]$  is a general basic belief assignment on  $\Theta$ ,  $gBel : D^\Theta \rightarrow [0, 1]$  is a general belief function on  $\Theta$ ,  $gPl : D^\Theta \rightarrow [0, 1]$  is a general plausibility function on  $\Theta$ , then:

$$m(X) = \sum_{Y \subseteq X} (-1)^{|X-Y|} gBel(Y) \forall X, Y \subseteq D^\Theta \quad (5)$$

$$gPl(X) = 1 - gBel(\bar{X}) = \sum_{Y \cap X} m(Y), \forall X, Y \subseteq D^\Theta \quad (6)$$

Where  $|X - Y|$  is the cardinal of set  $X$  minus set  $Y$ .

#### REQUIREMENT R

Two frames of discernment  $X$  and  $\Theta$ . Our knowledge on  $X$  given  $\theta_i$  is represented by  $gBel_X(\cdot|\theta_i)$ ,  $gPl_X(\cdot|\theta_i)$  and  $m_X(\cdot|\theta_i)$ ,  $\forall \theta_i \in D^\Theta$ .  $X$  are conditionally independent given  $\theta_i$ .

**Theorem 1** *The DDRC is in normalized beliefs. Given the Requirement R and  $gBel_X(X|\theta_i) = 1$ ,  $\forall x \subseteq X$ ,  $\forall \theta \subseteq D^\Theta$ . Then:*

$$gPl_X(x|\theta) = 1 - \prod_{\theta_i \in \theta} (1 - gPl_X(x|\theta_i)) \quad (7)$$

$$m_X(x|\theta) = \sum_{\cup_{i:\theta_i \in \theta} x_i = x} \prod_{i:\theta_i \in \theta} m_X(x_i|\theta_i) \quad (8)$$

The relation (9) shows the nature of the disjunctive of combination. Let us suppose two general belief functions with their general basic belief assignments  $m_1$  and  $m_2$  on  $D^\Theta$ . When combined, the product  $m_1(A)m_2(B)$ ,  $A \subseteq D^\Theta$ ,  $B \subseteq D^\Theta$ , is allocated to  $A \cup B$  in the disjunctive rule of combination.

**Theorem 2** *The DGBT is in normalized beliefs. Given the Requirement R and  $gBel_X(X|\theta_i) = 1$ ,  $\forall x \subseteq X$ ,  $\forall \theta \subseteq D^\Theta$ . Then:*

$$gPl_\Theta(\theta|x) = 1 - \prod_{\theta_i \in \theta} (1 - gPl_X(x|\theta_i)) \quad (9)$$

The DGBT permits to build  $gPl_\Theta(\cdot|x)$ , for any  $x \subseteq X$  from the conditional belief functions  $gPl_X(\cdot|\theta_i)$ .

## 4 Improvement by Using DS<sub>m</sub>H in Target Identification Evidential Network

### 4.1 Targets identification evidence network

In experiments, some sensors are used to identify the aerial targets. They can be classified into two groups: one is in charge of classifying foe and ally, another is used to identify the type of aircraft. The specific sensors in group one and the truths for classification are listed in Table 1. The sensors in group two and what the contents to be identified are listed in Table 2. According to the two tables, two core nodes are presented, Target Types (TT) node and Foe-Ally (FA) node. The two discernment frames of nodes are shown as below:

$$\Gamma_{\text{TargetType}} = \{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6, \tau_7, \tau_8\}$$

Table 1: The specific sensors in group one and what they can identify

Sensors	Identification
IFF	Ally, Unknown
Deceive Enquire (DE)	Foe, Unknown

Table 2: The specific sensors in group two and what they can identify

Sensors	Identification
HRRP	Long $\in$ [30m,+ $\infty$ ]; Medium (20m,30m); Short $\in$ [10m,20m]
JEM	Two Propellers at Fixed Wind(TPFW), One Top Propeller(OTP), Two Top Propellers(TTP), Jet-propelled with One Aero-engine at Tail(JPOAT), Jet-propelled with Two Aero-engines at Tail(JPTAT)
Horizontal Maneuver (HM)	High $\in$ [1200km/h,+ $\infty$ ]; Medium $\in$ (300km/h,1200km/h); Low $\in$ [100km/h,300km/h]
Vertical Maneuver (VM)	Can Vertical Takeoff and Landing(VTL); Cant Vertical Takeoff and Landing(NVTL)
ESM	$e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8$

$$F_{Foe-Allly} = \{Foe, Ally, Neutral, Unknown\}$$

Since horizontal maneuver and vertical maneuver cannot identify the target types directly, we take an Aircraft Type node between target types and maneuver sensors into account. The discernment frame of Aircraft Type (AT) is presented as below. For convenience, *H* stand for Helicopter, *JA* stand for Jet Airliner, *JF* stand for Jet Fighter, *PA* stand for Propeller Airliner.

$$A_{AircraftType} = \{H,JA,JF,PA\}$$

In the target identification evidential network, the two core nodes and the Aircraft Type node are centre nodes, the sensors listed in Table 1 and Table 2 are terminal nodes. Arcs are placed between the three centre nodes according to their relationship. Moreover, they are also placed between centre nodes and terminal nodes, since the identifications are relevant with the centre nodes. The target identification evidential network is shown in Fig. 1, and the classifications, known as the origins, the targets belonging to are shown in Table 3.

## 4.2 Comparison and analysing

General BBAs are different between DS and DSmH, we will evaluate the gBBAs at nodes Target Type and Foe-Ally for targets  $\tau_3$  and  $\tau_7$ .

When the 4 sensors are fused together using evidential network based on DSmH for target  $\tau_3$ , the results are shown in Fig. 2, Fig. 3, where the fusion results of evidential network based DS are

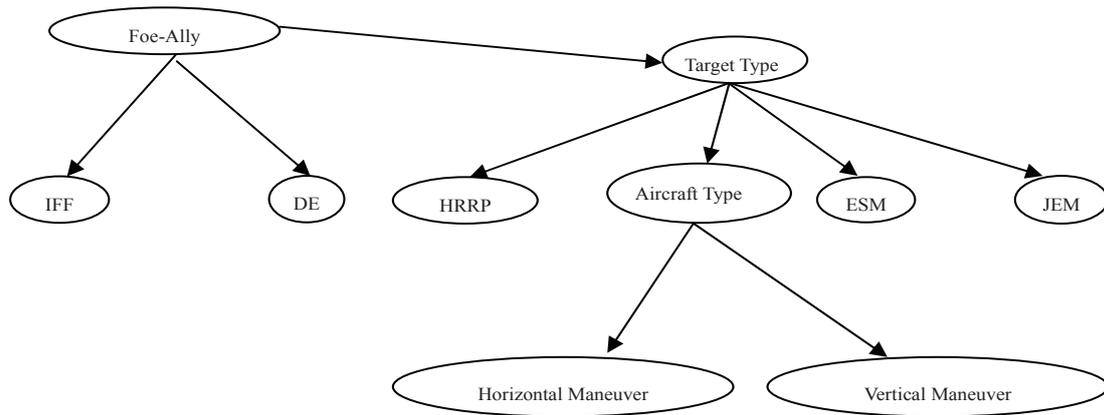


Fig. 1: The target identification evidential network

Table 3: The classification targets belong to

TT	FA	AT	HRRP	JEM	HM	VM	ESM	IIF	DE
$\tau_1$	Ally	JF	Short	JPOAT	Hight	NVTL	$e_1$	Ally	Unknown
$\tau_2$	Ally	JF	Medium	JPTAT	Hight	NVTL	$e_2$	Ally	Unknown
$\tau_3$	Foe	JF&H	Short	JPOAT	Hight	VTL	$e_3$	Unknown	Foe
$\tau_4$	Foe	H	Short	OTP	Low	VTL	$e_4$	Unknown	Foe
$\tau_5$	Neutral	JA	Long	JPTAT	Mdium	NVTL	$e_5$	Unknown	Unknown
$\tau_6$	Neutral	JA	Medium	TPFW	Mdium	NVTL	$e_6$	Unknown	Unknown
$\tau_7$	Foe	PA&H	Short	TPFW	Mdium	VTL	$e_7$	Unknown	Foe
$\tau_8$	Unknown	H	Short	TPFW TTP	Low	VTL	$e_8$	Unknown	Foe

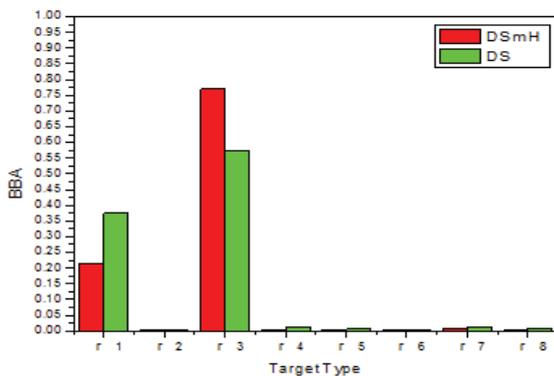


Fig. 2: Result of fusing four sensors at target type node

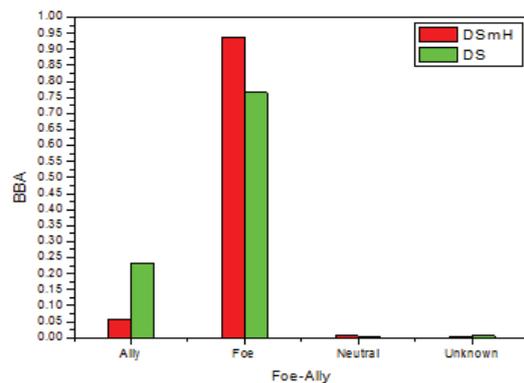


Fig. 3: Result of fusing four sensors at Foe-Ally node

presented for comparison. The indications of the accuracy of the identification at nodes Target Type and Foe-Ally are  $AoI_{TargetType}$  and  $AoI_{Foe-Ally}$ , and the indications of the confusion of the identification at nodes Target Type and Foe-Ally are  $CoI_{TargetType}$  and  $CoI_{Foe-Ally}$ . They can be

Table 4: AoIs and CoIs of  $\tau_3$  of DSmH and DS

	DSmH	DS
$AoI_{TargetType}$	0.7694	0.5731
$AoI_{Foe}$	0.9261	0.6401
$CoI_{TargetType}$	0.2598	0.4721
$CoI_{Foe}$	0.0827	0.3065

Table 5: AoIs and CoIs of  $\tau_7$  of DSmH and DS

	DSmH	DS
$AoI_{TargetType}$	0.8423	0.6731
$AoI_{Foe}$	0.8906	0.5431
$CoI_{TargetType}$	0.1763	0.4725
$CoI_{Foe}$	0.1227	0.2836

formulized in (11) to (15).

$$AoI_{TargetType} = m_{TargetType}(\tau_3) \tag{10}$$

$$AoI_{Foe-Allly} = m_{Foe-Allly}(Foe) \tag{11}$$

$$CoI_{TargetType} = 1 - \sqrt{\frac{\sum_{\tau_i \in \Gamma_{TargetType}} [m_{TargetType}(\tau_3) - m_{TargetType}(\tau_i)]^2}{|\Gamma_{TargetType}| - 1}} \tag{12}$$

$$CoI_{Foe-Allly} = 1 - \sqrt{\frac{\sum_{\tau_i \in \Gamma_{Foe-Allly}} [m_{Foe-Allly}(\tau_3) - m_{Foe-Allly}(\tau_i)]^2}{|\Gamma_{Foe-Allly}| - 1}} \tag{13}$$

After the 4 sensors were fused for  $\tau_3$ , the obtained AoIs and CoIs are listed in Table 4. For target  $\tau_7$ , the AoIs and CoIs are also listed in Table 5. In the identification, a high AoI and a low CoI will lead to a better identification performance. Therefore, from Table 4 and Table 5 one can conclude that Evidential Network based on DSmH are advantageous over the purely base on DS in fusing conjunctive sets.

## 5 Conclusions

We presented an evidential network based on DSmH by using DDRC and DGBT for the knowledge representation and reasoning. Compared to the evidential network based on DS, some conjunctive sets that did not cause an actual conflict can be propagated by using DSmH. In this paper, the proposed model is applied to improve targets identification of multi-sensors. By comparing the results obtained from DSmH and DS, it is shown that more gBBAs have been assigned into the right focal elements when the DSmH model is taken.

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